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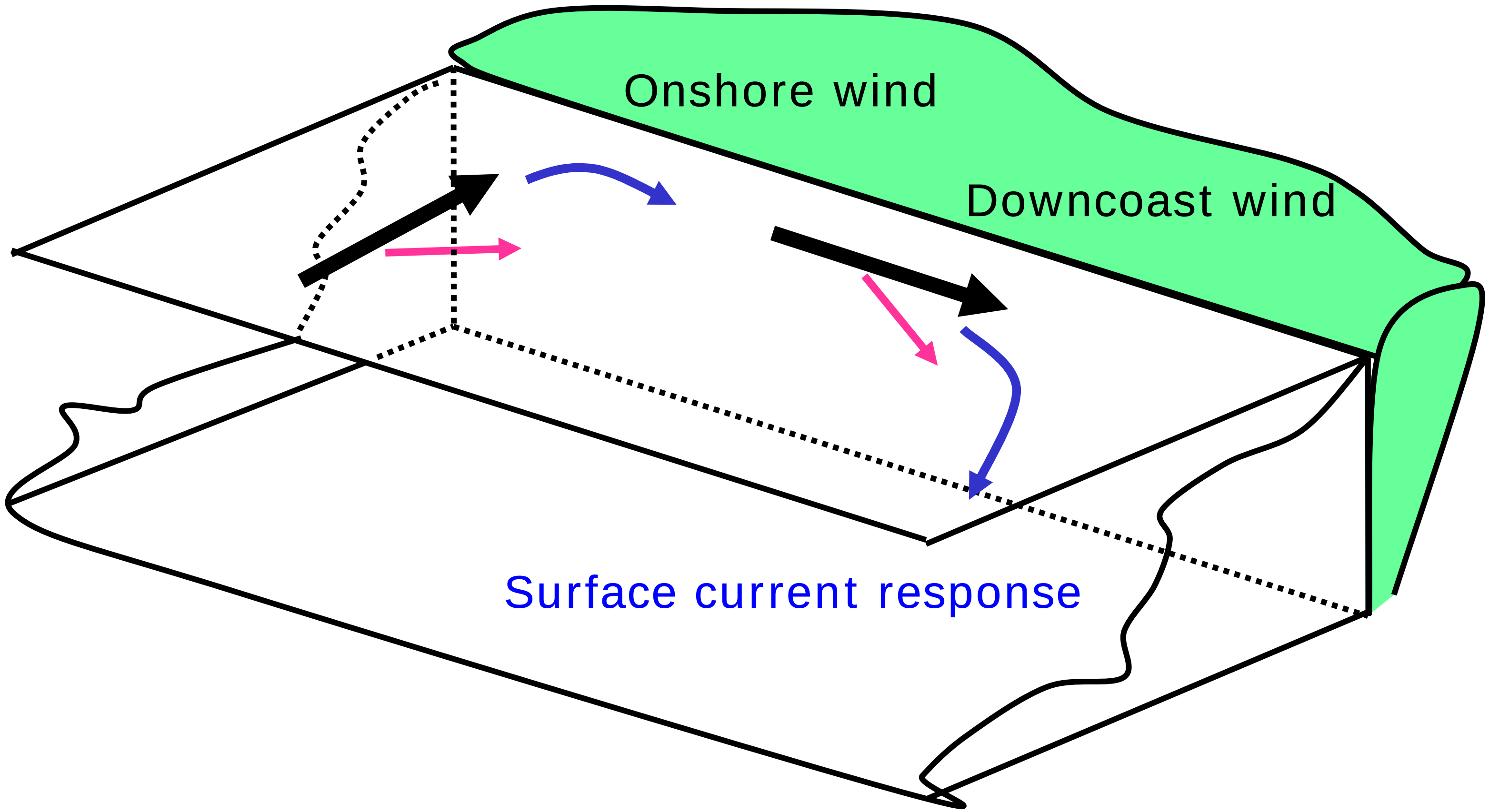
Anisotropic surface current response to the wind in a coastal region

Sung Yong Kim, Bruce Cornuelle, and Eric Terrill
Scripps Institution of Oceanography

SOUTHERN CALIFORNIA COASTAL OCEAN OBSERVING SYSTEM

Anisotropic surface current response



- Anisotropic response means the current response depends on the wind direction.
- Surface current response to the wind in a coastal region is different from the isotropic response in the open ocean.

Outline

- Ekman solutions (iso- and aniso-tropic cases)
- Statistical impulse response function estimate
  - Regression using observation data (surface current and shore station wind)
- Response to steady wind
  - Wind-driven current estimate (time integration).
  - Linear/Nonlinear response function

Isotropic and anisotropic views

$$\frac{\partial u}{\partial t} - f_c v = \frac{1}{\rho}\frac{\partial \tau_x}{\partial z},$$
$$\frac{\partial v}{\partial t} + f_c u = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z},$$

$$\mathbf{u} = u + iv \quad \boldsymbol{\tau} = \tau_x + i\tau_y$$

$$\lambda^2 \hat{\mathbf{u}}(z,\omega) = \frac{\partial^2 \hat{\mathbf{u}}(z,\omega)}{\partial z^2},$$

where $\lambda = \sqrt{i(\omega + f_c)/\nu},$

$\nu =$ Depth independent eddy viscosity

With BCs (finite or infinite depth)

$$\mathbf{H}(z,\omega) = \frac{\hat{\mathbf{u}}(z,\omega)}{\hat{\boldsymbol{\tau}}(\omega)} = \frac{e^{-\lambda z}}{\lambda\rho\nu},$$

where **H = wind impulse response function (WIRF)**

$$\frac{\partial u}{\partial t} - f_c v + A_x = \frac{1}{\rho}\frac{\partial \tau_x}{\partial z},$$
$$\frac{\partial v}{\partial t} + f_c u + A_y = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z},$$

where

$$A_x = a_{xx} * u + a_{xy} * v,$$
$$A_y = a_{yx} * u + a_{yy} * v,$$

*: time domain convolution

Fourth order PDE is solved with BCs
using Ferrai-Cardan method
for the quartic characteristic equation.

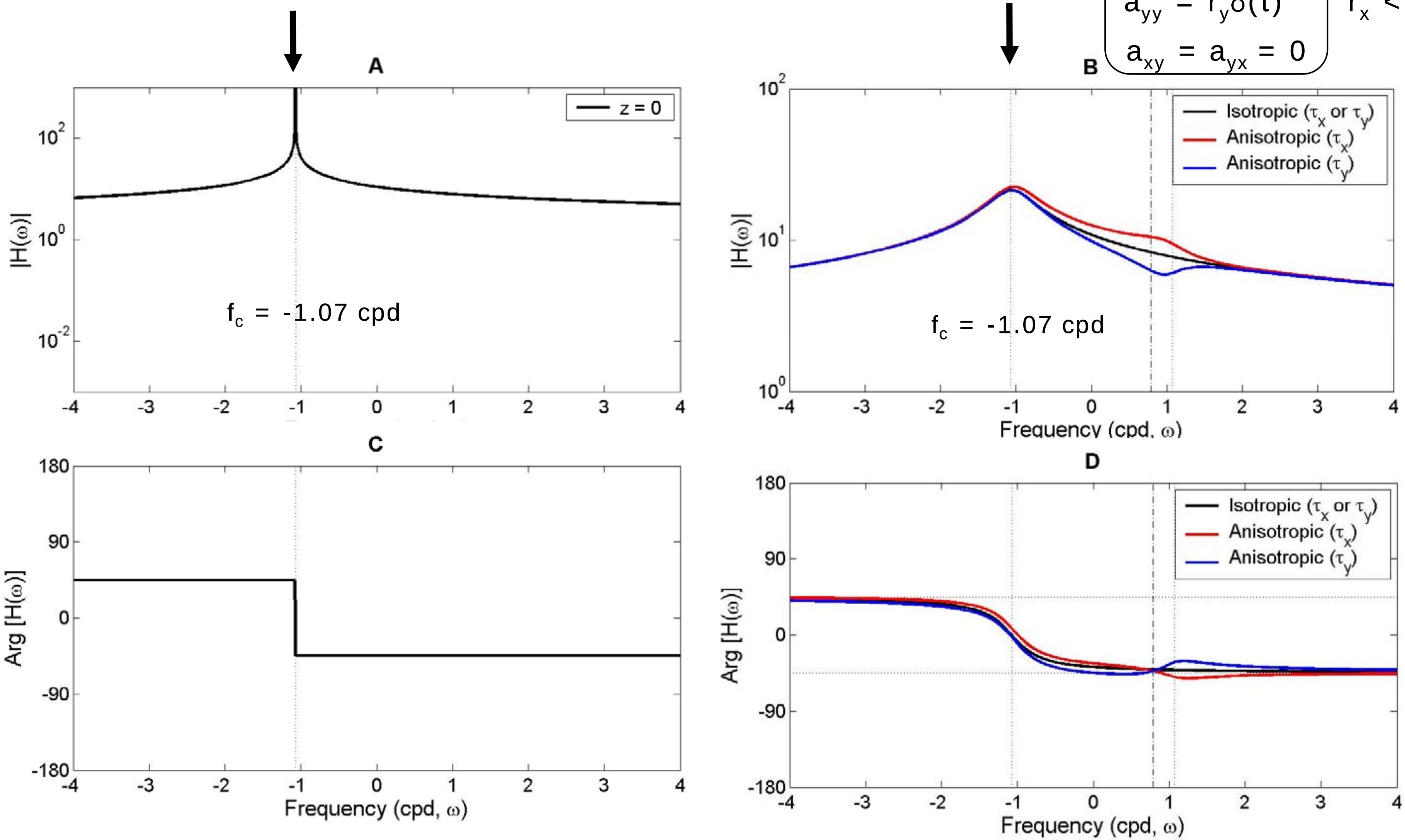
Freq. Iso/Anisotropic WIRF (Ekman @ surface)

$$a_{xx} = r_x \delta(t)$$

$$a_{yy} = r_y \delta(t)$$

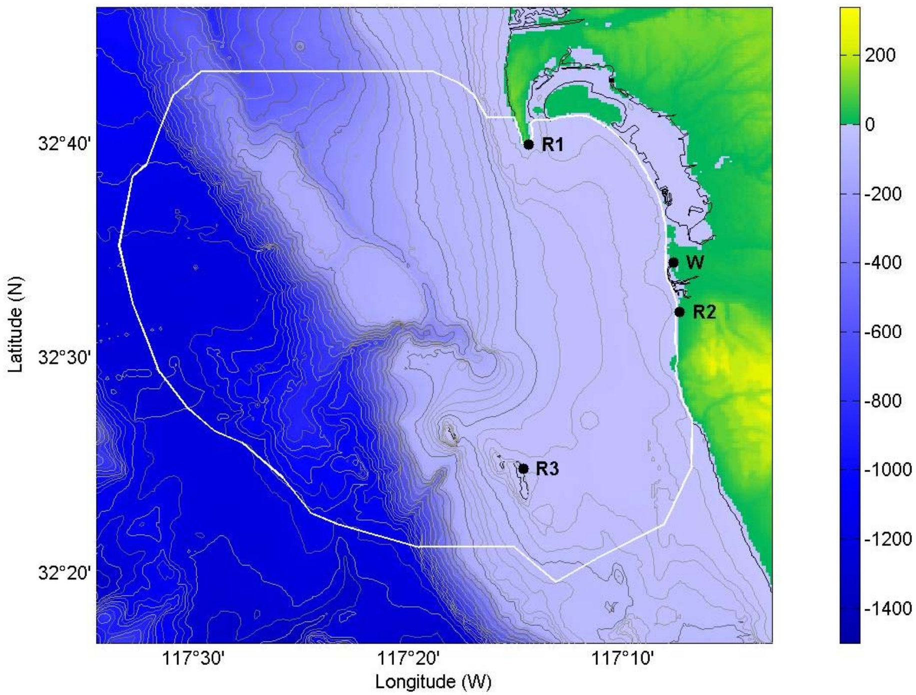
$$a_{xy} = a_{yx} = 0$$

$r_x < r_y$

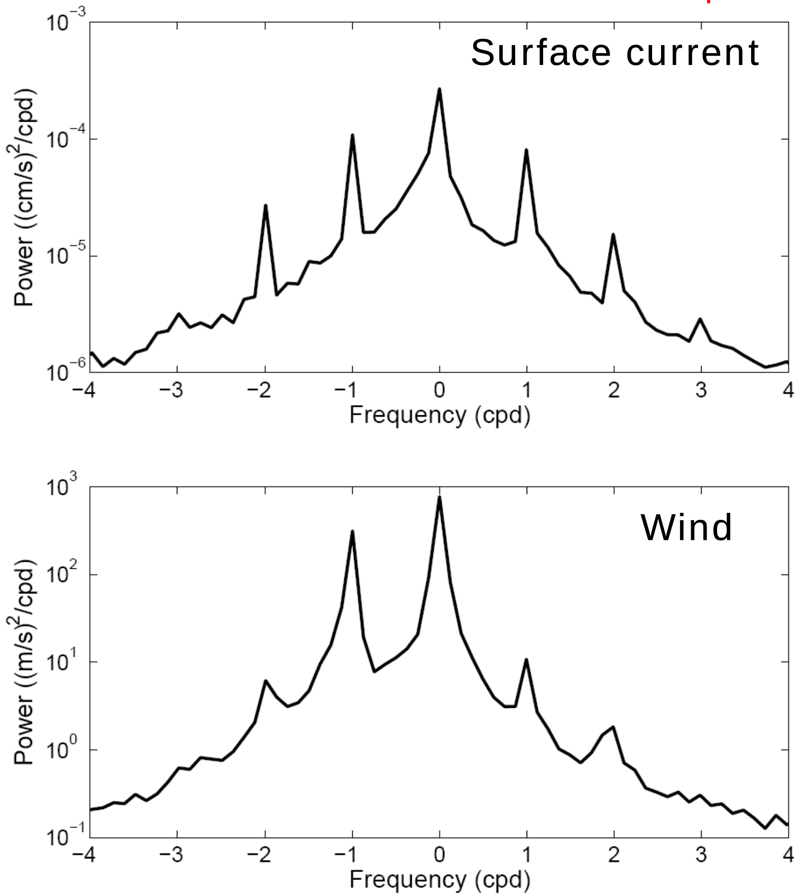


Study domain & data

- A single hourly surface current (spatially averaged) and wind observation near Tijuana River during two years are used.
- Major tides (K1, P1, O1, M2, and S2) are removed.



90 subsamples



Statistical estimate

Isotropic

w/ complex notation:

$\mathbf{u} = u + iv, \ \tau = \tau_x + i\tau_y$

$H = H_x + iH_y$

Time domain

$$\mathbf{u}(z,t) = \int_{t'} \mathbf{G}(z, t-t')\boldsymbol{\tau}(t')\,\mathrm{d}t',$$

$$\mathbf{G}(z,t) = \left(\langle \mathbf{u}(z,t)\,\boldsymbol{\tau}_N^\dagger(t)\rangle\right)\cdot \left(\langle \boldsymbol{\tau}_N(t)\,\boldsymbol{\tau}_N^\dagger(t)\rangle + \mathbf{R_b}\right)^{-1}$$

$\boldsymbol{\tau}_N$: Wind stress matrix stacked with N time lags

Linear

Symmetric responses for the opposite sign of impulses

$$\mathbf{u}(t) = \mathbf{G}\boldsymbol{\tau}_N$$

$$\hat{\mathbf{u}}(\omega) = \mathbf{H}\hat{\boldsymbol{\tau}}$$

Anisotropic

w/ real notation (four terms):

H_{xx}, H_{xy}, H_{yx}, and H_{yy}

Frequency domain

$$\hat{\mathbf{u}}(z,\omega) = \mathbf{H}(z,\omega)\hat{\boldsymbol{\tau}}(\omega).$$

$$\mathbf{H}(z,\omega) = \left(\langle \hat{\mathbf{u}}(z,\omega)\,\hat{\boldsymbol{\tau}}^\dagger(\omega)\rangle\right)\cdot \left(\langle \hat{\boldsymbol{\tau}}(\omega)\,\hat{\boldsymbol{\tau}}^\dagger(\omega)\rangle + \mathbf{R_a}(\omega)\right)^{-1}$$

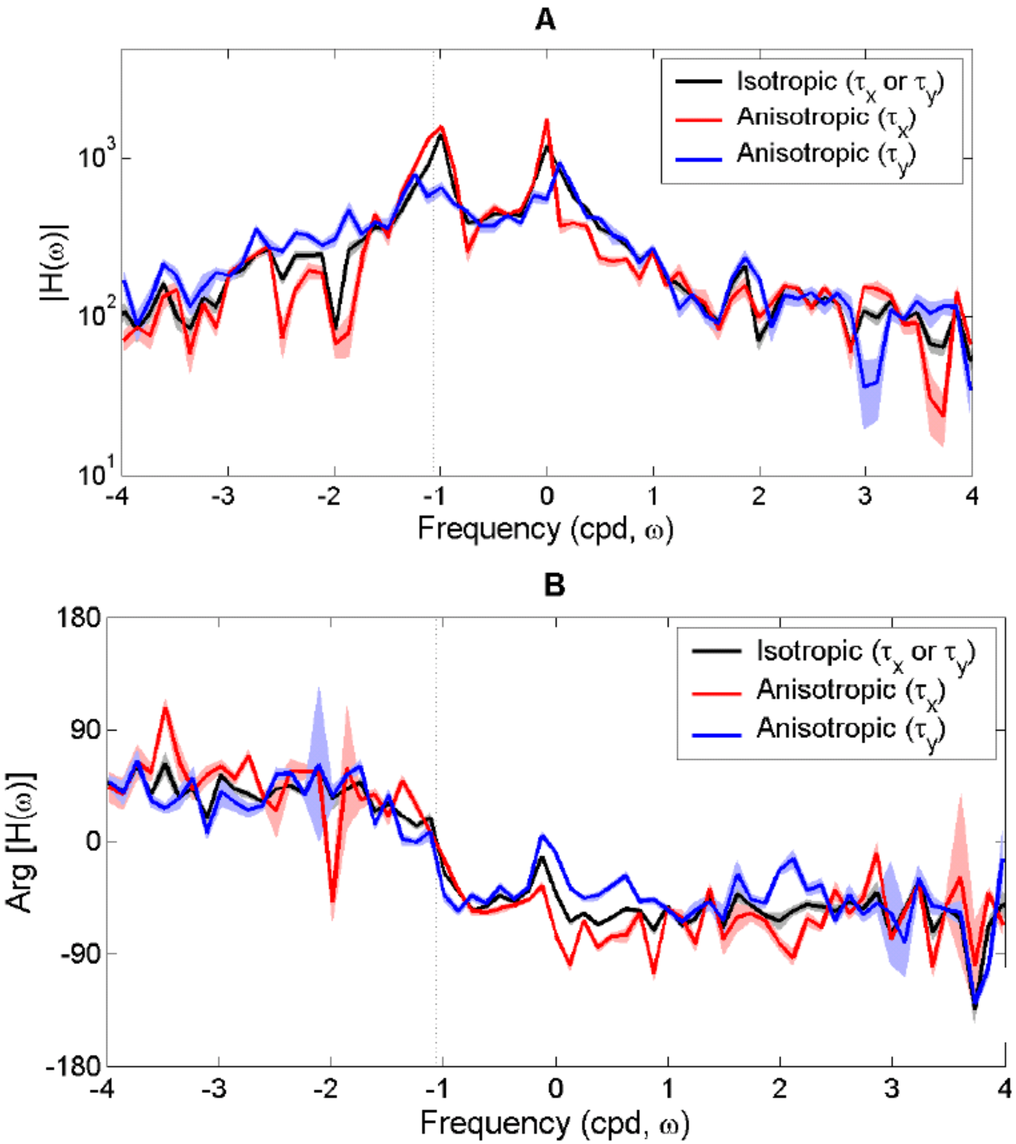
Nonlinear

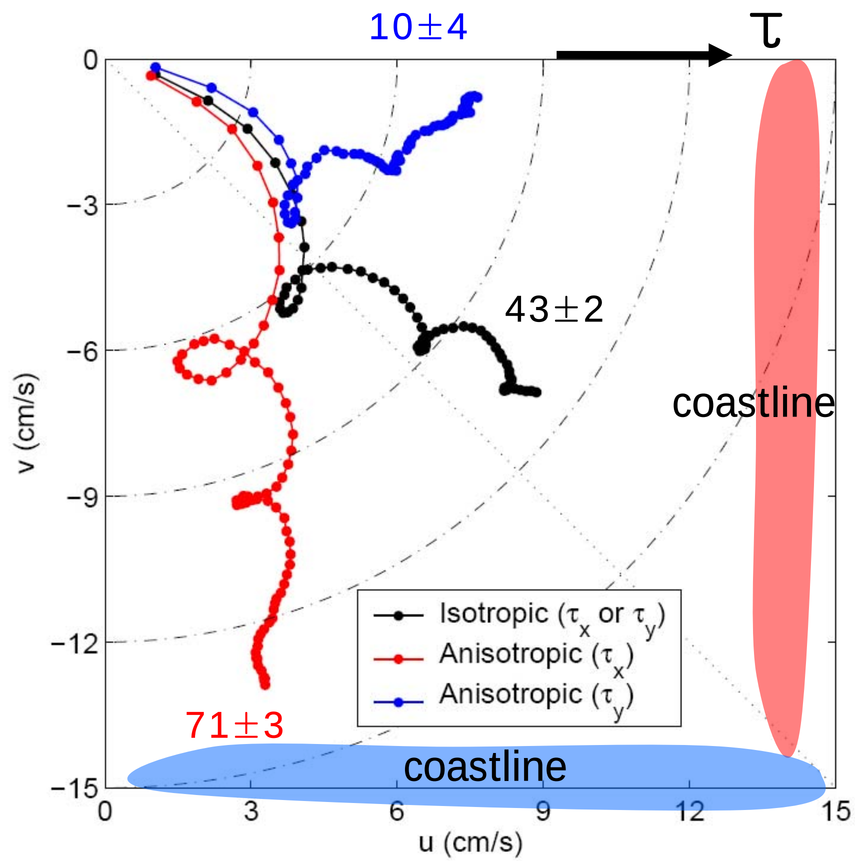
Break symmetric responses

$$\mathbf{u}(t) = \mathbf{G_1}\boldsymbol{\tau}_N + \mathbf{G_2}|\boldsymbol{\tau}_N|,$$

$$\hat{\mathbf{u}}(\omega) = \mathbf{H_1}\hat{\boldsymbol{\tau}} + \mathbf{H_2}|\hat{\boldsymbol{\tau}}|$$

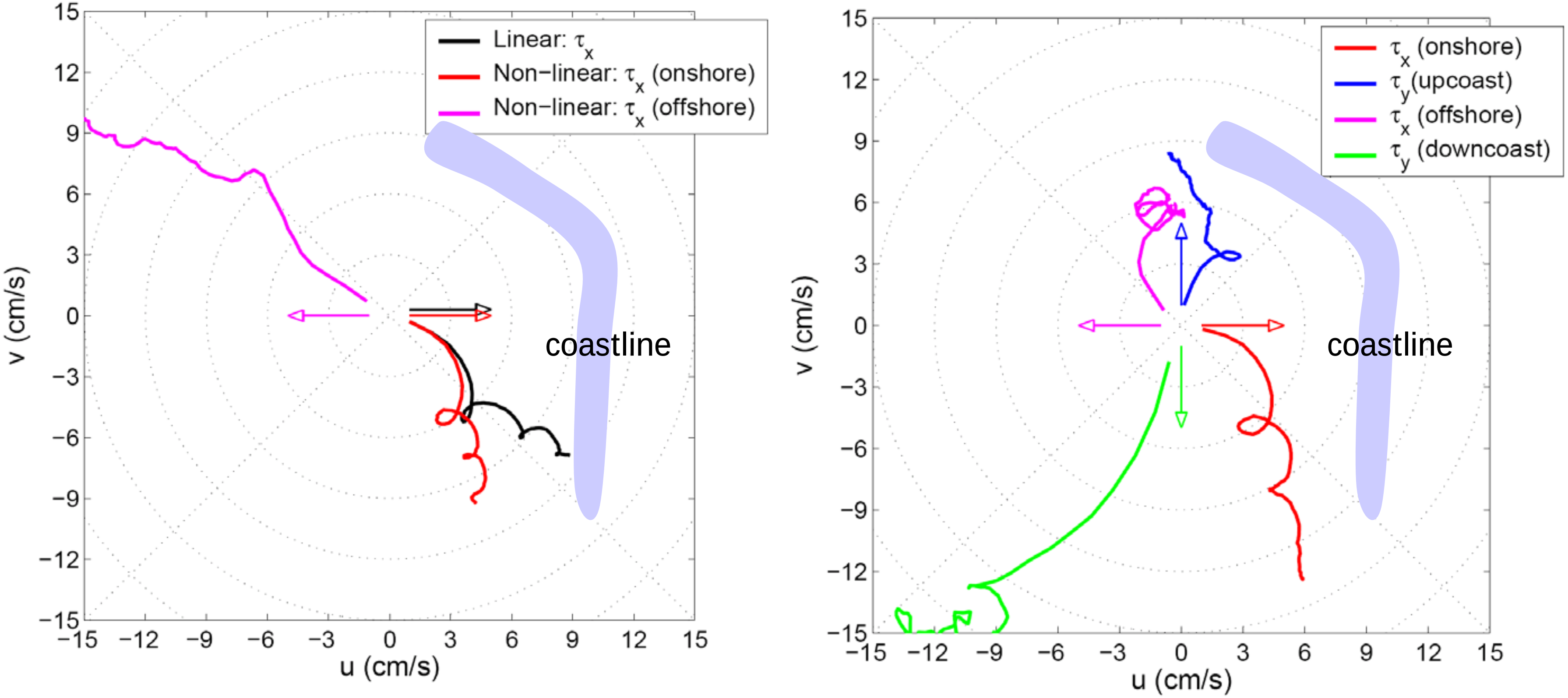
Freq. Iso/Anisotropic Linear WIRF





Steady wind (3 m/s) is applied to both WIRFs

Freq. Iso/Anisotropic Nonlinear WIRF



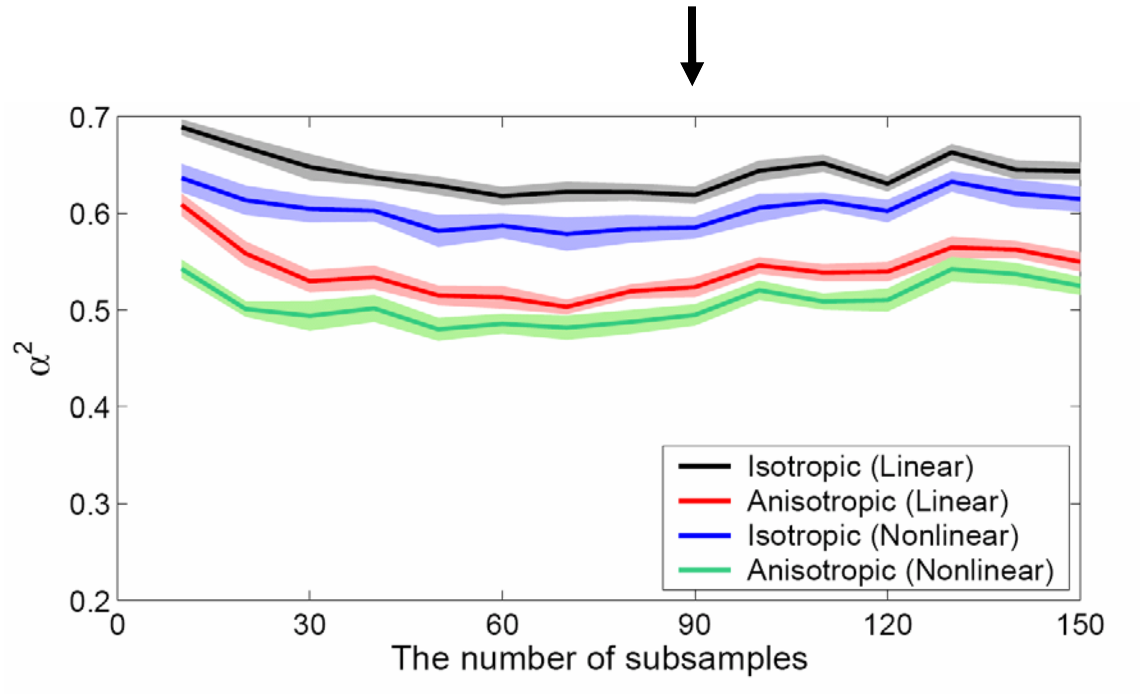
- Nonlinear WIRF breaks symmetric responses.
- Isotropic nonlinear WIRF to the upcoast and onshore wind yields the identical response.

Summary

- Anisotropic surface current response to the wind in a coastal region was investigated with the impulse response function of the observation data.
- The response function is used to estimate the wind-driven surface currents.
- The asymmetric and anisotropic response in a coastal region may result from the pressure setup due to wind and bottom/coastline friction.

Extra slides

Residual variance ratio



$$\alpha^2 = \frac{\sum_k |\hat{\mathbf{r}}(\omega_k)|^2}{\sum_k |\hat{\mathbf{u}}(\omega_k)|^2}.$$

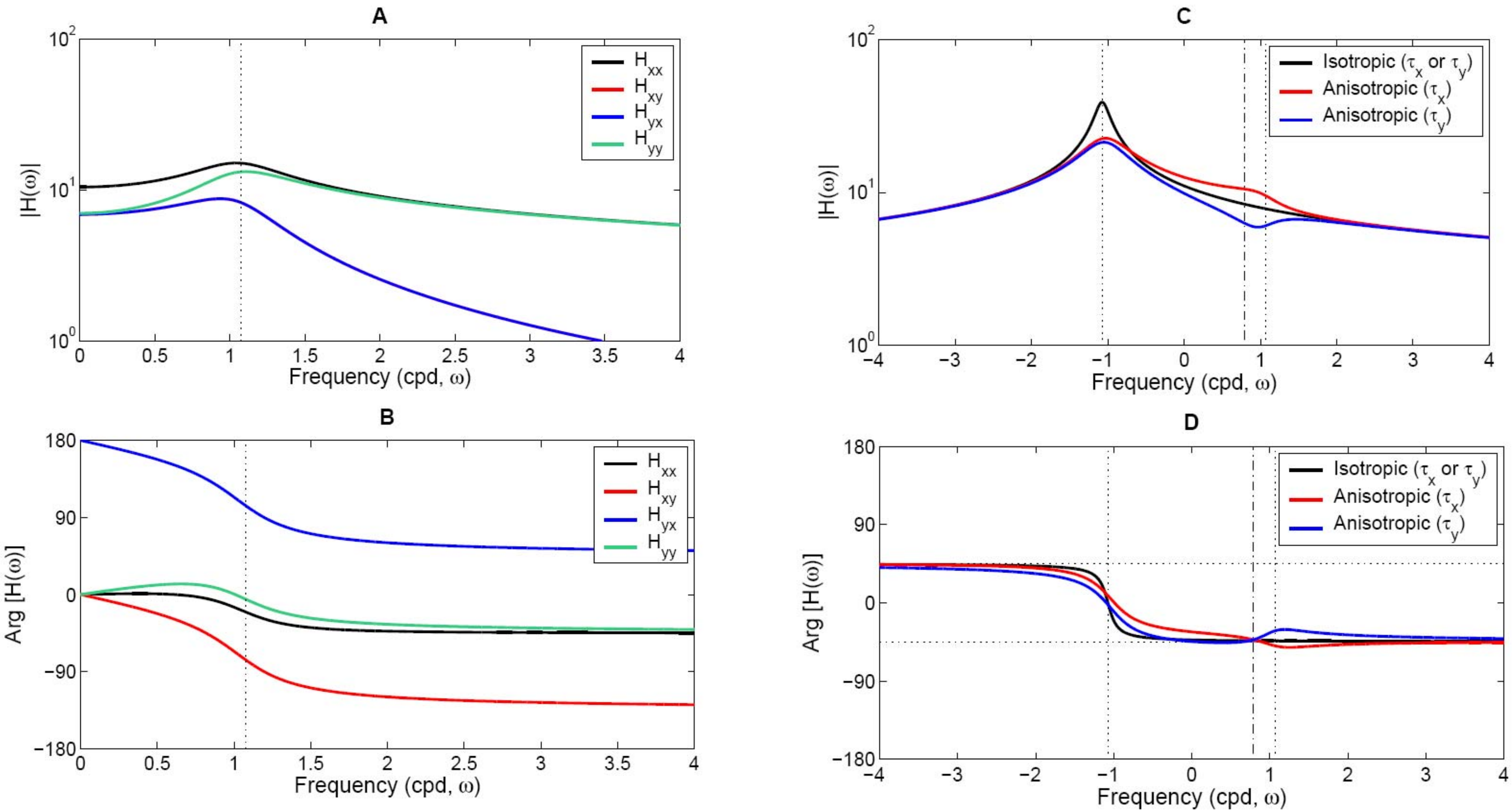
$$\hat{\mathbf{r}}(\omega) = \mathbf{H}(\omega)\hat{\boldsymbol{\tau}}(\omega) - \hat{\mathbf{u}}(\omega)$$

- 90 subsamples are chosen for WIRF estimate in freq. domain.
  - ~8.12 days period
- Residual variance cross-validated on the training data.
- Similar pattern to the sample data.

Comparison of iso/anisotropic WIRF

$$\hat{\mathbf{u}}(\omega < 0) = \left(H_{xx}^{\dagger} + iH_{yx}^{\dagger}\right) \hat{\tau}_x(\omega), \qquad \hat{\mathbf{u}}(\omega < 0) = \left(H_{xy}^{\dagger} + iH_{yy}^{\dagger}\right) \hat{\tau}_y(\omega)$$

$$\hat{\mathbf{u}}(\omega > 0) = \left(H_{xx} + iH_{yx}\right) \hat{\tau}_x(\omega), \qquad \hat{\mathbf{u}}(\omega > 0) = \left(H_{xy} + iH_{yy}\right) \hat{\tau}_y(\omega),$$



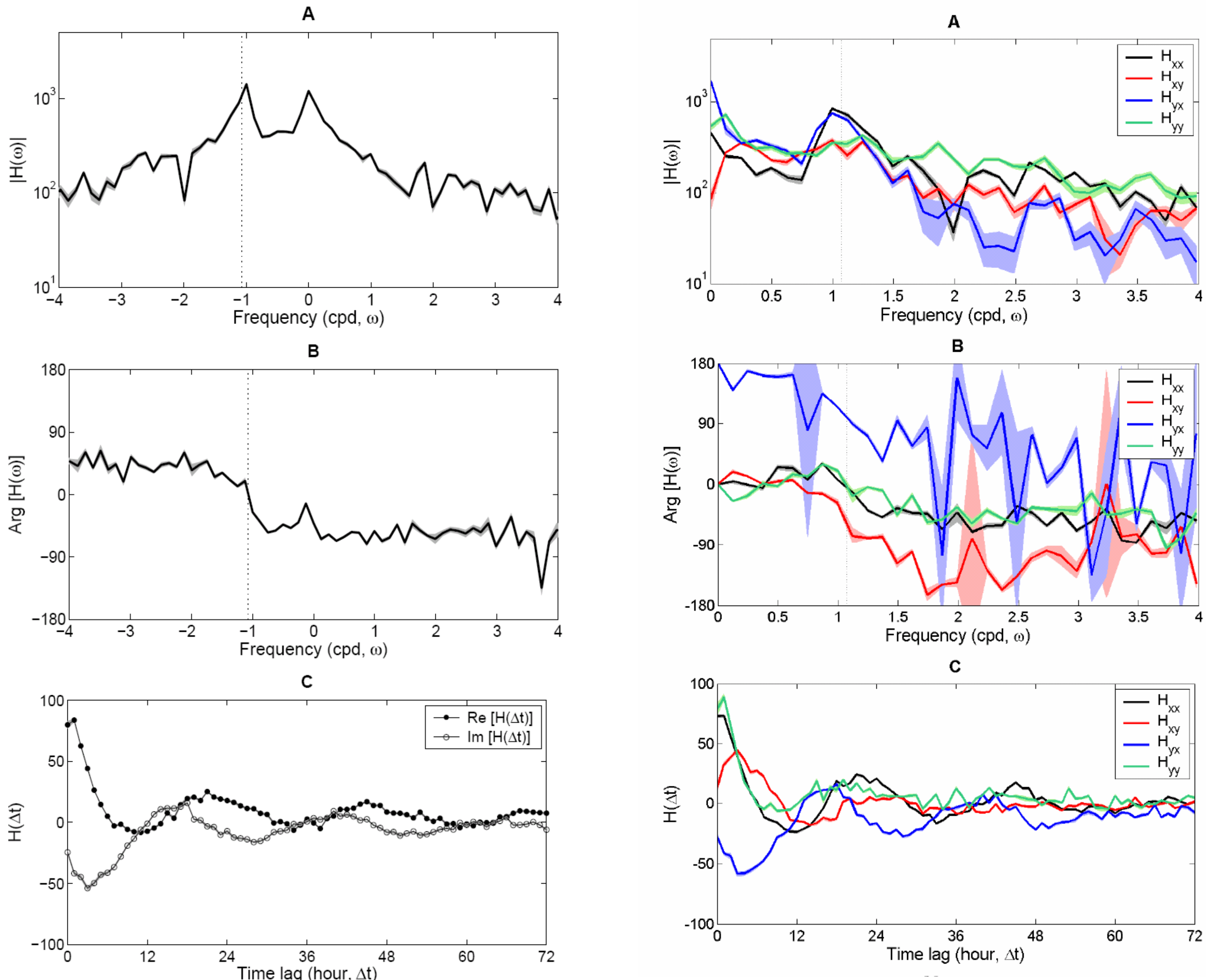
Test-out cross validation

- 30 times repeated
- Comparable to results with systematic subsampling.
- The regularization matrix (R_b) may need for the non-positive definite wind stress covariance matrix due to missing data.
- The x-validation has minimum at κ^2 = 10-40% of mean eigenvalue when Rb = κ^2I.

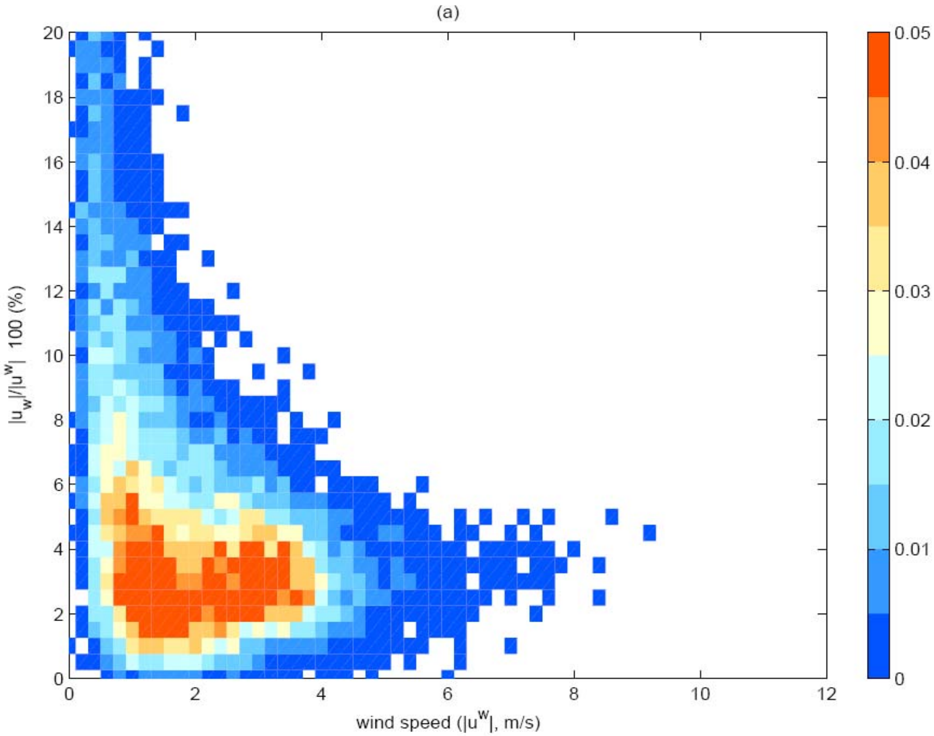
$$\mathbf{G}(z,t) = \left(\langle \mathbf{u}(z,t)\,\boldsymbol{\tau}_N^\dagger(t)\rangle\right)\left(\langle \boldsymbol{\tau}_N(t)\,\boldsymbol{\tau}_N^\dagger(t)\rangle + \mathbf{R_b}\right)^{-1}$$

Time domain WIRF

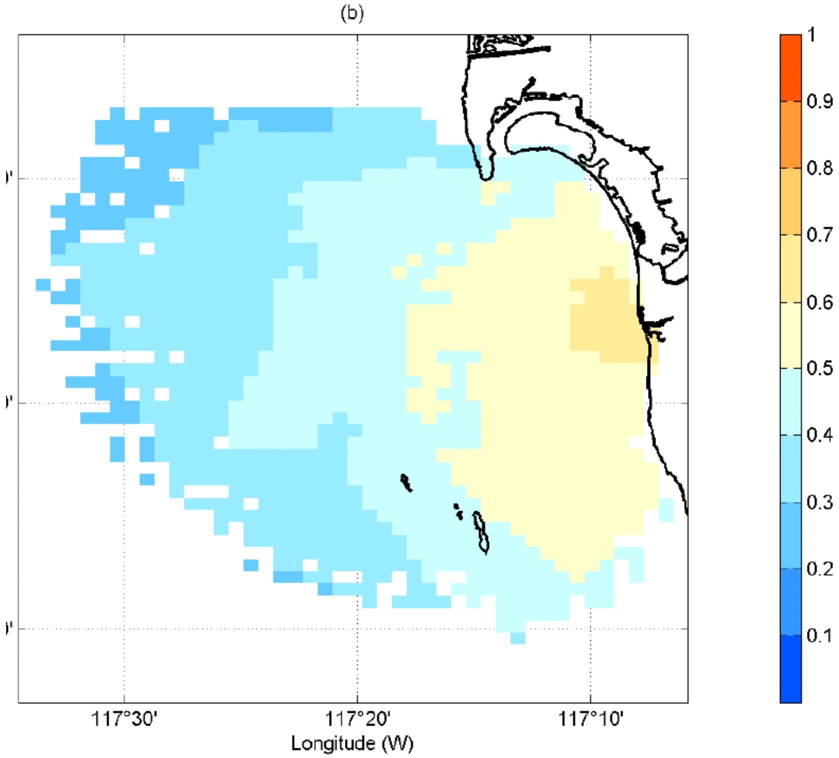
Freq. domain iso/anisotropic WIRF



Characteristics of wind-driven currents (Magnitude & Coherence)



$$\frac{|\text{Wind-driven currents}|}{|\text{Wind speed}|} \times 100$$



Averaged coherence ($|\omega| < 2$ cpd)

END.