Energy spectra + energy fluxes of submesoscale coastal ocean currents

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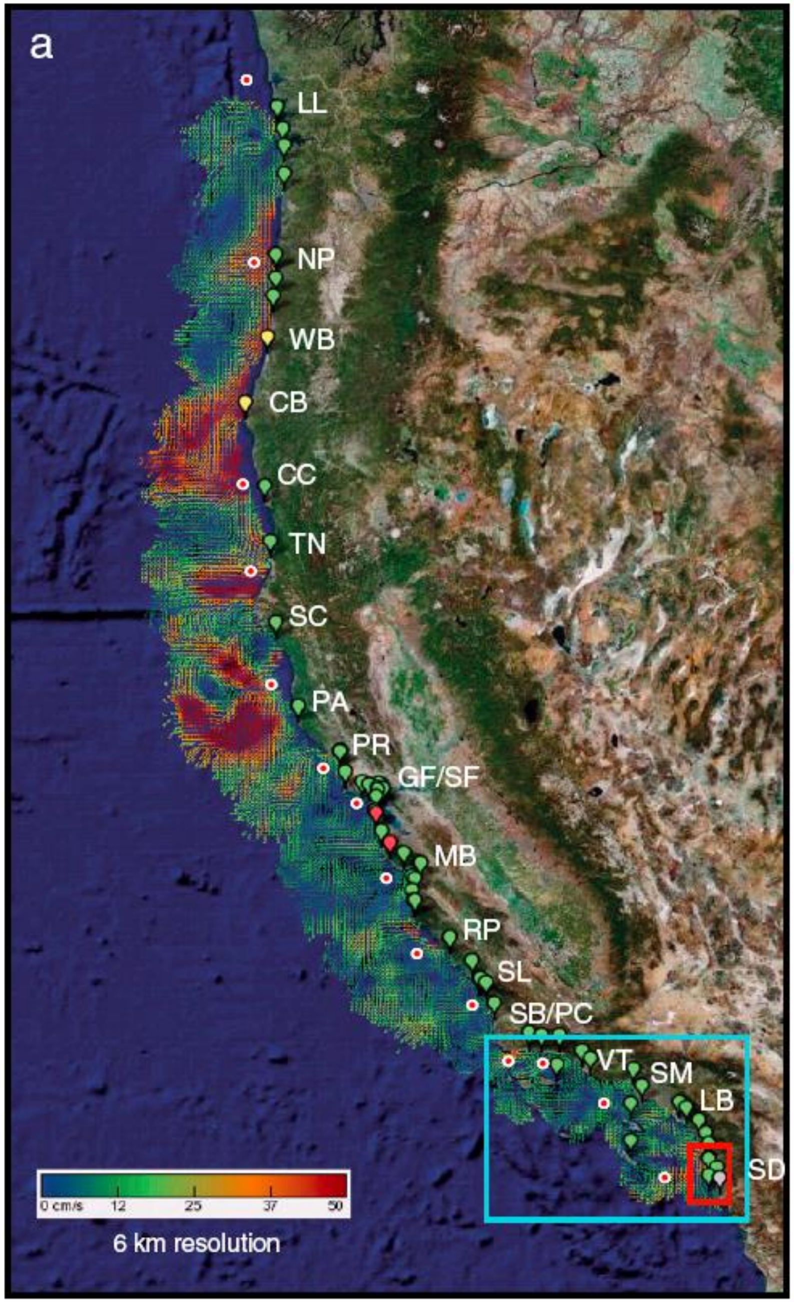

(Kim and Crawford, *GRL* 2015)

Energy spectra (+ energy fluxes) of submesoscale coastal ocean currents

- 3-30 MHz frequency (HFR)
- Using Doppler shift of backscattered signals of surface gravity waves to estimate the background currents
- Upper 1 m depth-averaged currents
- Hourly and O(1) km scale surface current maps
- A network of HFRs along the coast over 2500 km of US West Coast provides km resolution and hourly surface current maps which cover about 150 km offshore from shoreline.

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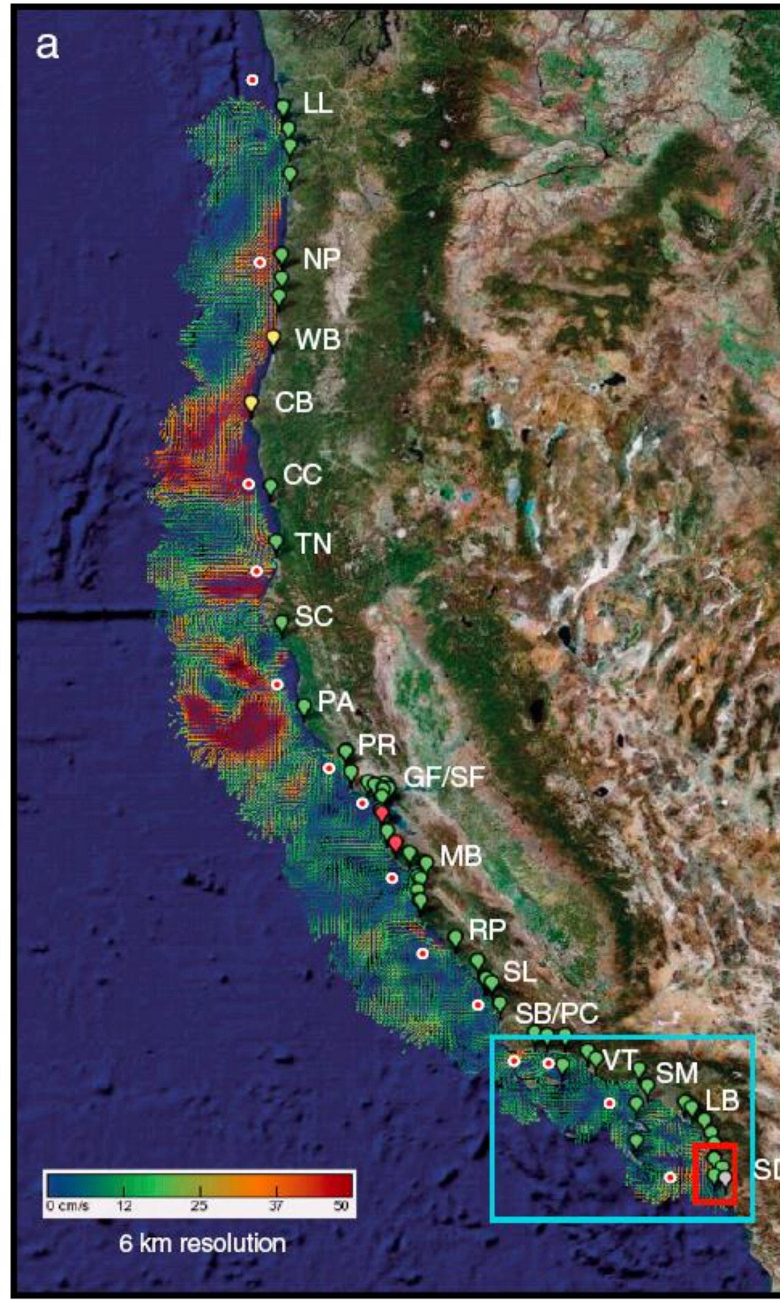
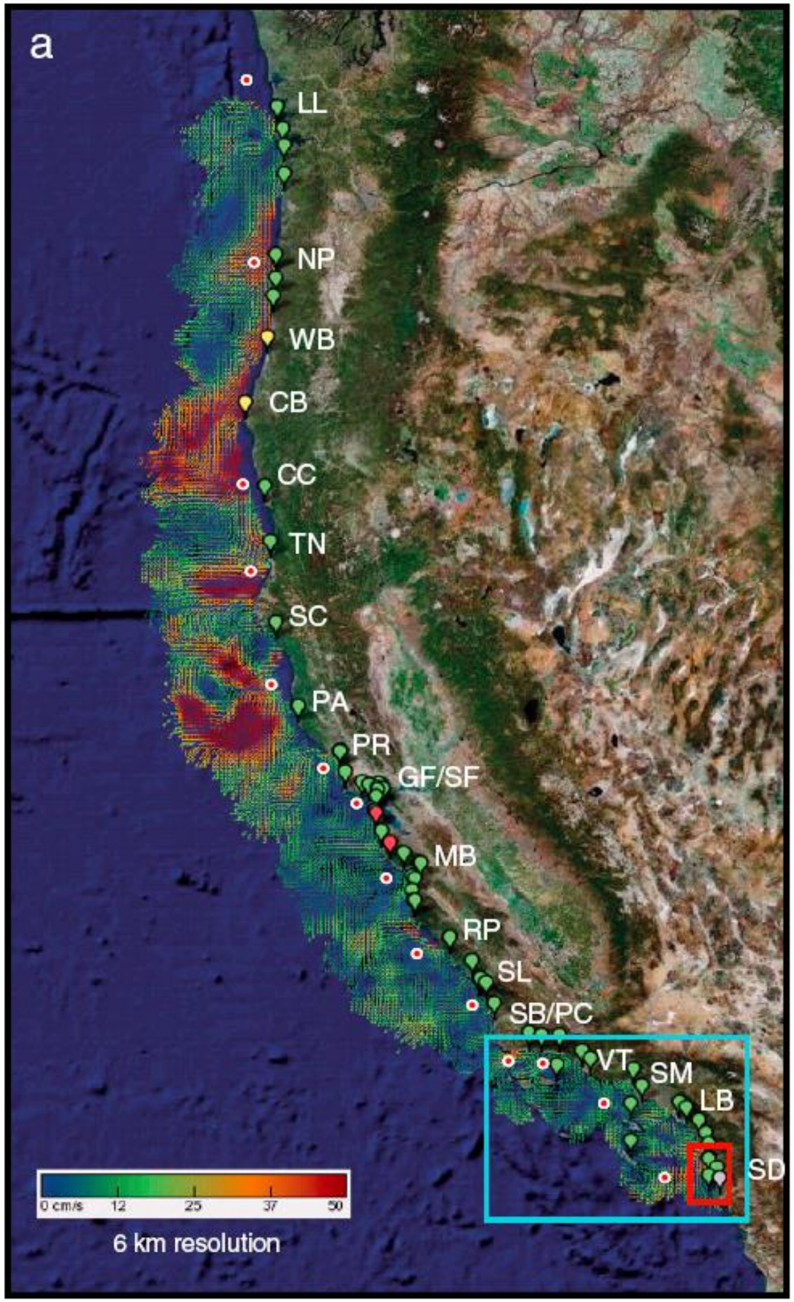


(Kim and Crawford, *GRL* 2015)

Energy spectra of submesoscale coastal ocean currents

+ energy fluxes

- O(1) Rossby number
  [Ro = U/(fL) = ζ/f]
- A horizontal scale smaller than the first baroclinic Rossby deformation radius; O(1-10) km
- Frequently observed as fronts, eddies, and filaments
- Contribute to the vertical transport of oceanic tracers, mass, and buoyancy and rectify the mixed-layer structure and upper-ocean stratification



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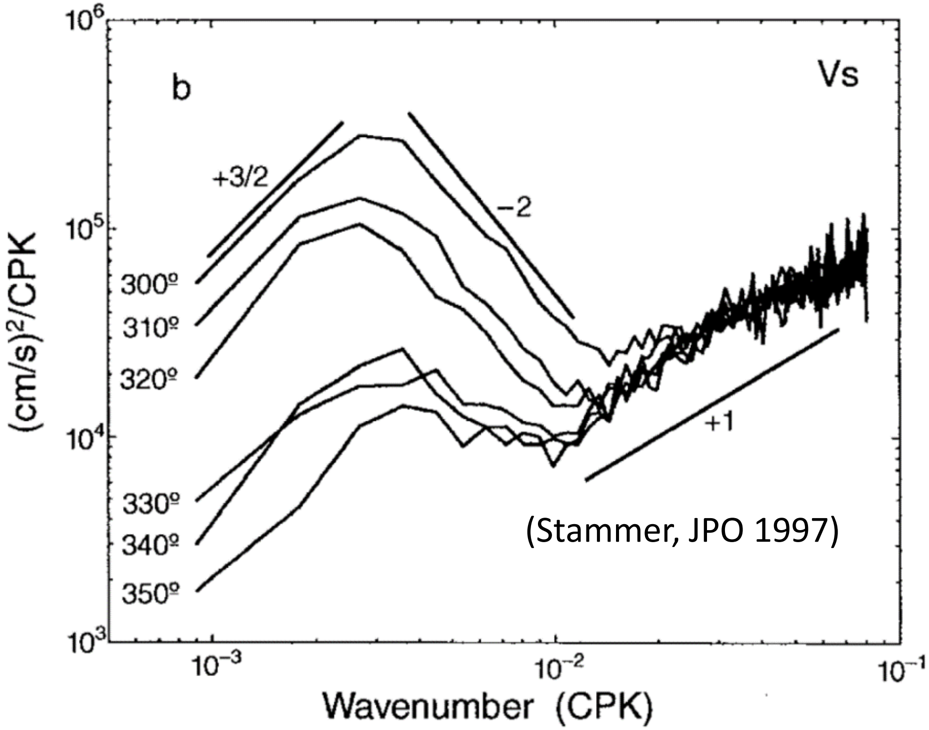
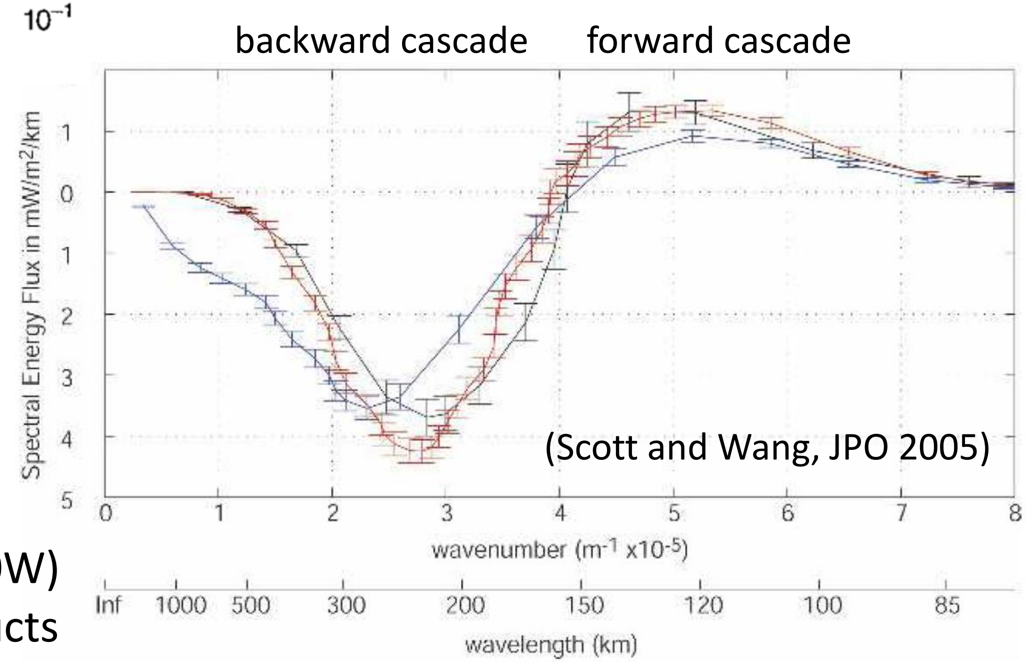
Motivation



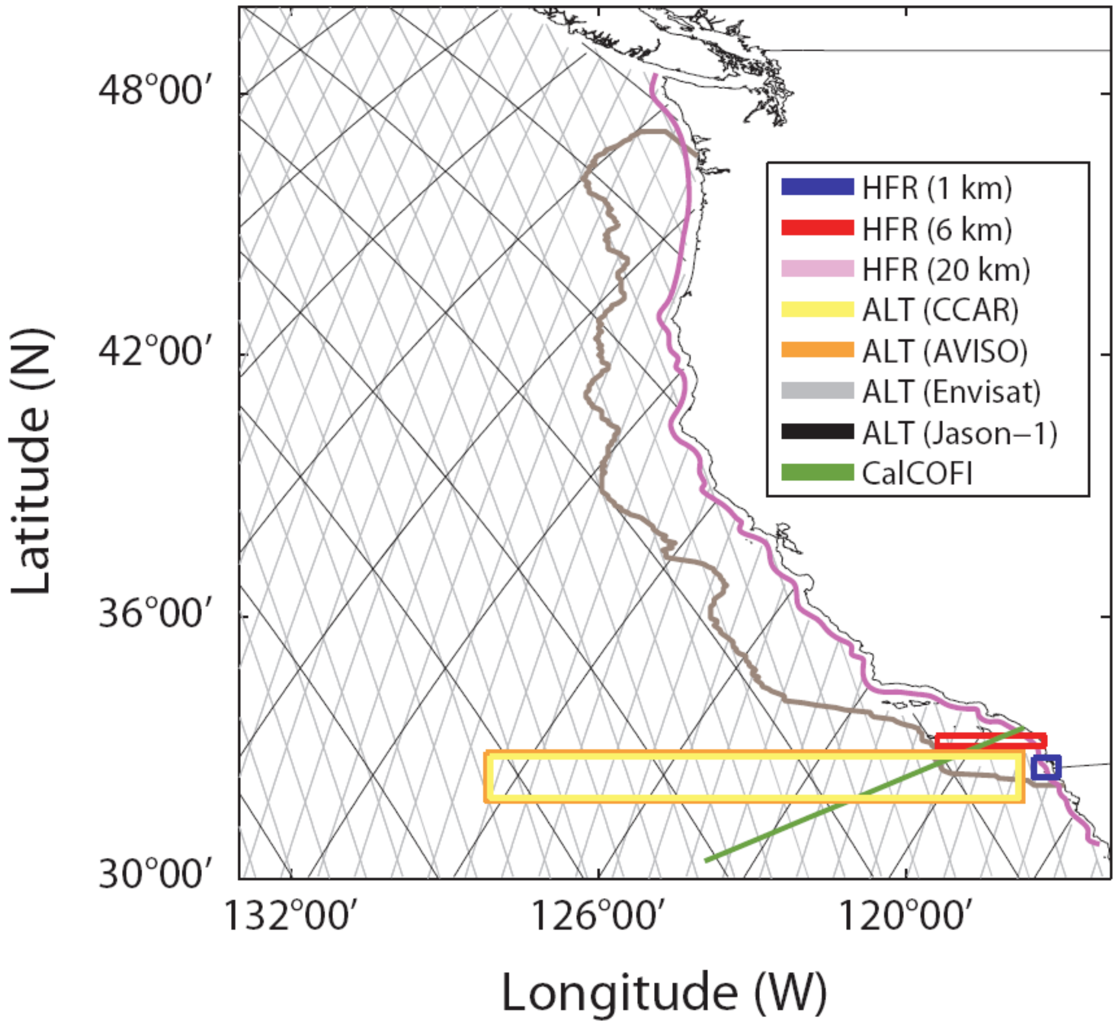
- What can be the slope of energy spectra and kinetic energy flux below 100 km scale?

Wavenumber spectra of altimeter-derived cross-track geostrophic currents (30N to 40 N)



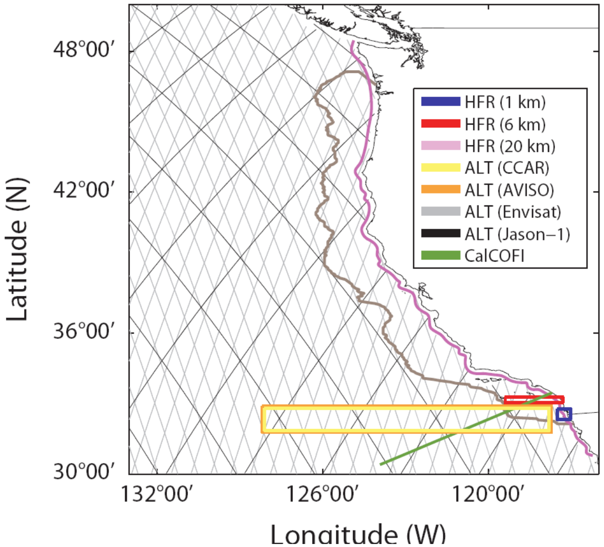
Kinetic energy flux in ACC region (57S, 120W)
Optimally interpolated 1/3° AVISO products

Sampling domain in computation of energy spectra

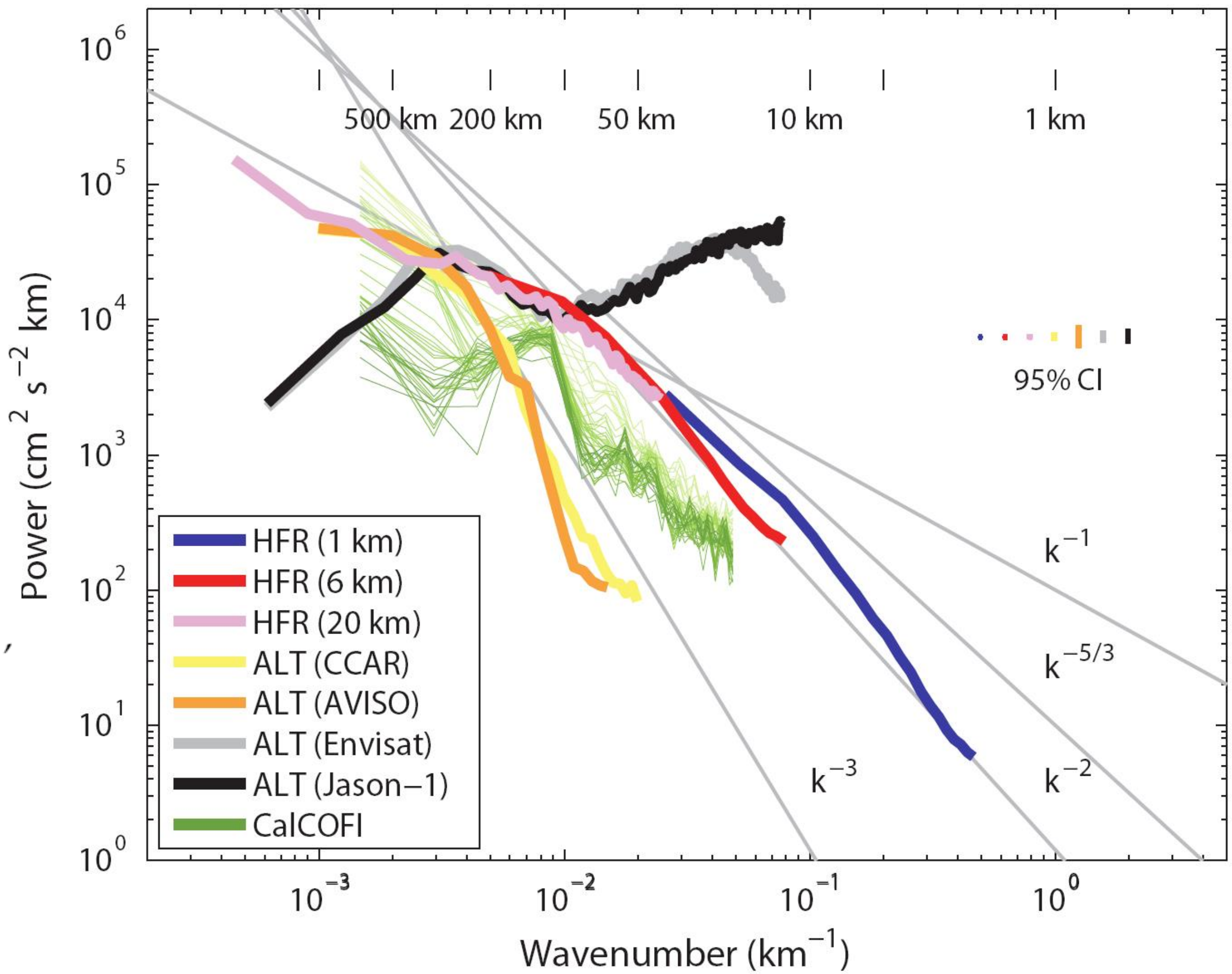


- HFR surface currents (1, 6, and 20 km resolution; hourly) off southern California and on coastline axis (USWC)
- Gridded ALT products [CCAR (daily) and AVISO (weekly)] and along-track altimeter (ALT; Envisat/Jason-1; weekly) on NE Pacific
- CalCOFI shipboard ADCP (Line 90; quarterly)
- SoCAL was chosen because it contains relatively minimum ageostrophic components.

Energy spectra in the wavenumber domain (1D)



$$S_{\mathbf{u}_\perp}(k_\parallel) = \left(\frac{g}{f_c}\right)^2 \left(2\pi k_\parallel\right)^2 S_{\boldsymbol{\eta}_\parallel}(k_\parallel),$$



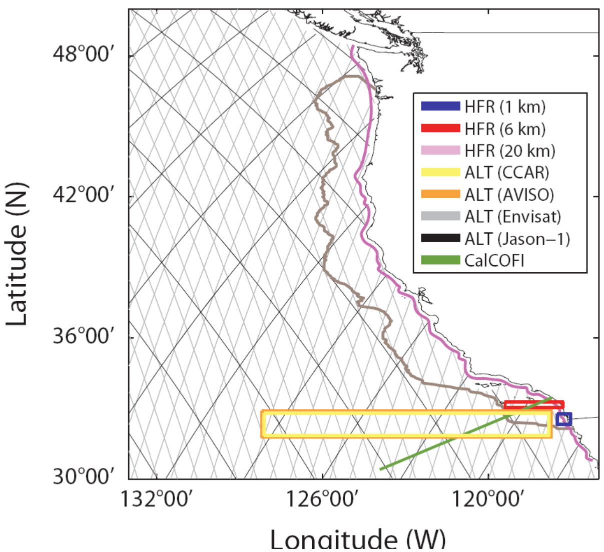
Power spectrum of cross-track geostropic currents from along-track SSHAs

K^{-2} power law related to sub-mesoscale.

Robust estimate on k-2 spectra with data in other regions.

Two kinds of ALT data: Envisat and Jason-1
HFR data with three resolutions:
1 km and 6 km data are sampled from SoCAL,
because minimum ageostropic components are expected.
20 km data are from the coastline axis.

Energy spectra in the wavenumber domain (1D; +Spray)



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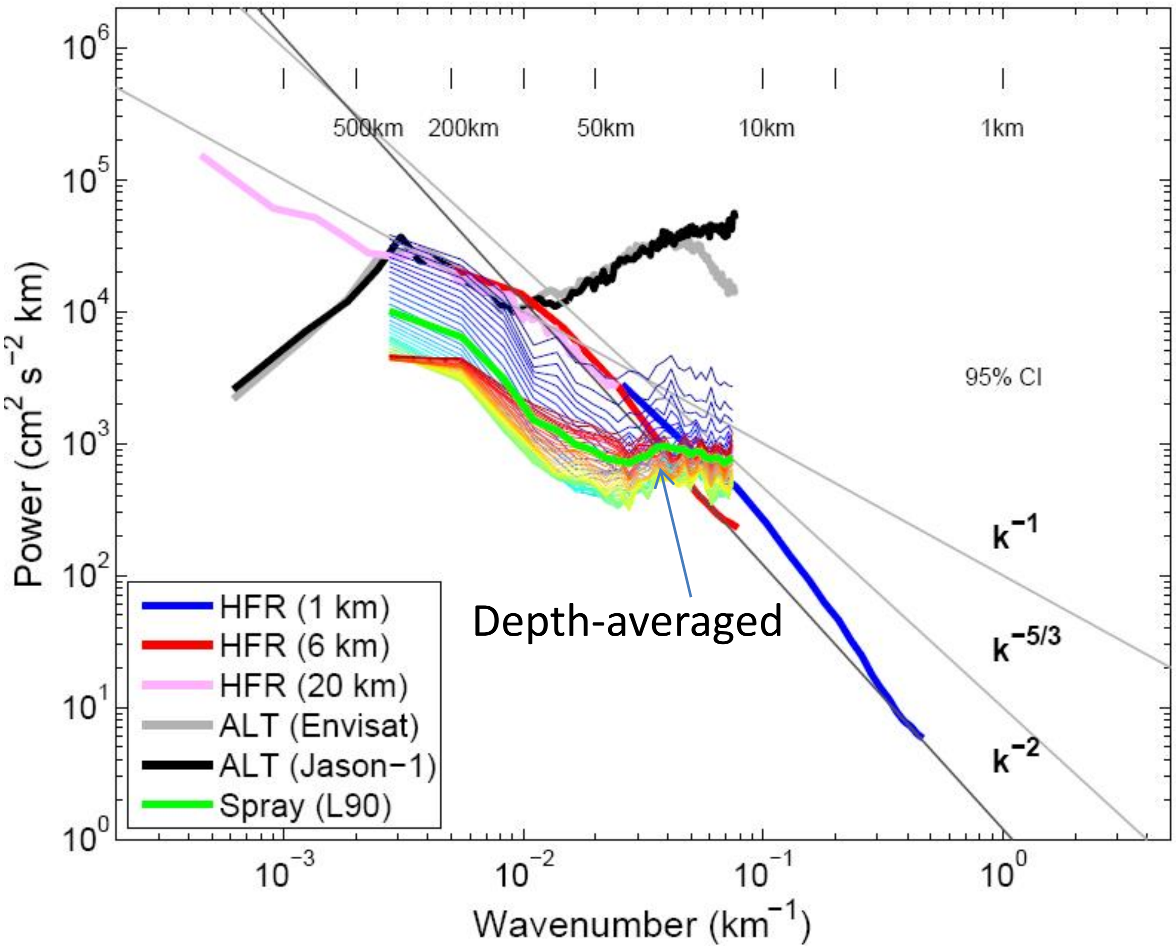

Two kinds of ALT data: Envisat and Jason-1
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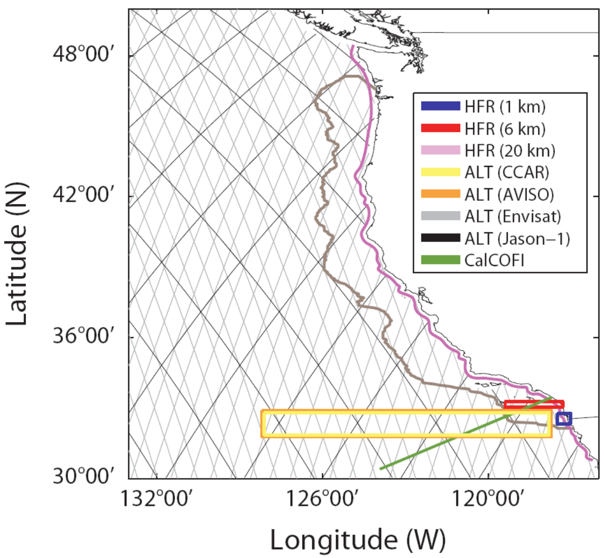
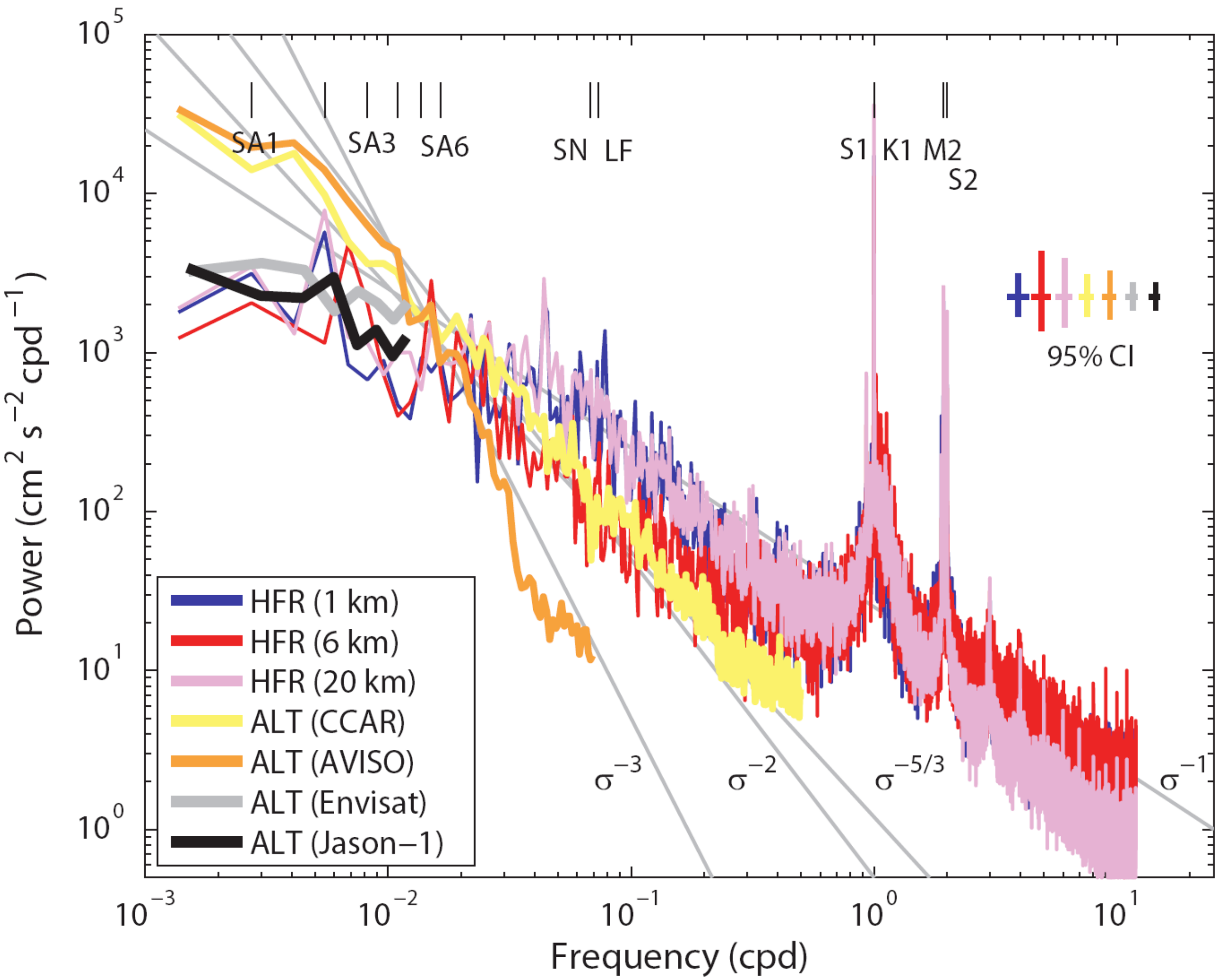
Energy spectra in the frequency domain





Along-track altimeter data are binned in 2° x 2° grid boxes and averaged in time (7-daily → 30 daily time series) to increase signal to noise ratio.

Conversion between covariance and power spectra

$$E_\circ(k_x, k_y) = \mathcal{F}(d)^\dagger \mathcal{F}(d),$$
$$= \frac{1}{\Delta k_x} \frac{1}{\Delta k_y} \left| \frac{1}{NM} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} d(x_n, y_m) e^{-ik_x x_n - ik_y y_m} \right|^2$$

$$E_\bullet(\alpha_x, \alpha_y) = |\mathcal{F}(c)|,$$
$$= \frac{1}{\Delta\alpha_x} \frac{1}{\Delta\alpha_y} \left| \frac{1}{N^* M^*} \sum_{n=N^-}^{N^+} \sum_{m=M^-}^{M^+} c(n\Delta x, m\Delta y) e^{-i\alpha_x n\Delta x - i\alpha_y m\Delta y} \right|$$

Wiener-Khinchin theorem

where

$$c(n\Delta x, m\Delta y) = \langle d(x,y,t) d(x+n\Delta x, y+m\Delta y, t)^\ddagger \rangle,$$
$$= \frac{1}{L} d(x,y,t) d(x+n\Delta x, y+m\Delta y, t)^\ddagger,$$

and k_x and α_x, respectively, and k_y and α_y are wave-numbers in the x and y directions. † and ‡ denote the complex conjugate and matrix transpose, respectively. L is the number of time records, N and M are the number of space in x and y directions ($N^* = 2N-1$ and $M^* = 2M-1$; X^+ and X^- indicate $-X+1$ and $X-1$).

Examples:

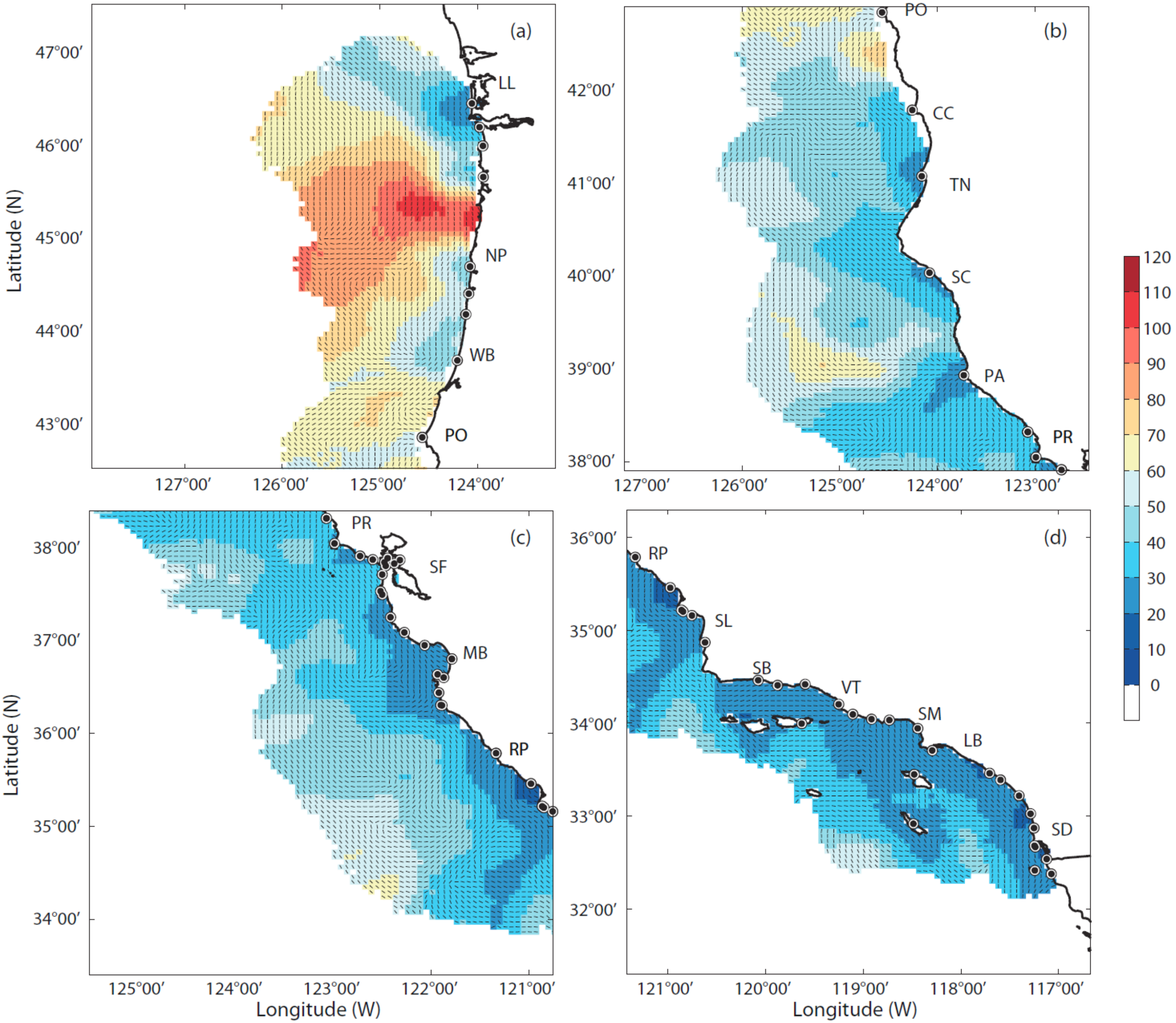
$$c(x) = e^{-\frac{x^2}{\lambda^2}},$$
$$E(k) = \sqrt{\pi}\lambda e^{-\frac{1}{4}k^2\lambda^2}$$

Gaussian covariance ⟷ Gaussian wavenumber spectra

$$c(x) = e^{-\frac{|x|}{\lambda}},$$
$$E(k) = \frac{2\lambda}{1+k^2\lambda^2}$$

Exponential covariance ⟷ (approximate) k-2 wavenumber spectra

Spatial covariance and decorrelation length scales



Scale-by-scale energy budget equation

$$\frac{\partial}{\partial t}E(k^*) + \Pi(k^*) = -2\nu\Omega(k^*) + F(k^*),$$

(Frisch 1995)

where

$$E(k^*) = \frac{1}{2}\sum_{|\mathbf{k}|<k^*} |\hat{\mathbf{u}}(\mathbf{k})|^2,$$ Cumulative kinetic energy

$$\Pi(k^*) = \langle \mathbf{u}_< \cdot (\mathbf{u} \cdot \nabla \mathbf{u})\rangle,$$ Cumulative advective kinetic energy flux

$$= \langle \mathbf{u}_< \cdot (\mathbf{u}_< \cdot \nabla \mathbf{u}_>)\rangle + \langle \mathbf{u}_< \cdot (\mathbf{u}_> \cdot \nabla \mathbf{u}_>)\rangle,$$

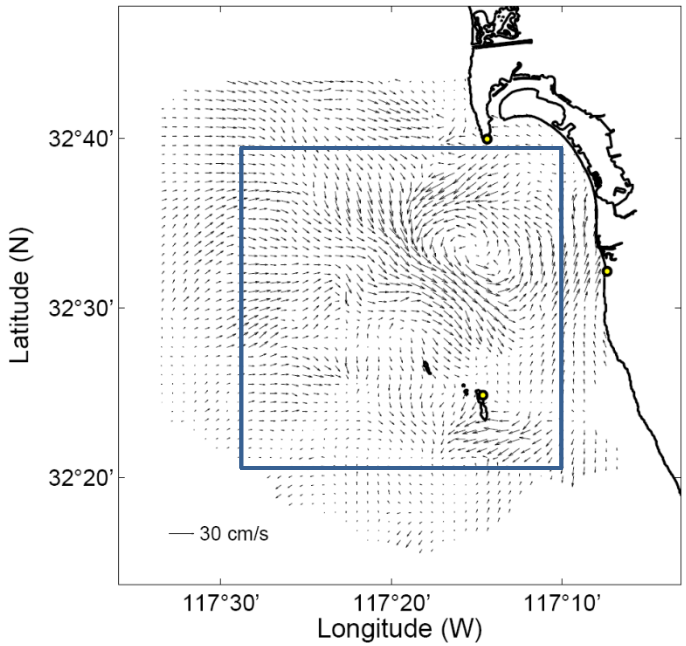
$$\Omega(k^*) = \frac{1}{2}\sum_{|\mathbf{k}|<k^*} \mathbf{k}^2|\hat{\mathbf{u}}(\mathbf{k})|^2,$$ Cumulative enstrophy

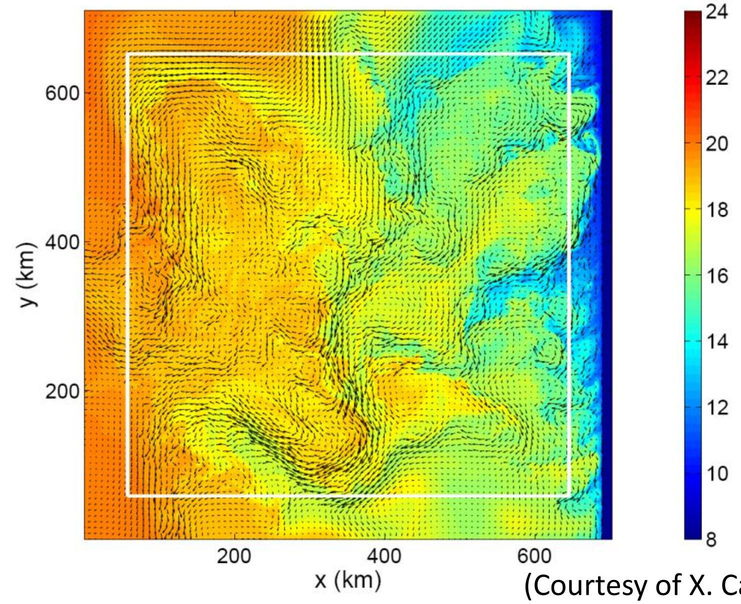
$$\mathbf{u}(\mathbf{x}) = \mathbf{u}_<(\mathbf{x}) + \mathbf{u}_>(\mathbf{x}),$$

$$= \sum_{|\mathbf{k}|<k^*} \hat{\mathbf{u}}(\mathbf{k})e^{i\mathbf{k}\mathbf{x}} + \sum_{|\mathbf{k}|>k^*} \hat{\mathbf{u}}(\mathbf{k})e^{i\mathbf{k}\mathbf{x}},$$

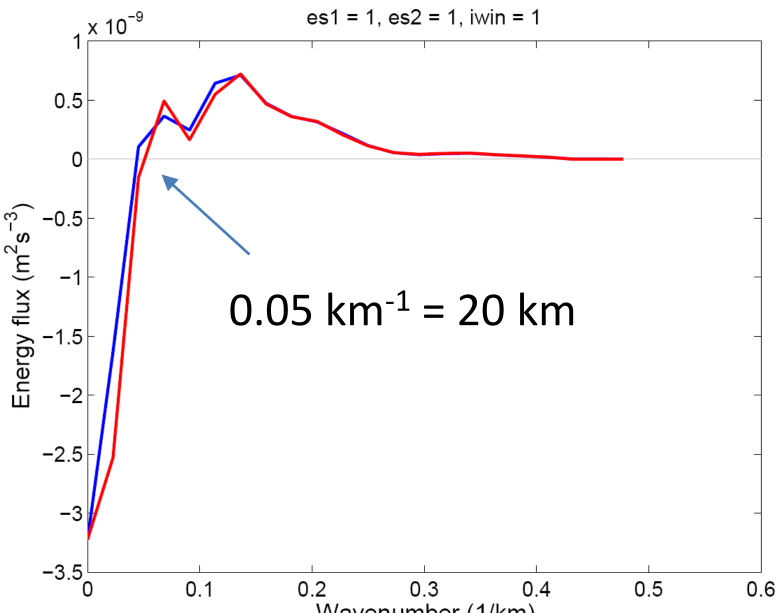
- Surface currents from HFR observations (1 km) and sub-mesoscale model (0.75 km; X. Capet *et al*, 2009) off southern California

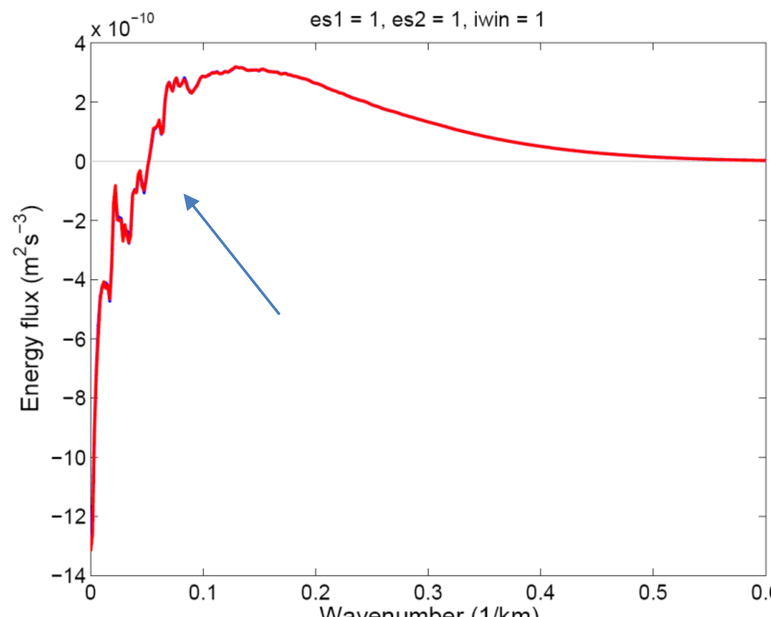
Comparison of advective kinetic energy flux [Π(k*)]





(Courtesy of X. Capet)





Summary

- Energy spectra at mesoscale and sub-mesoscale are examined with altimeter-, high-frequency radar-, shipboard ADCP-derived (coastal) currents.
- The operational HFR network provides the detailed aspects of coastal surface circulation and ocean dynamics at a resolution (km in space and hourly in time) containing responses to the low frequency, tides, wind forcing, and Earth rotation.
- The spatial covariance appears as an anisotropic exponential shape with decorrelation length scales of 20 km nearshore and 100 km offshore parallel to the shoreline, consistent with approximate k-2 and k-3 decay behavior.
- Energy fluxes computed from sub-mesoscale [O(1) km] HFR observations and numerical model results show consistent forward cascades at O(1-10) km scale.


Thank you!
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