Resonant ocean current responses driven by coastal winds near the critical latitude

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Resonant **ocean current responses** driven by **coastal winds** near the critical latitude

- Wind and current responses
  - Ekman theory
Resonant ocean current responses driven by coastal winds near the critical latitude

- Wind and current responses
  - Ekman theory
- Resonance
  - Forcing-response in the frequency domain
  - Natural frequency – Coriolis frequency

\[ f_c = 2 \sin \text{(latitude)} \text{ [cycles per day]} \]
Resonant ocean current responses driven by coastal winds near the critical latitude

- Wind and current responses
  - Ekman theory
- Resonance
  - Forcing-response in the frequency domain
  - Natural frequency – Coriolis frequency
- Critical latitude
  - Observations at different latitudes – wind and surface currents off the USWC

Shaffer, 1972; Ekman model
Resonant ocean current responses driven by coastal winds near the critical latitude

• At a given latitude, what would be the wind-current response in the frequency domain?
• At a given frequency, what would be the wind-current response as a function of latitude?
Wind-current responses in the freq. domain

\[ \frac{\partial \mathbf{u}}{\partial t} + if_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]

Ekman theory
Wind-current responses in the freq. domain

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{i} f_c \mathbf{u} + r \mathbf{u} = -\frac{1}{\rho} \frac{\partial \tau}{\partial z}
\]

\[
H_E(z, \sigma) = \frac{\hat{u}(z, \sigma)}{\hat{\tau}(\sigma)} = \frac{e^{i z}}{\lambda \rho v}
\]

\[
\lambda = \sqrt{[i(\sigma + f_c) + r]/v}
\]

At a given latitude, the relationship between wind stress and surface currents is given as a transfer function in the frequency domain.
Wind-current responses in latitude

\[ \frac{\partial \mathbf{u}}{\partial t} + if_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]

Ekman theory

\[ H_E(z,f_c) = \frac{\hat{u}(z,f_c)}{\hat{\tau}(f_c)} = \frac{e^{i\lambda z}}{\lambda \rho v} \]

as a function of Coriolis freq. (latitude)

\[ \lambda = \sqrt{[i(\sigma + f_c) + r]/v} \]
Wind-current responses in latitude

\[
\frac{\partial \mathbf{u}}{\partial t} + if_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z}
\]

Ekman theory

\[
H_{E}(z, f_c) = \frac{\hat{u}(z, f_c)}{\hat{\tau}(f_c)} = \frac{e^{iz}}{\lambda \rho v}
\]

as a function of Coriolis freq. (latitude)

\[
\lambda = \sqrt{[i(\sigma + f_c) + r]/v}
\]

\[r = 0;\]
\[\sigma = 1 \text{ cpd}\]
(diurnal frequency).

Simpson et al, JPO 2002 (Slab layer model)

Resonant latitude due to land/sea breeze: ±30°N
Latitudinal coastal observations

- US West Coast high-frequency radar network-derived surface currents and wind stress (red dots) at NDBC buoys.
- Latitudinal variation of 32°N to 47°N

\[
\hat{u}(z, \omega) = H(z, \omega) \hat{\tau}(\omega)
\]

\[
H(z, \omega) = \left( \langle \hat{u}(z, \omega) \hat{\tau}^\dagger(\omega) \rangle \right) \left( \langle \hat{\tau}(\omega) \hat{\tau}^\dagger(\omega) \rangle + R_a \right)^{-1}
\]

\(R_a\) : Regularization matrix
Variability of surface currents and wind

- Wind- and tide-coherent, low-frequency variance, and inertial variance

Kim et al (JGR, 2011)
Variability of surface currents and wind

- Wind- and tide-coherent, low-frequency variance, and inertial variance
- Variance of the diurnal wind does not vary that much in the along-shore direction, but it is given as a function of distance from the shoreline (cross-shore direction).

Kim et al (JGR, 2011)
Coast-wide wind transfer functions

\[ \hat{u}(\tau, \omega) = H(\tau, \omega) \hat{\tau}(\omega) \]

(Kim and Crawford, GRL 2014)
Coast-wide wind transfer functions

At a given latitude, what would be the wind-current response in the frequency domain?

At a given frequency, what would be the wind-current response as a function of latitude?

(Kim and Crawford, GRL 2014)
Coast-wide wind transfer functions

(Kim and Crawford, GRL 2014)
Coast-wide wind transfer functions

(Kim and Crawford, GRL 2014)
Resonant responses near the critical latitude

**(a)**

- Slab layer model
- $Z = 0$ (Ekman)
- $Z = 0.35\delta_E$ (Near-surface avg. Ekman)

**Resonant latitude due to land/sea breeze:** ±30°N
Summary

- Wind-current responses are examined in the frequency domain and latitude using analytic solutions of Ekman model (and slab layer and surface-averaged Ekman models) and observations off the US West Coast.
- The current responses are enhanced at the local inertial frequency.
- Resonant responses can be expected at the +/-30° latitude in the diurnal land-sea breeze environment.
- Energetic mixing and potential internal motions near the critical latitude are expected.
BACKUP SLIDES
Wind variability

- Variance of the diurnal wind does not vary that much in the along-shore direction, but it is given as a function of distance from the shoreline (cross-shore direction).
Resonant responses at the critical latitude

\[ \frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]

\[ H_E(z, \sigma) = \frac{\hat{u}(z, \sigma)}{\hat{\tau}(\sigma)} = \frac{e^{iz}}{\lambda \rho \nu} \]

\[ \lambda = \sqrt{i(\sigma + f_c) + r} / \nu \]

\[ \frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{\tau^w}{\rho h} \]

\[ H_S(\sigma) = \frac{\hat{u}(\sigma)}{\hat{\tau}(\sigma)} = \frac{1}{\rho h [i(\sigma + f_c) + r]} \]

\( f_c = 2 \sin \text{ (latitude), } r = 0 \);

at \( \sigma = 1 \) cpd (diurnal frequency).

Shaffer, 1972; Ekman model

Simpson et al, JPO 2002 (Slab layer model)

Resonant latitude due to land/sea breeze: ±30°N
Resonant responses at the critical latitude

\[ \frac{\partial \mathbf{u}}{\partial t} + if_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]

\[ H_E(z, \sigma) = \frac{\hat{u}(z, \sigma)}{\hat{\tau}(\sigma)} = \frac{e^{jz}}{\lambda \rho v} \]

\[ \lambda = \sqrt{[i(\sigma + f_c) + r]/v} \]

\[ H_A(\sigma) = \frac{1}{Z^*} \int_0^{Z^*} H_E(z, \sigma) \, dz, \]

\[ = \frac{1}{Z^*} \frac{1}{\lambda^2 \rho v} \left( e^{\lambda Z^*} - 1 \right), \]

\( f_c = 2 \sin (\text{latitude}) \)

When \( \sigma = 1 \) cpd.

Near-surface averaged Ekman model can be appropriate to explain the HFR-derived surface currents.

Shaffer, 1972; Ekman model

Resonant latitude due to land/sea breeze: \( \pm 30^\circ \text{N} \)