Cautionary remarks on the correlation analysis of non-Gaussian self-similar time series

(particularly, near-inertial amplitudes)

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Questions

- When we describe the variability of the spatial-temporal data (fields or system) or characterize them, we may use decorrelation scales.
- Could we apply the correlation analysis to self-similar time/spatial series (or non-Gaussian variables)?
- Are there any other reasonable approaches to quantify the decorrelation scales?

Outline

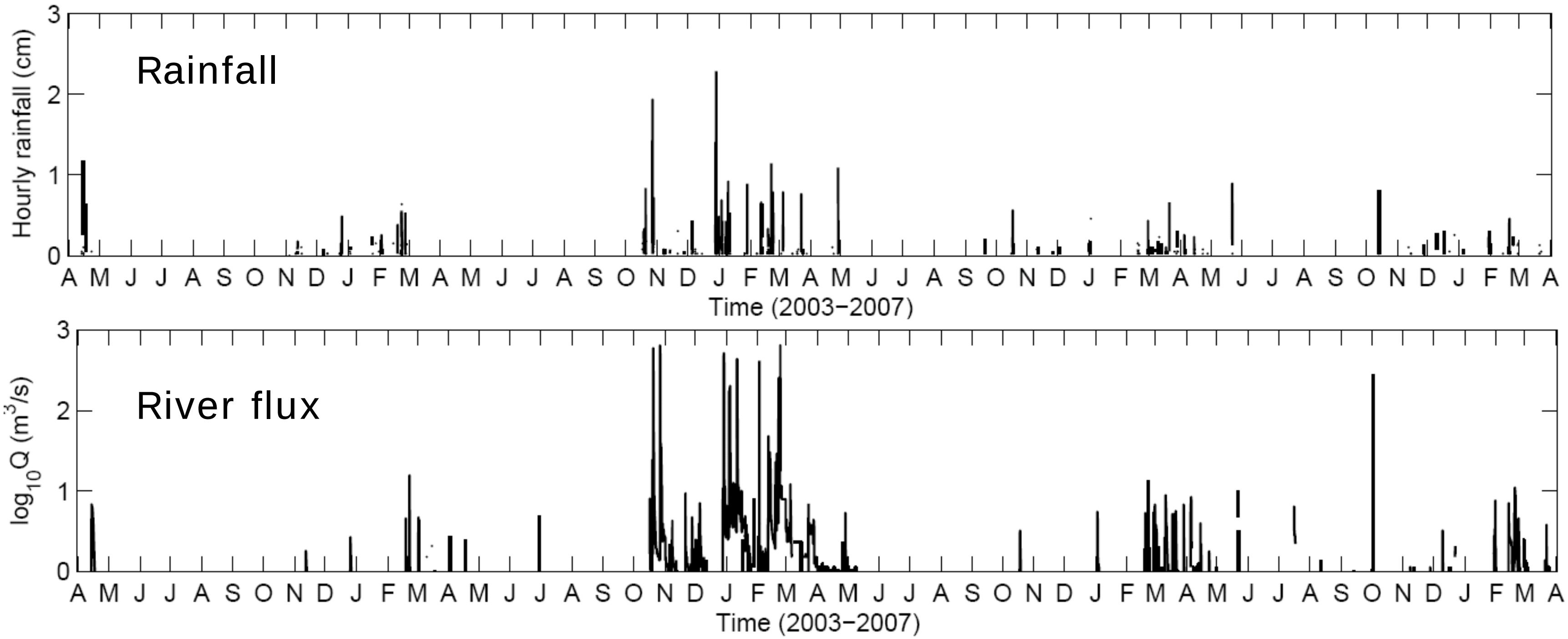
- Self-similar time series and correlation analysis
  - Examples of self-similar time series
  - Definition of correlation analysis
  - Near-inertial motions and their amplitude time series

- A simple simulation with synthetic datasets
  - Generation of self-similar time series with exponential, Gaussian, and linearly decay patterns (single-sided and double-sided pulses)
  - Cross-correlation analysis to quantify the decay scales
  - Another approaches?

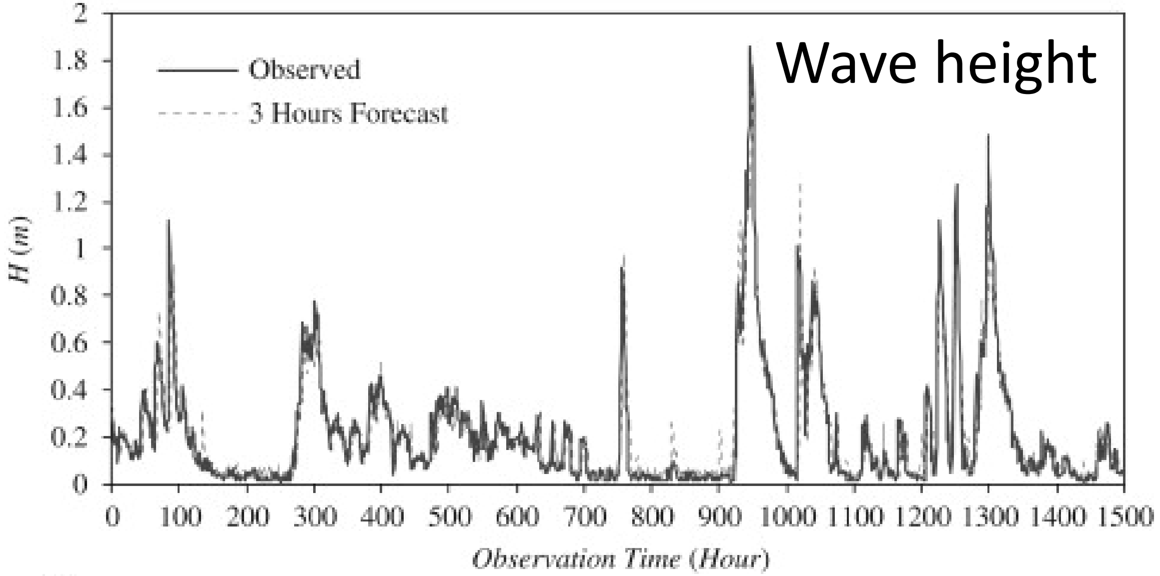
- Summary

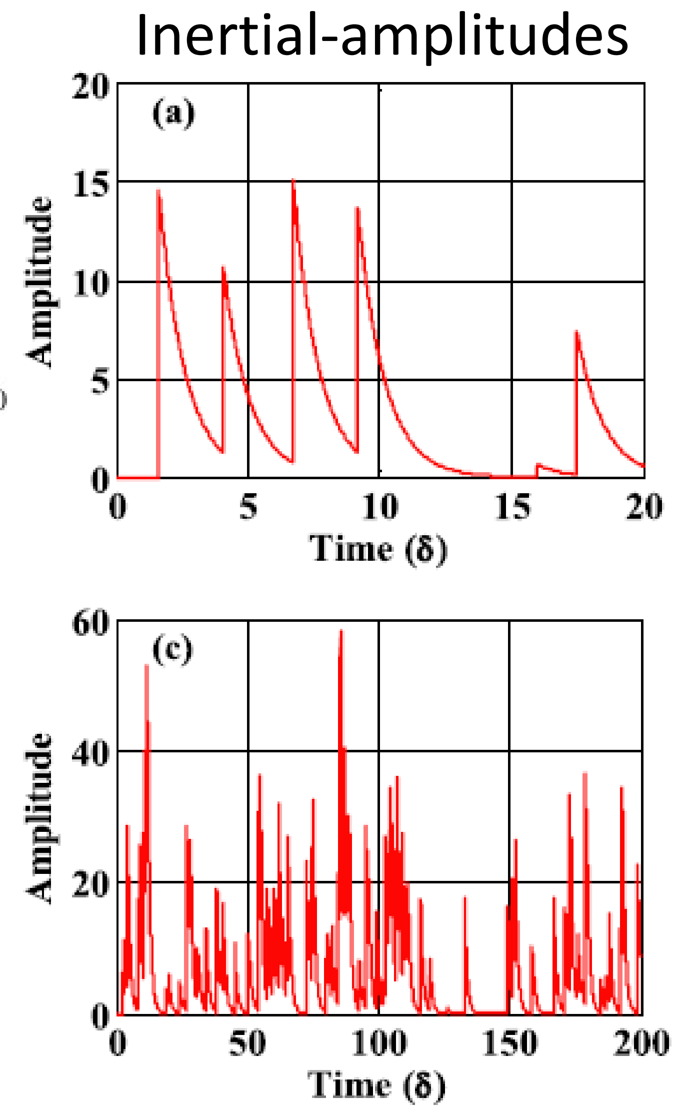
Self-similar time series?

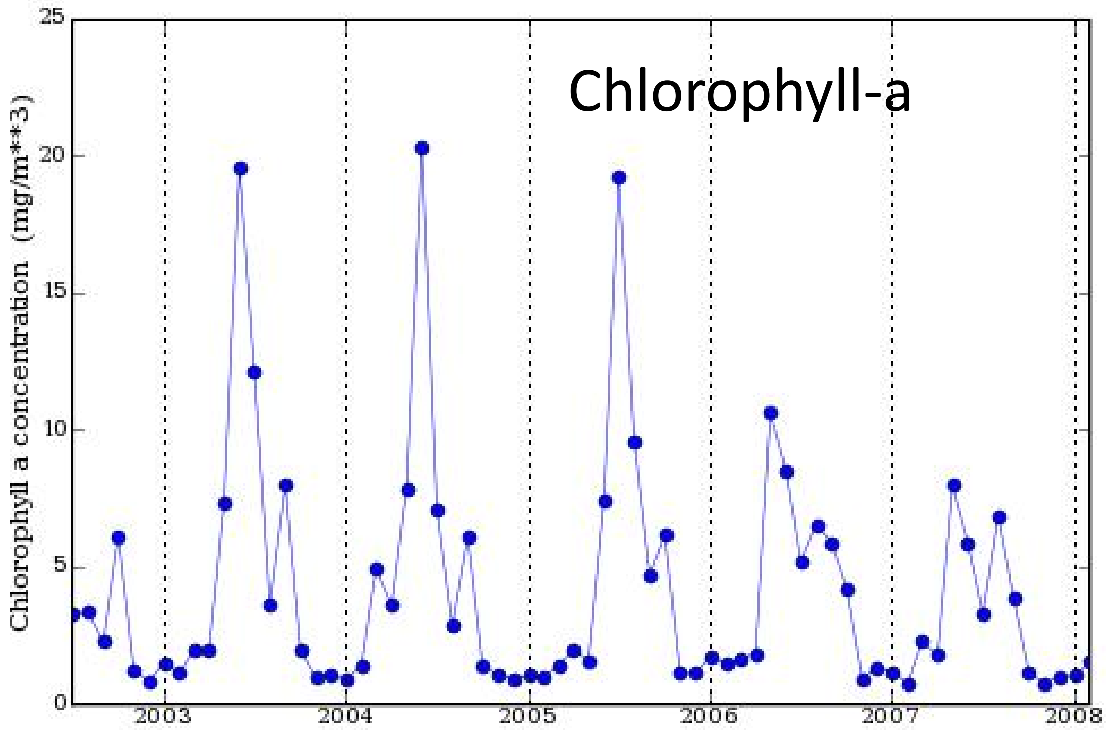
- Continuous time series with similar shapes of disturbances, pluses, or amplitudes, which can be governed by non-Gaussian statistics
  - e.g., River flows, rain fall, wind speed, wave height, concentration of Chlorophyll, and (near-) inertial amplitudes



Examples of self-similar time series





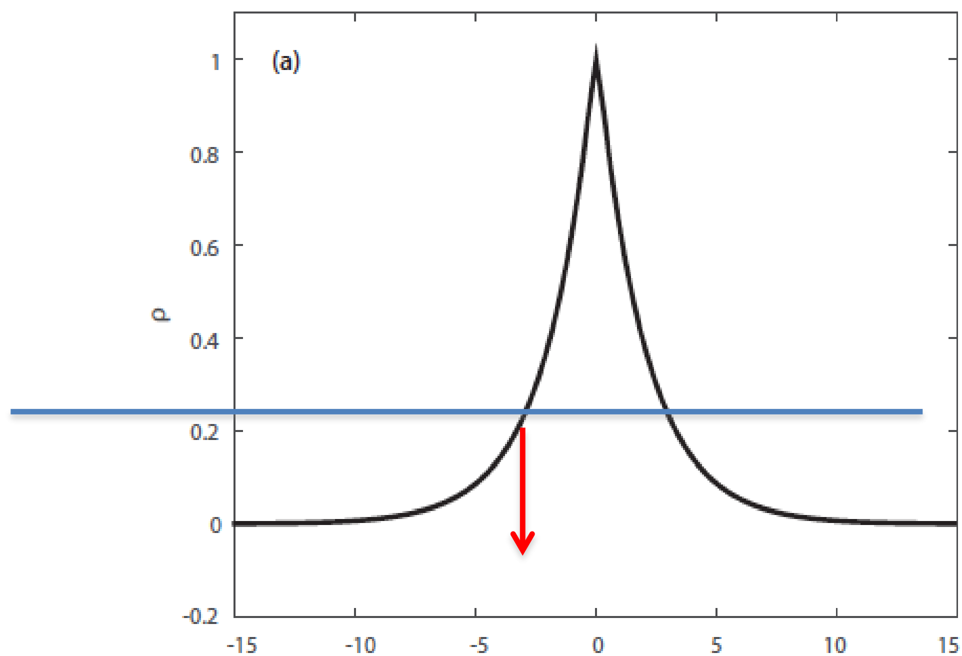


(Park et al 2009, JGR)

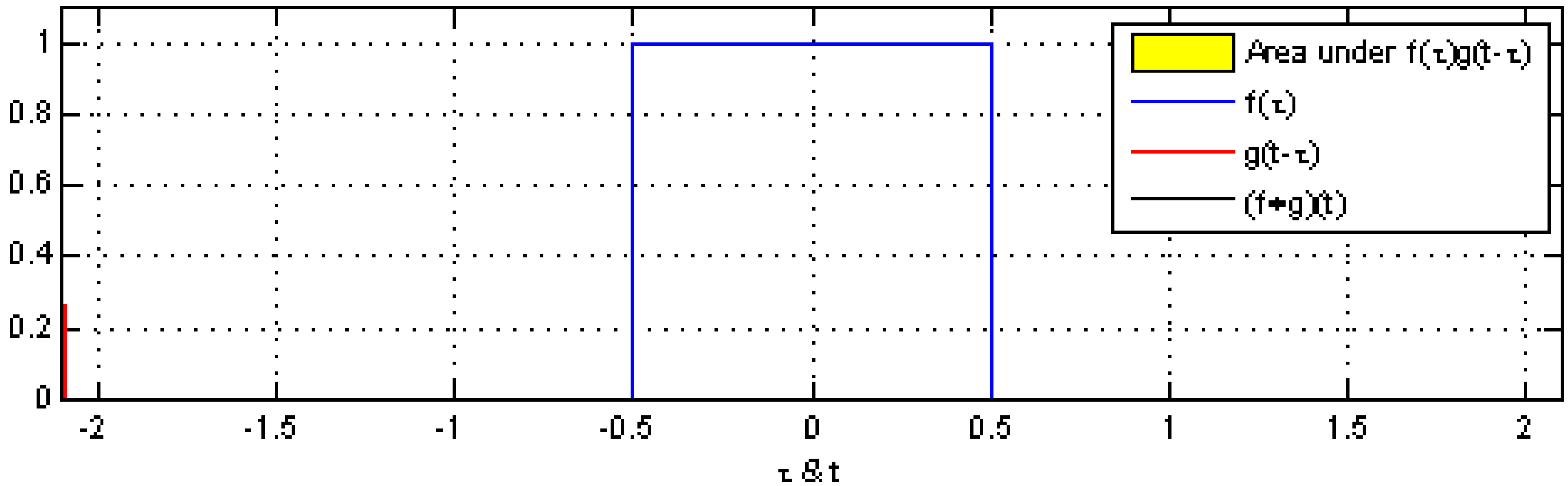
Decorrelation scales (or e-folding scales)

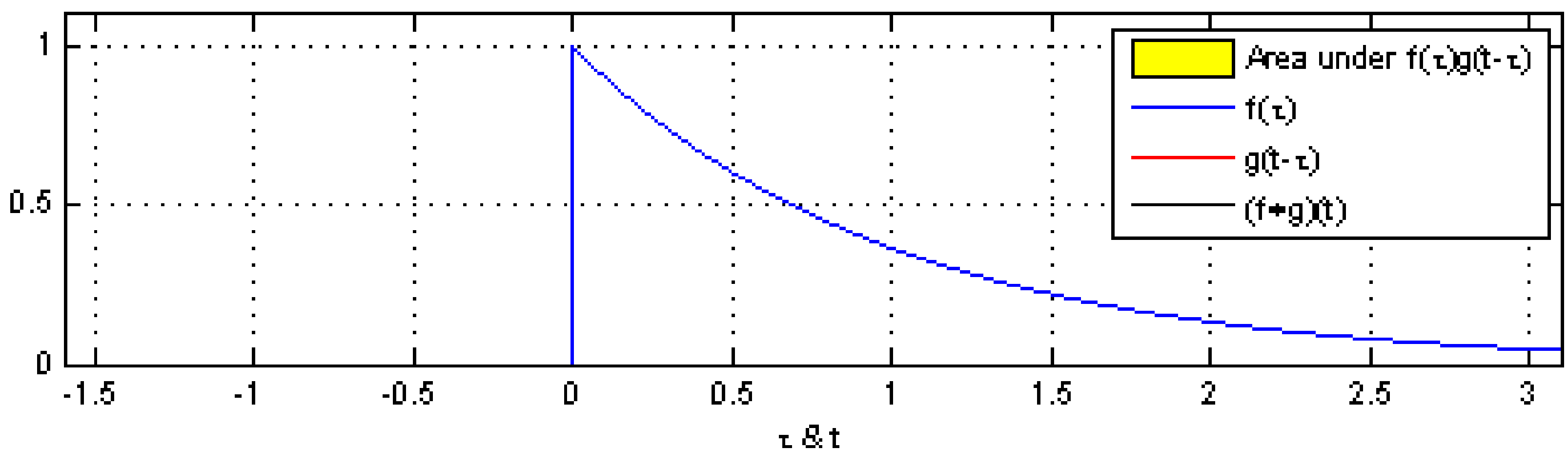
- Provide a convenient description on the structure of variables and a unique value for the system identification.
  - e.g.) decorrelation time scale is 3 days….
    E-folding length scales are 4 km and 10 km in the x- and y-directions.
- System design and analysis, sampling techniques and optimization
- How can we quantify these scales?

1/exp(1) = 0.3679



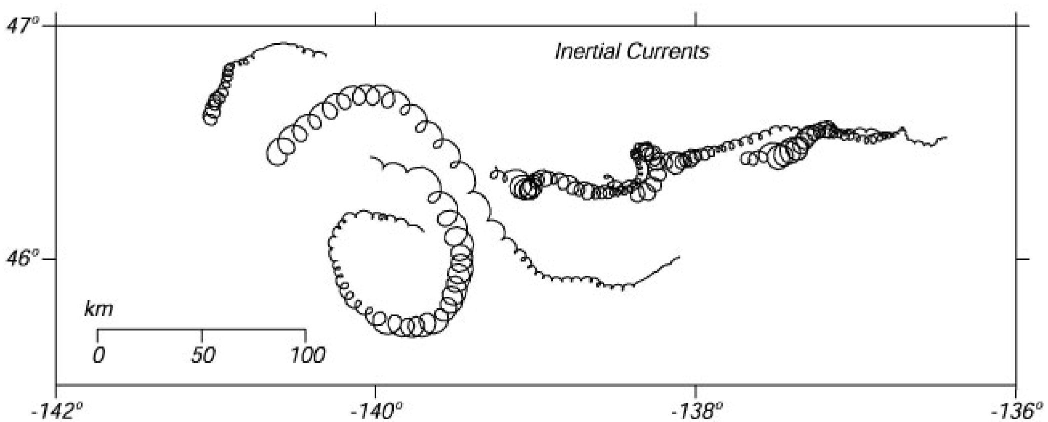
Cross-correlation (vs. convolution)





(Near-) Inertial motions and decay scales

- As ubiquitous motions in the upper ocean, flows under a rotating framework, near-inertial currents/motions appear rotational motions as a function of Coriolis freq. (=2 sin latitude [cycles per day]), which are typically driven by the broad wind stress.
- Can be used to examine the pathway of wind energy (at global scale) and kinetic energy flux in the mixed layer.



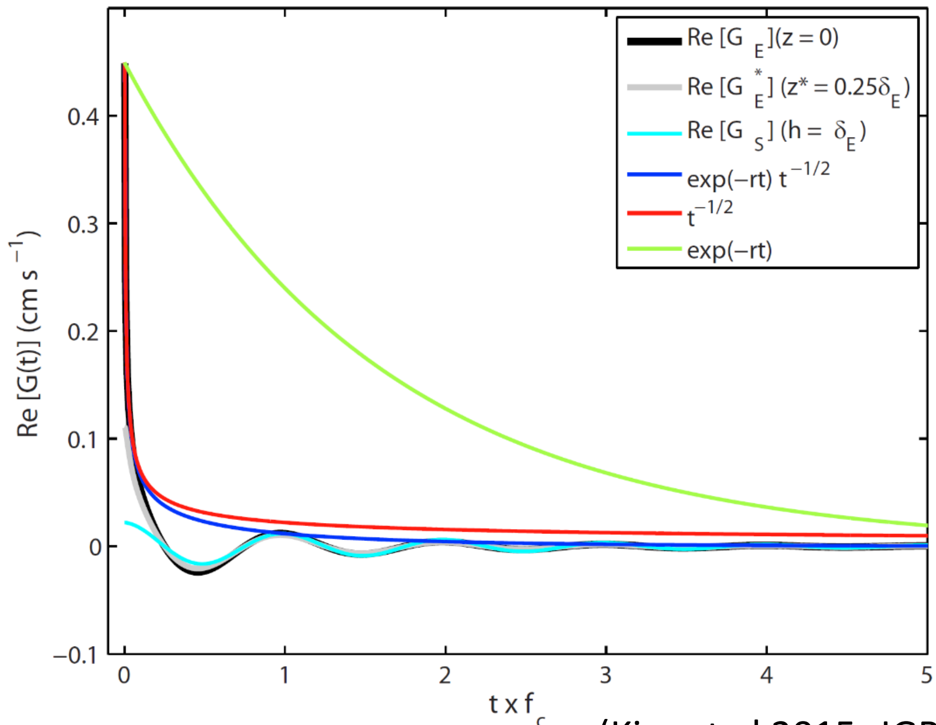
$$\frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r\mathbf{u} = \frac{1}{\rho}\frac{\partial \tau}{\partial z}$$

$$\mathbf{H_S}(\sigma) = \frac{\hat{\mathbf{u}}(\sigma)}{\hat{\tau}(\sigma)} = \frac{1}{\rho h[i(\sigma + f_c) + r]}$$

$$\mathbf{G_S}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{H_S}(\sigma) e^{i\sigma t} d\sigma,$$

$$= \frac{1}{2\rho h} e^{-i f_c t} e^{-rt}.$$

(Near-) Inertial motions and decay scales



(Kim et al 2015, JGR)

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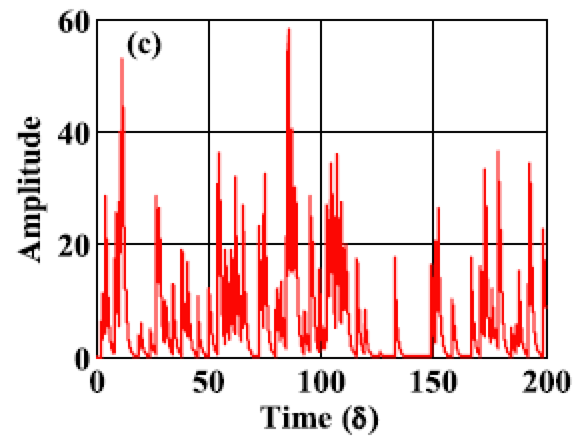


Examples of decay time scales

(Park et al 2009, JGR)

	Location	Observation Period	Decay Timescale
Pollard and Millard [1970]	(39.1N, 70.2W, 39.1N, 69.6W) North Atlantic	Oct ∼ Dec 1968 and Oct ∼ Nov 1965	4 days
D'Asaro [1985]	(51N, 136W) North Pacific	Dec ∼ Feb 1979	4 days
D'Asaro [1995a, 1995b] and *D'Asaro et al.* [1995]	(47.5N, 140W) North Pacific	Oct 1987	20 days
Shay and Elsberry [1987]	(24N, 94W) North Atlantic	Sep 1988	5 days
Hisaki and Naruke [2003]	(26N, 128E) North Pacific	Aug ∼ Sep 1995	4 days
Plueddemann and Farrar [2006]	(25.5N, 29.0W) North Atlantic	Feb ∼ Oct 1992	3.7 days
	(27.1N, 69.8W) North Atlantic	Feb – May 1986	3.5 days
	(34.0N, 70.0W) North Atlantic	May – Oct 1982	4.1 days
	(59.5N, 20.8W) North Atlantic	Jun – Sep 1991	1.5 days

Issues of correlation analysis of self-similar time series

- However, the auto-correlation of self-similar times series produces spurious structures which are not relevant the raw time series.

- Let’s take a look at what we examined with synthetic data sets:

Time series with multiple pulses

$$d^e(t) = \sum_{n=1}^{N} a_n^e b_n^e(t) = \sum_{n=1}^{N} a_n^e \exp\left[-\frac{|t-t_n|}{\lambda_n}\right],$$

$$d^g(t) = \sum_{n=1}^{N} a_n^g b_n^g(t) = \sum_{n=1}^{N} a_n^g \exp\left[-\frac{(t-t_n)^2}{\lambda_n^2}\right],$$

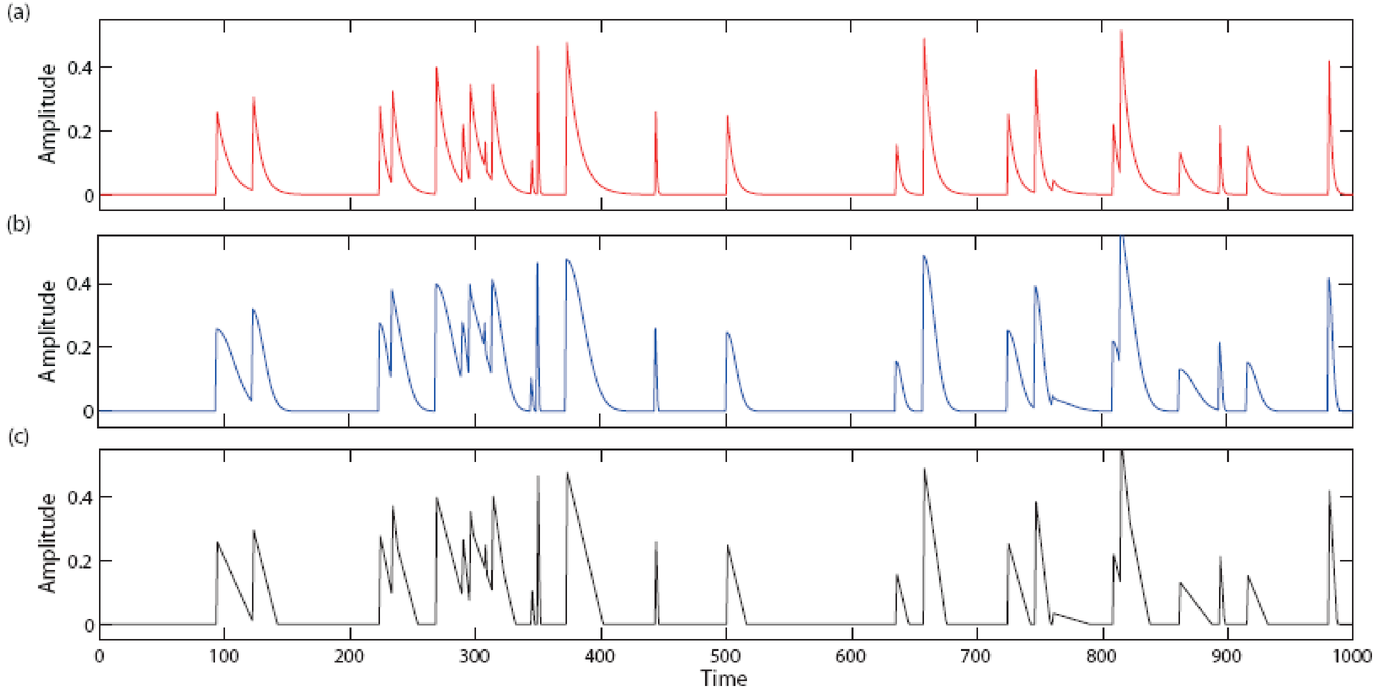
$$d^l(t) = \sum_{n=1}^{N} a_n^l b_n^l(t) = \sum_{n=1}^{N} a_n^l \left[\frac{t-t_n}{t_n-\beta}+1\right],$$

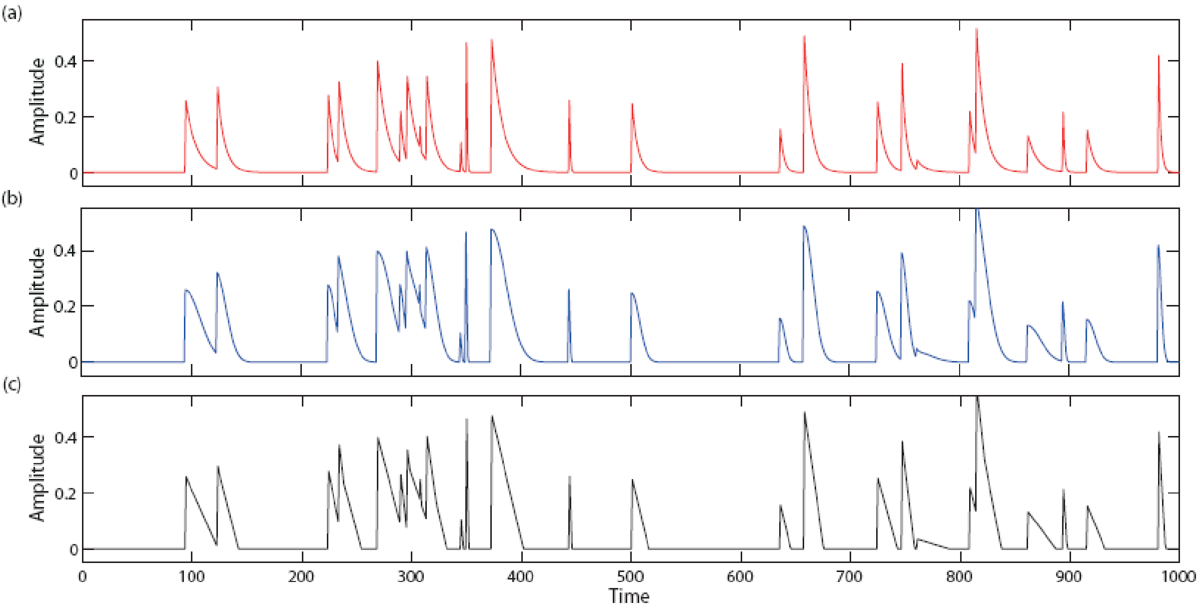
- an: Amplitude of each pulse
- bn: Decay shape of each pulse.

Exponential

Gaussian

Linear



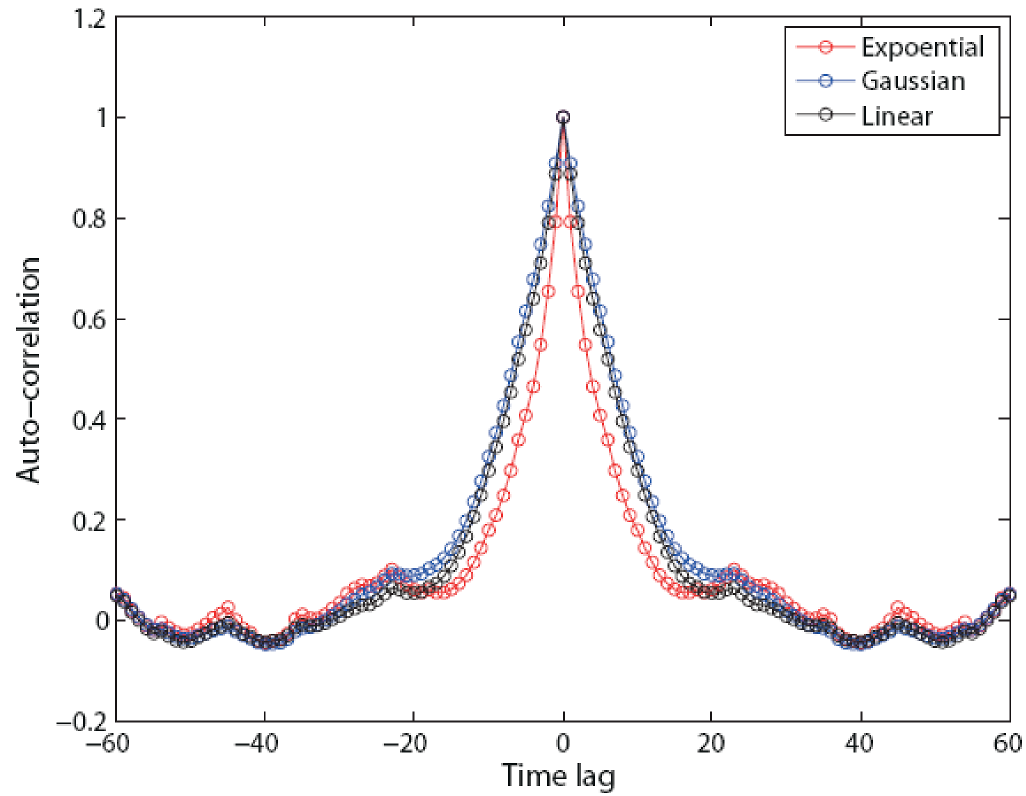


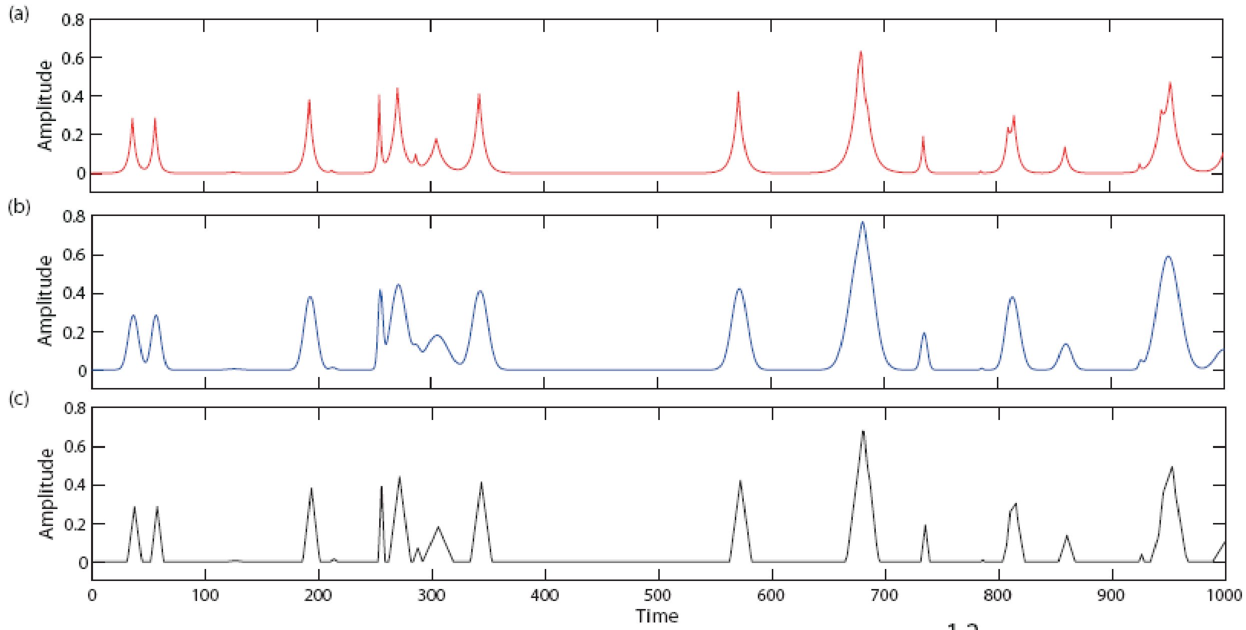
Exponential

Gaussian

Linear

Single-side pulses



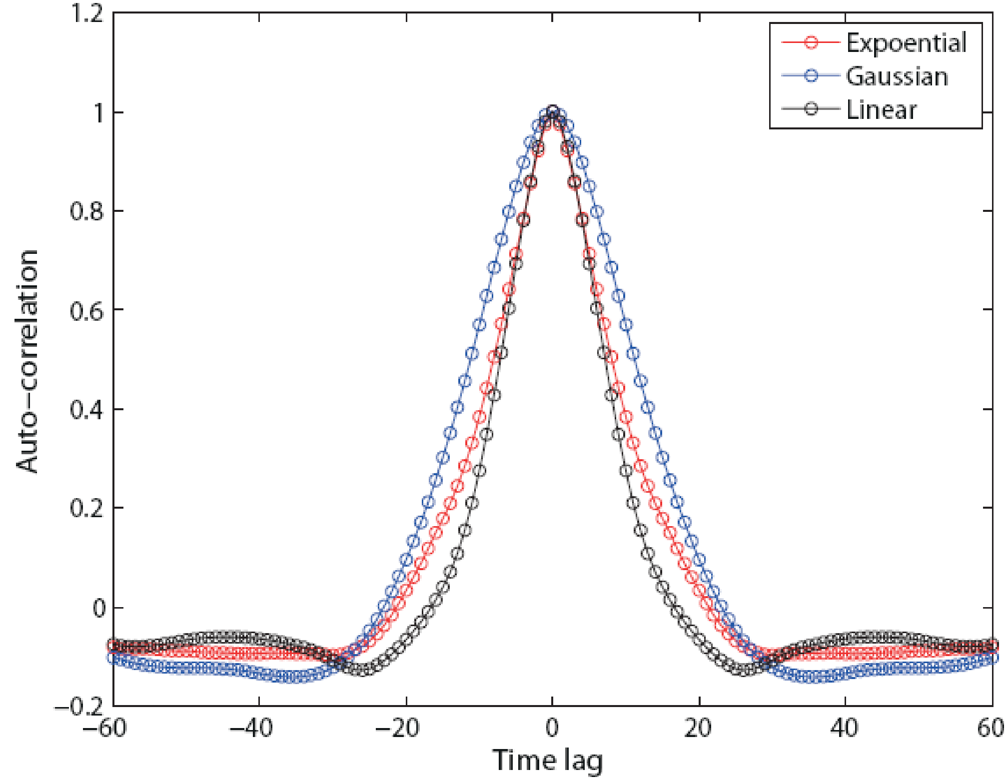


Exponential

Gaussian

Linear

Two-side pulses



Another approach to estimate decorrelation scales?

- Direct auto-correlation analysis of self-similar time series can mislead the estimate of decorrelaiton scales.

- Decorrelation scales of Individual pulses can be averaged or their statistics can identify the system.

- Dynamical constraints can be used.
  - e.g., near-inertial amplitudes can be addressed with the response function estimated from observations of wind stress and currents instead of conducting the correlation analysis on the (self-similar) time series of amplitudes of near-inertial currents.

Summary

- Cross-correlation analysis of self-similar time series may generate the spurious results in the estimate of decorrelation scales because convolving two time series does not guarantee their shape based on the original datasets.

- Using cross-correlation analysis the time lag having a maximum correlation can be found.

- Dynamical data analysis or composite mean of individual pulses in the self-similar time series can be used to identify the system or quantify the decorrelation scales.

Cross-correlations

- For the finite and evenly spaced data [d(t)]:

$$\rho(n\Delta t) = \frac{c(n\Delta t)}{\sqrt{\langle d^2(t)\rangle}\sqrt{\langle d^2(t+n\Delta t)\rangle}},$$

$$c(n\Delta t) = \langle d(t)d(t+n\Delta t)^\dagger\rangle = \frac{d(t)d(t+n\Delta t)^\dagger}{N}$$