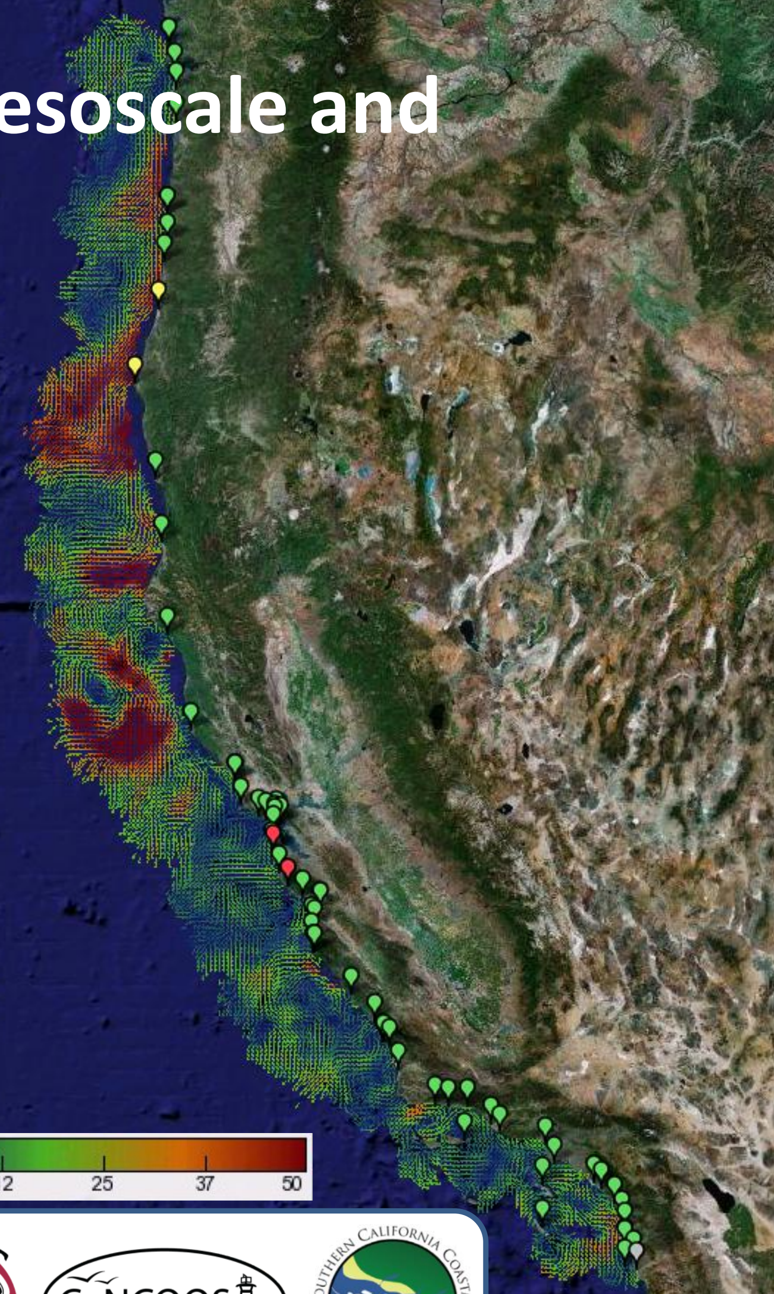
Characterization of oceanic mesoscale and sub-mesoscale energy spectra

Sung Yong Kim and Eric Terrill
Marine Physical Laboratory
Scripps Institution of Oceanography
syongkim@mpl.ucsd.edu

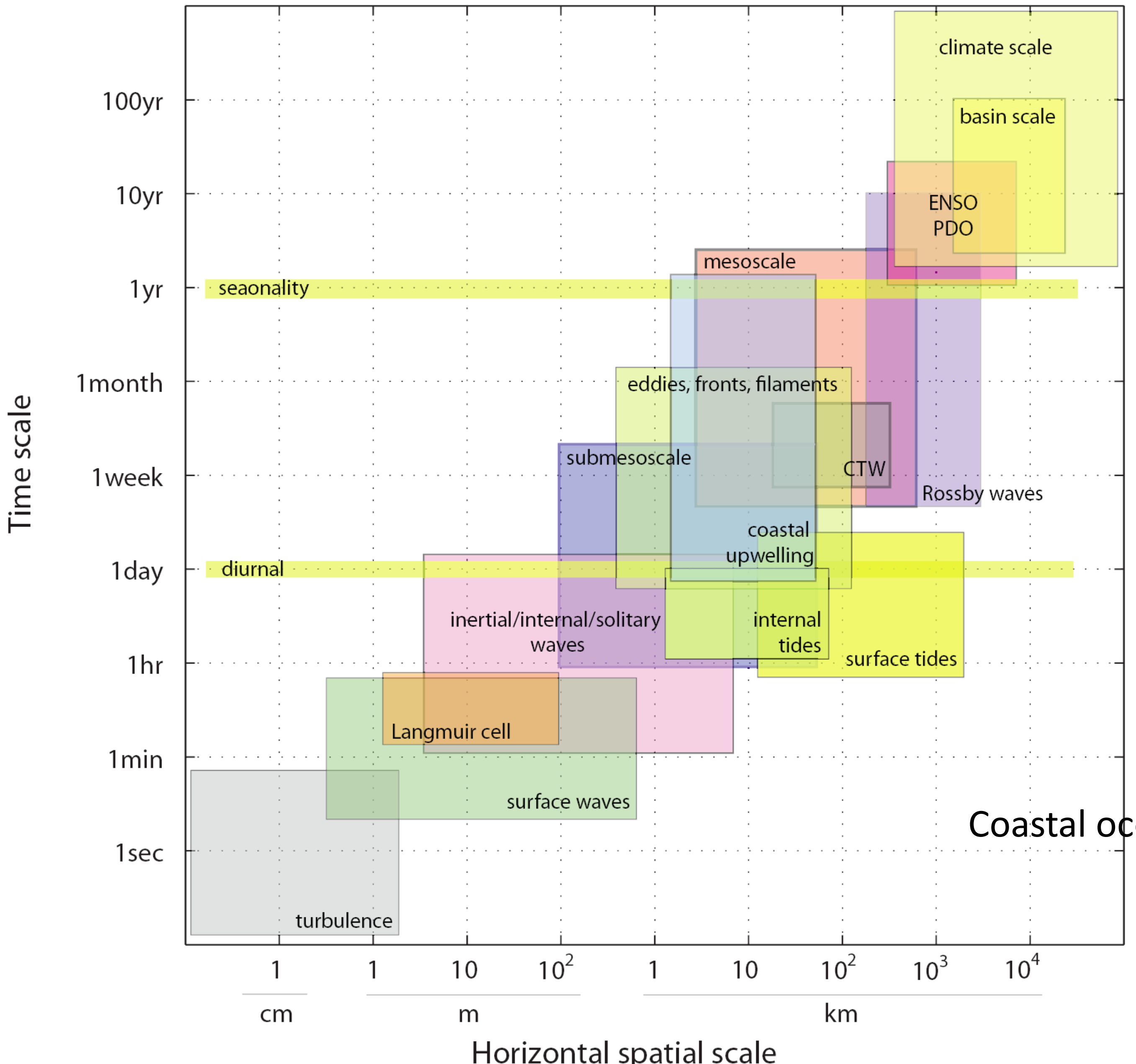
Collaborators:
Bruce Cornuelle, Burt Jones, Libe Washburn, Mark Moline,
Jeffrey Paduan, Toby Garfield, John L. Largier, Greg Crawford
P. Michael Kosro, and Xavier Capet



Outline

- Motivation
  - Oceanic processes in time and spatial scales
  - Wavenumber energy spectra derived from satellite altimeters
- Overview
  - Coastal surface current measurements using high-frequency radar (HFR)
  - Spectral contents in coastal surface currents off the U.S. West Coast
- Comparison of energy spectra
  - Energy spectra in wavenumber (1D) domain
  - Conversion between covariance and spectra (2D; anisotropy?)
  - Decorrelation length scales
- Summary

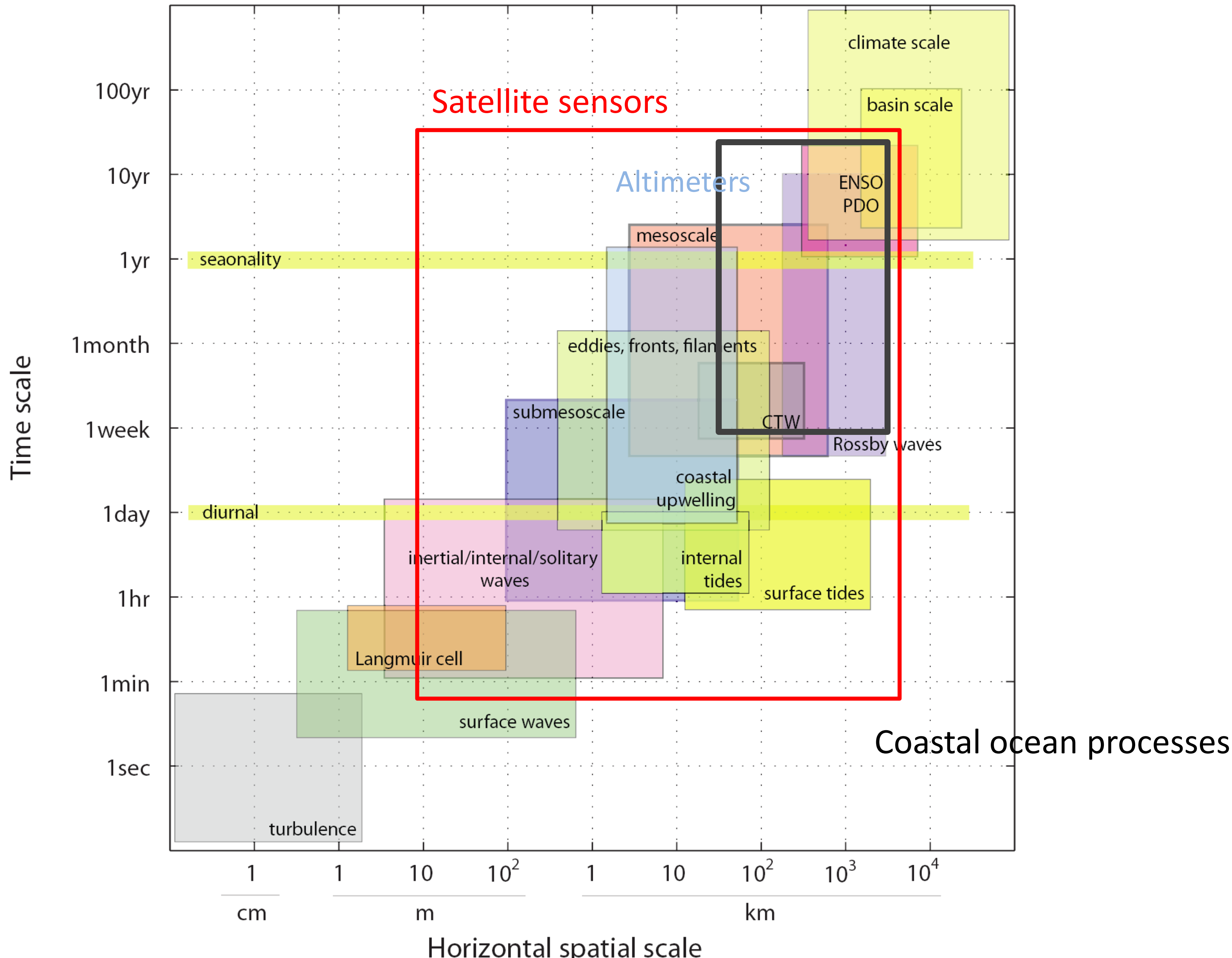
Oceanic processes in time and spatial scales



Coastal ocean processes

(Chelton 2001, Dickey *et al*, RG 2006)

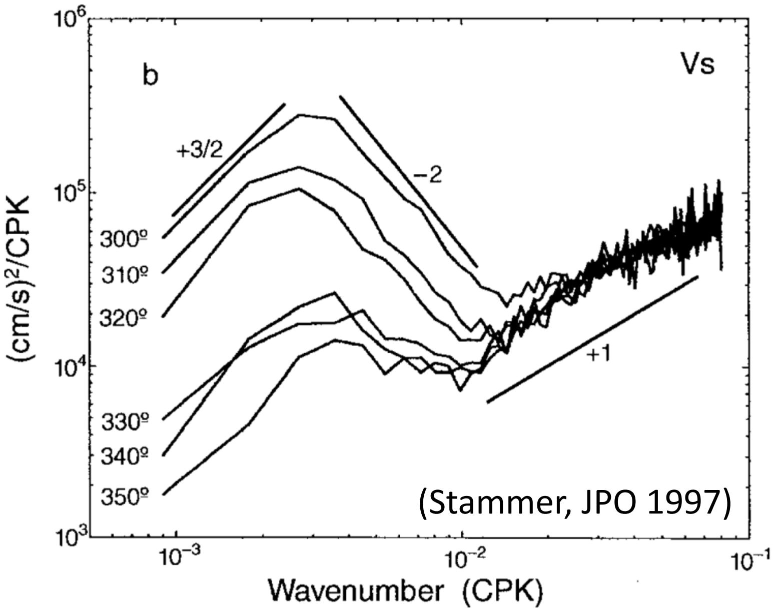
Oceanic processes in time and spatial scales



(Chelton 2001, Dickey *et al*, RG 2006)

Motivation

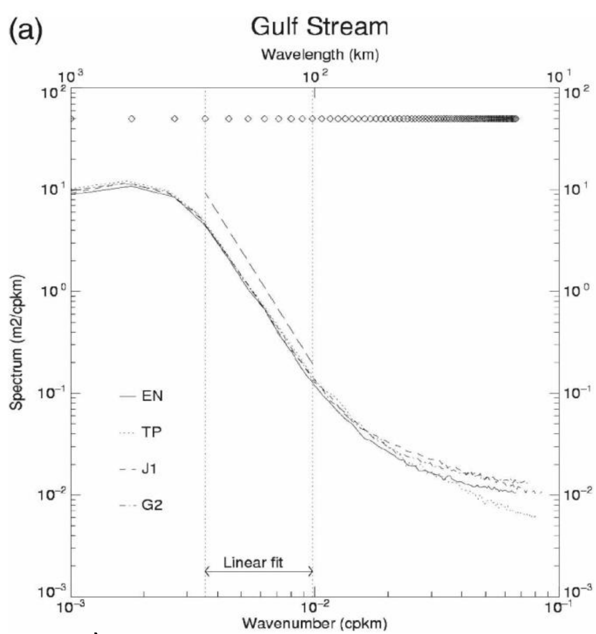
- Slope of energy spectra at sub-mesoscale ?
- Quasi-geostrophic (QG) vs. surface QG (sQG) theory?



(Stammer, JPO 1997)

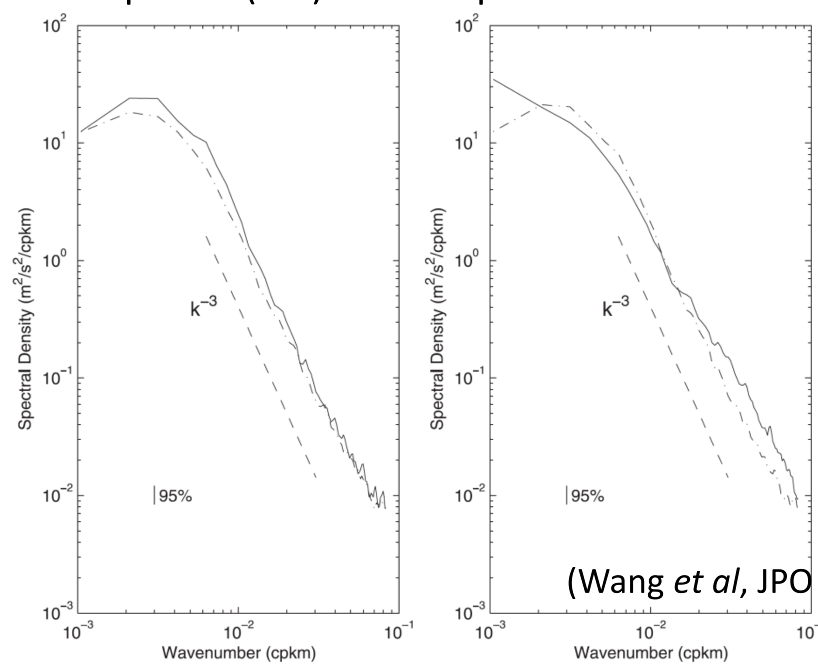
Wavenumber spectra of altimeter-derived cross-track geostrophic currents (30N to 40 N)

K-5/3 spectra at high eddy energy region



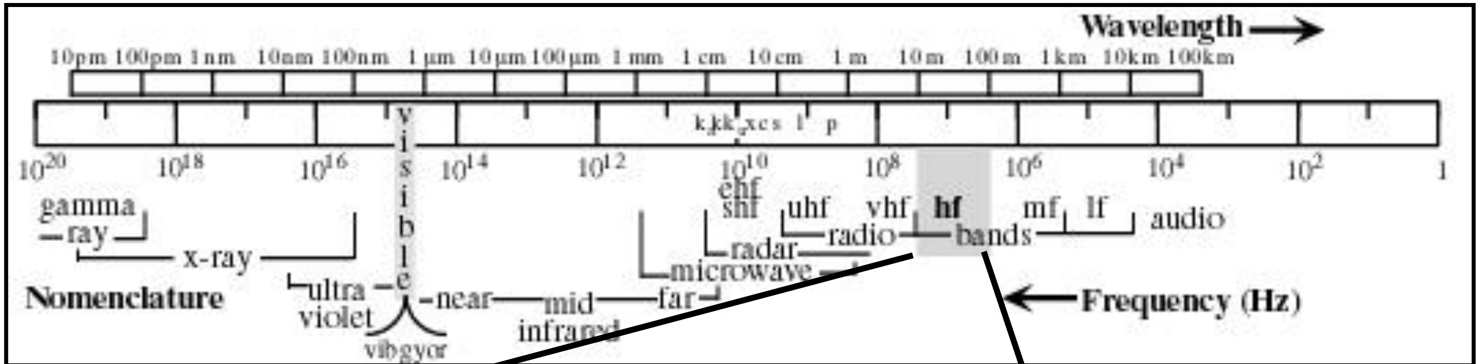
(Le Traon *et al*, JPO 2008)

Spectra (k-3) from shipboard ADCP off MAB



(Wang *et al*, JPO 2010)

Radio signals used in high-frequency radar



(Paduan and Graber, Oceanography 1997)

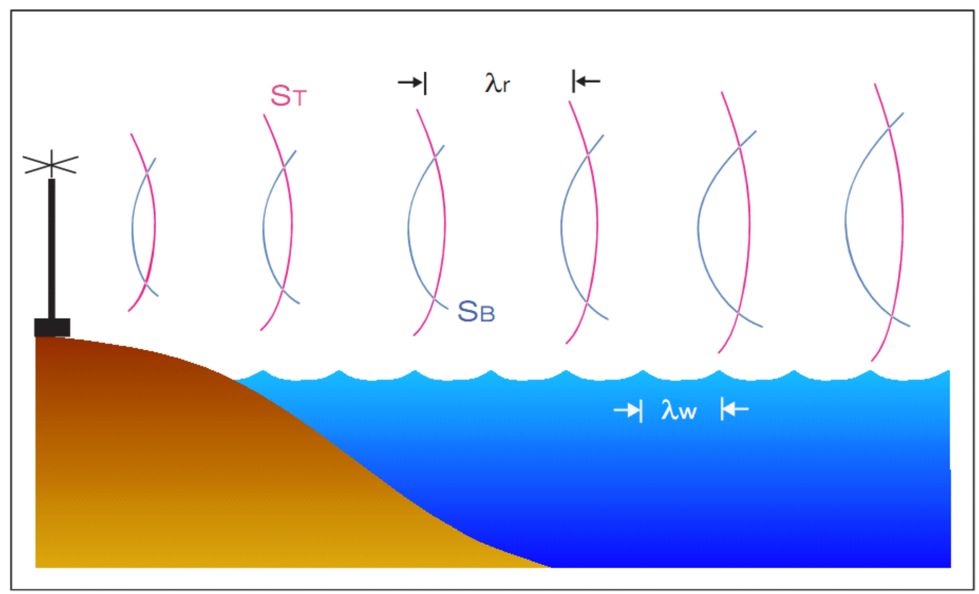
3-30 MHz (between AM radio and TV)

Wavelength (λ_r) : 10 ~ 100 (m)

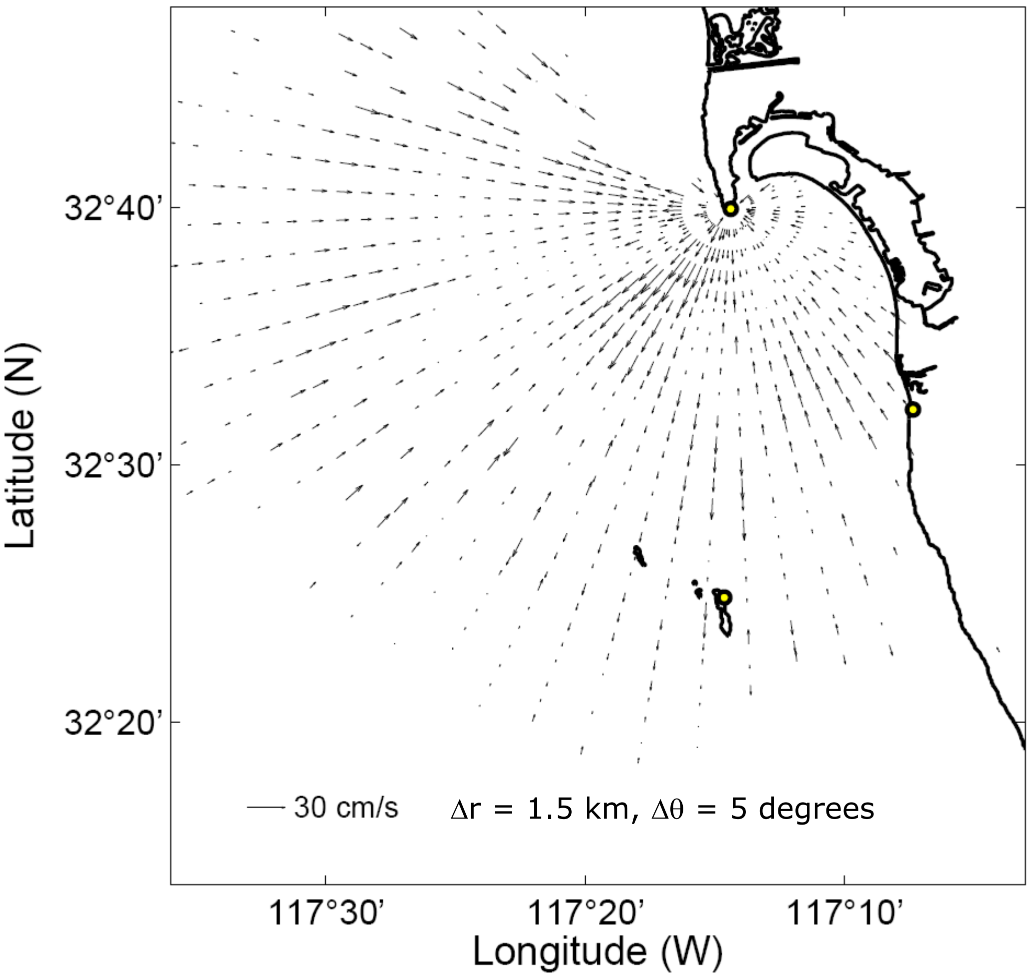
Bragg backscattering

When the radar signals are backscattered in phase,

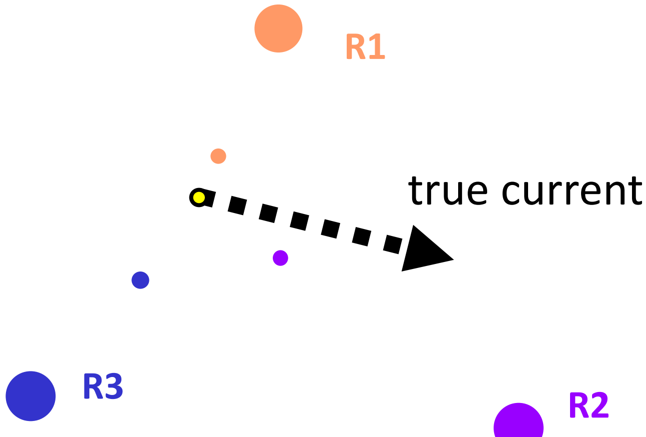
$$\lambda_w = \lambda_r / 2$$



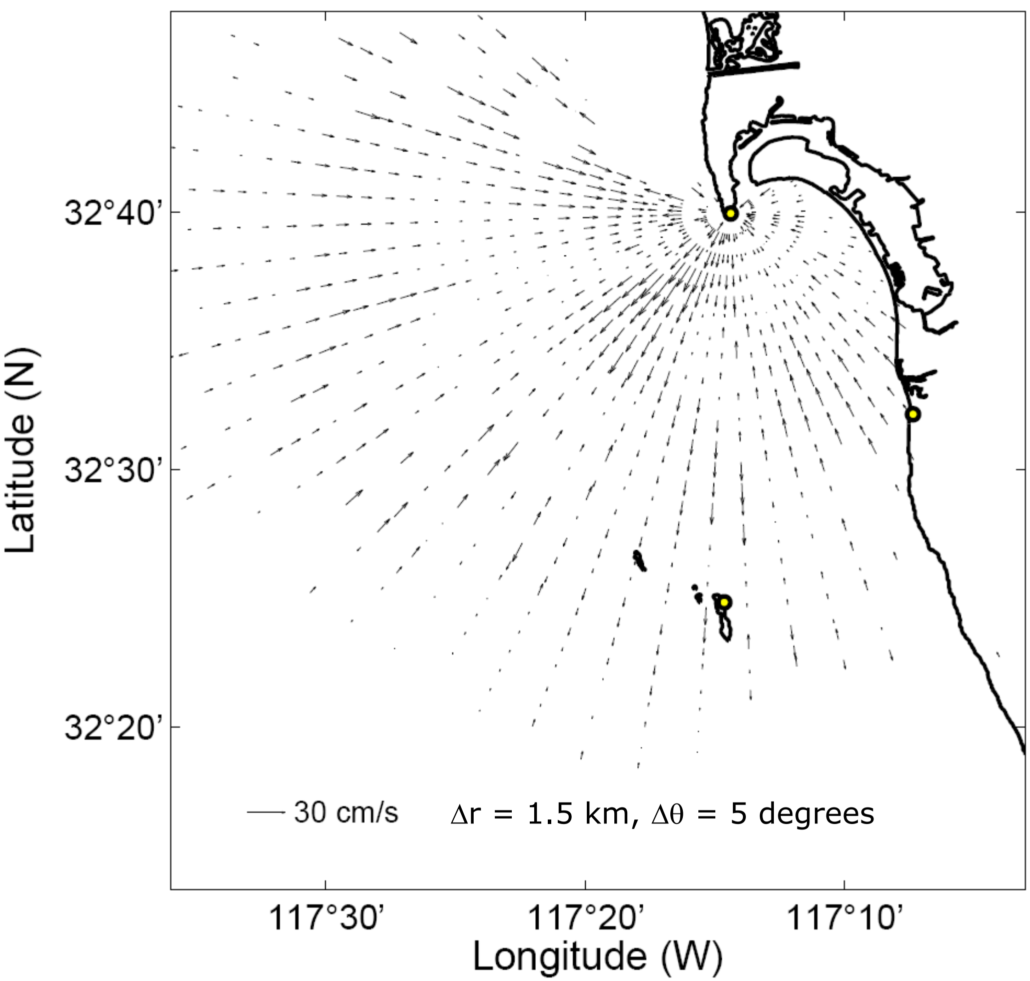
Surface radial current map



- Range
  - Operating and sweeping frequency
- Angle
  - Direction finding v.s. MUSIC
- Radial velocity
  - Doppler shift
  - Projected current component



Surface radial current map



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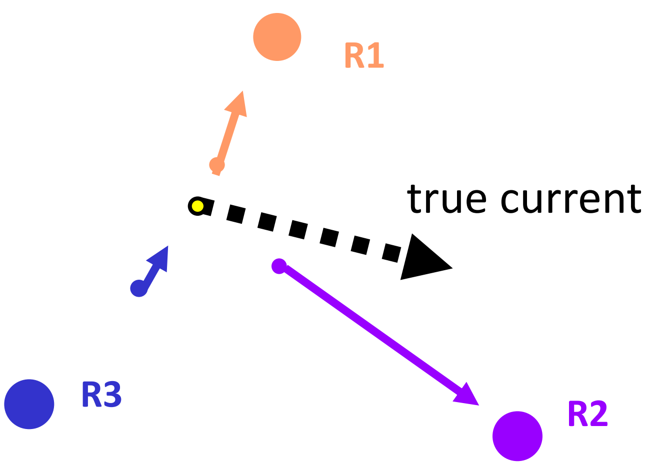
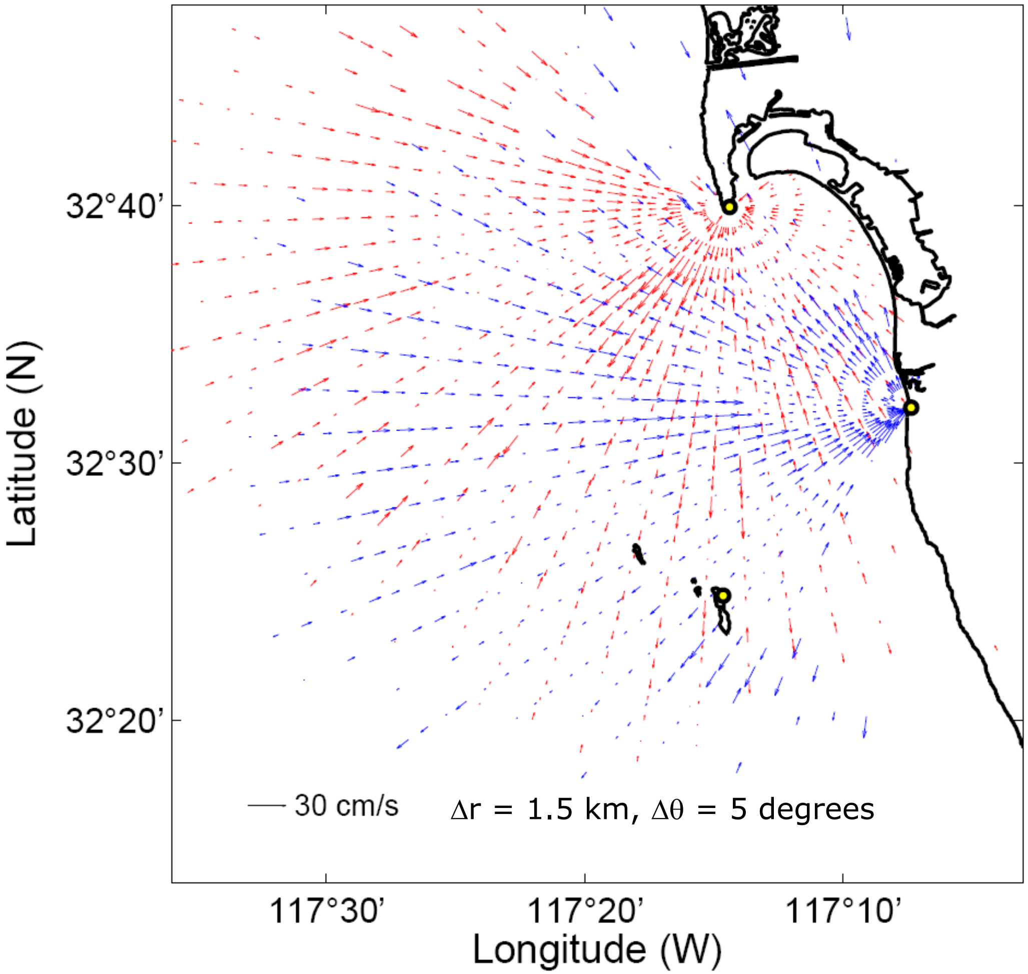


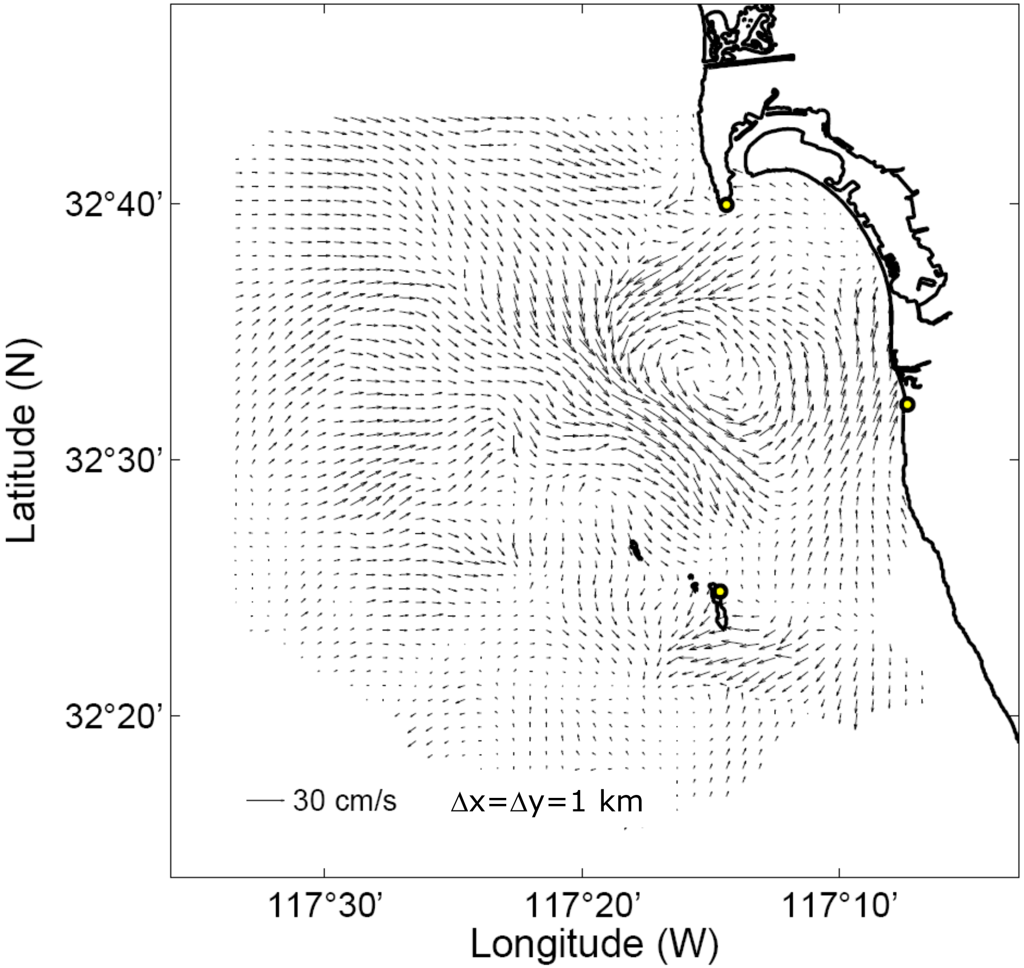
Multiple surface radial current maps



- Vector current map estimates
  - Un-weighted least squares fit (UWLS)
  - Optimal interpolation (OI)

(Kim *et al*, JGR 2008; Kim, CSR 2010)

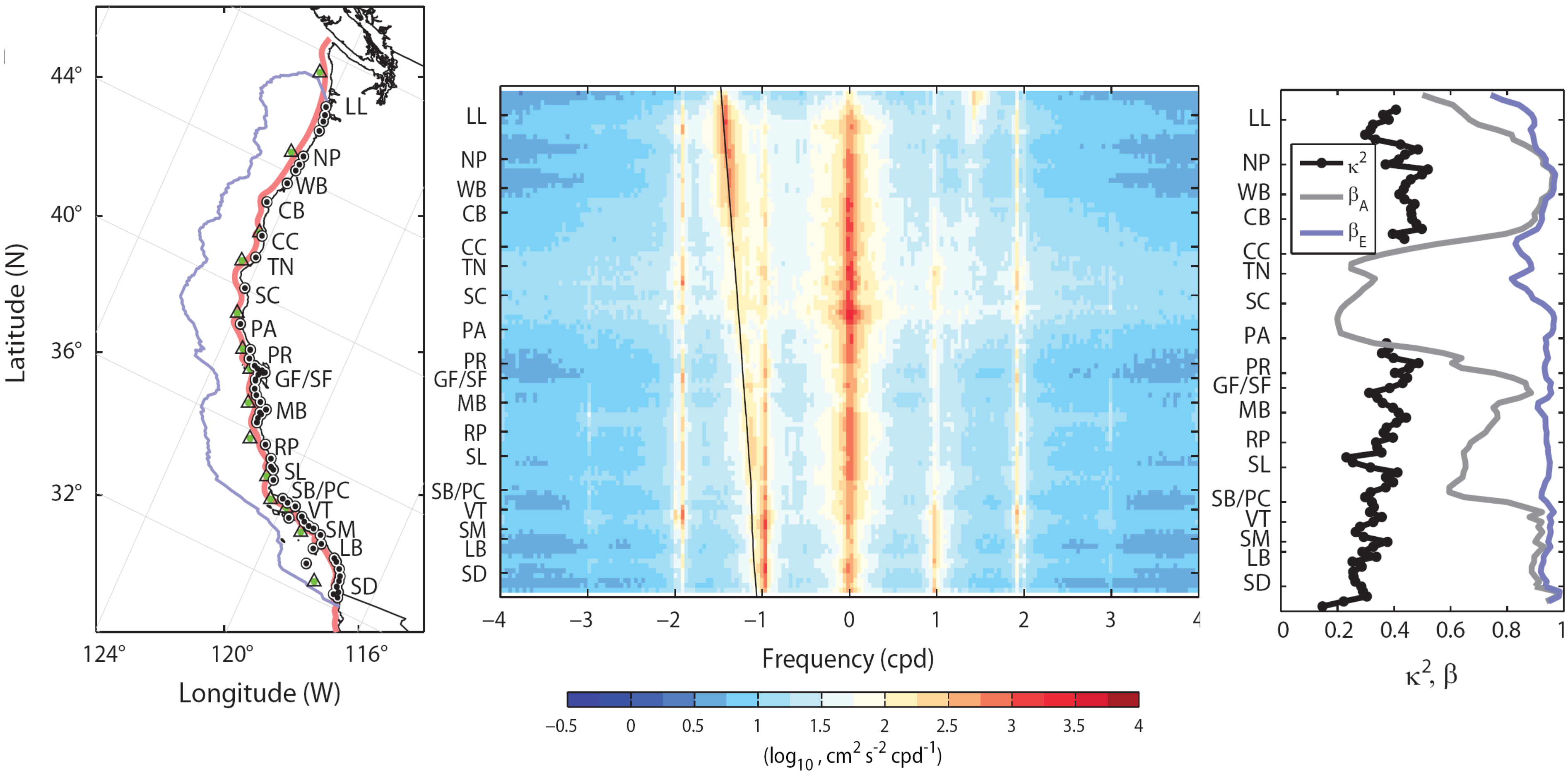
Improved vector current map



- Optimal interpolation
  - Minimize baseline inconsistency
  - A unified uncertainty definition
  - Divergence and vorticity
  - Velocity potential and stream function
  - Exponential correlation function (based on data covariance matrix) leads to minimum level of spatial smoothing.

(Kim *et al*, JGR 2008; Kim, CSR 2010)

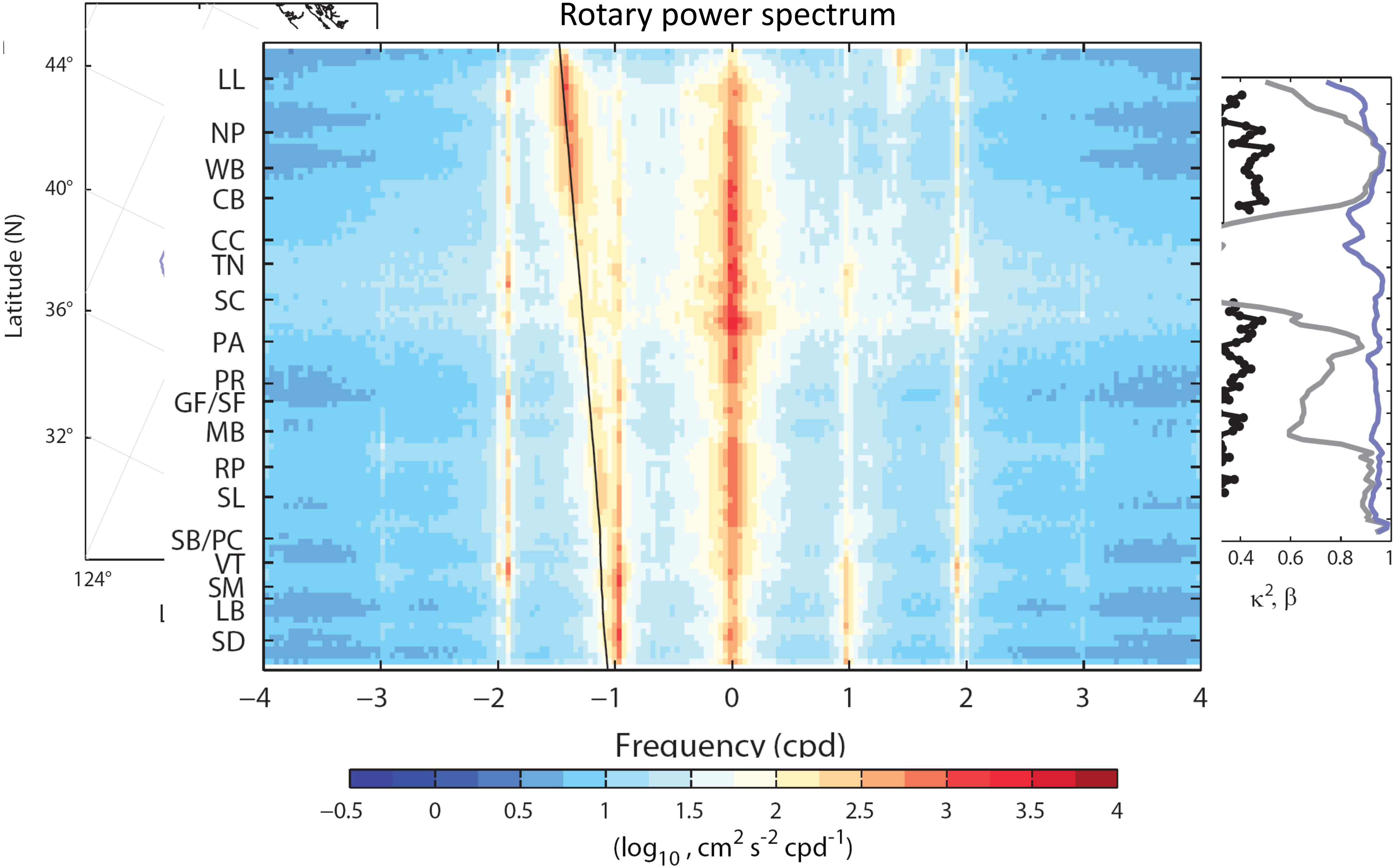
Variance of surface currents (alongshore view)



- 60+ compact array HFR (CODAR) system
- Hourly surface current maps (0.5, 1, 2, and 6 km resolution)
- Upper 1 m depth averaged currents
- From nearshore to 50 - 150 km offshore

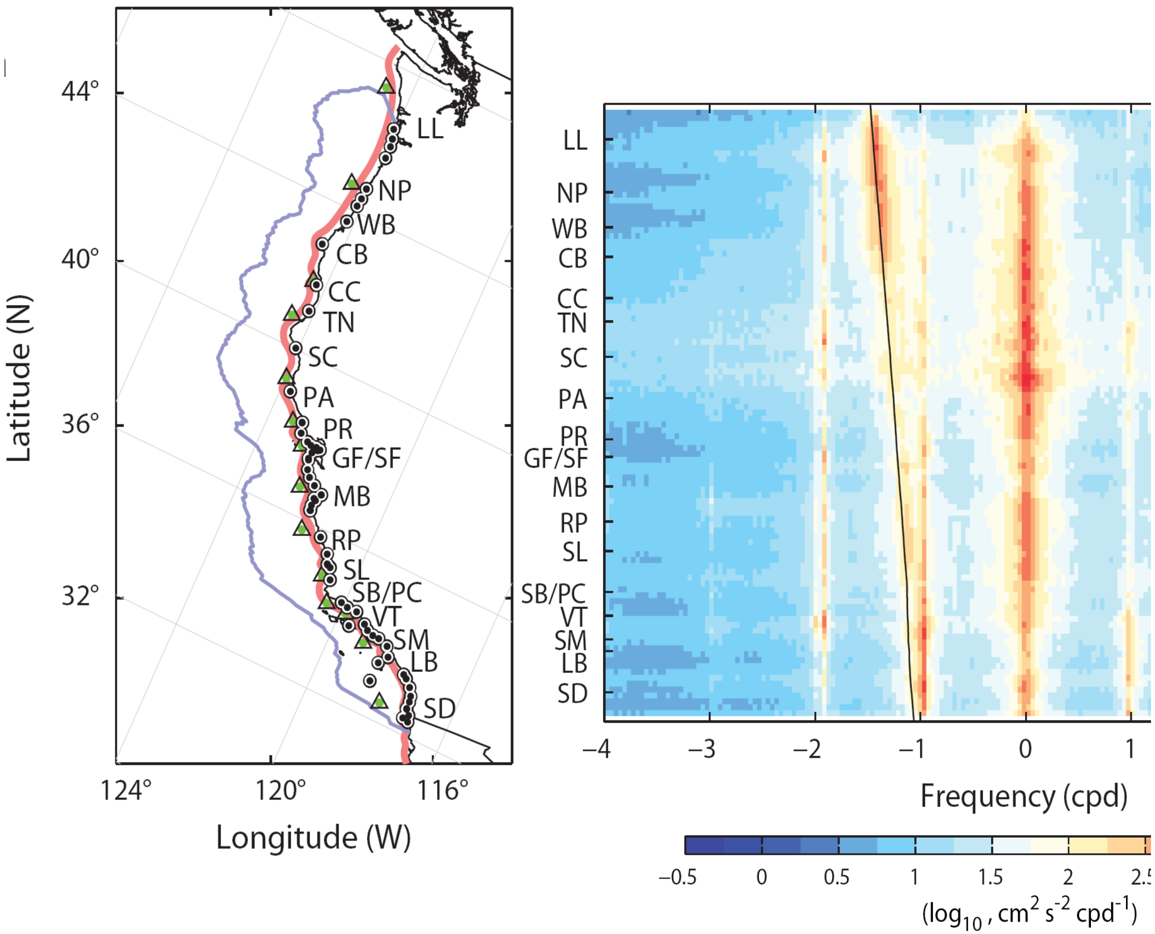
(Kim *et al*, JGR 2011)

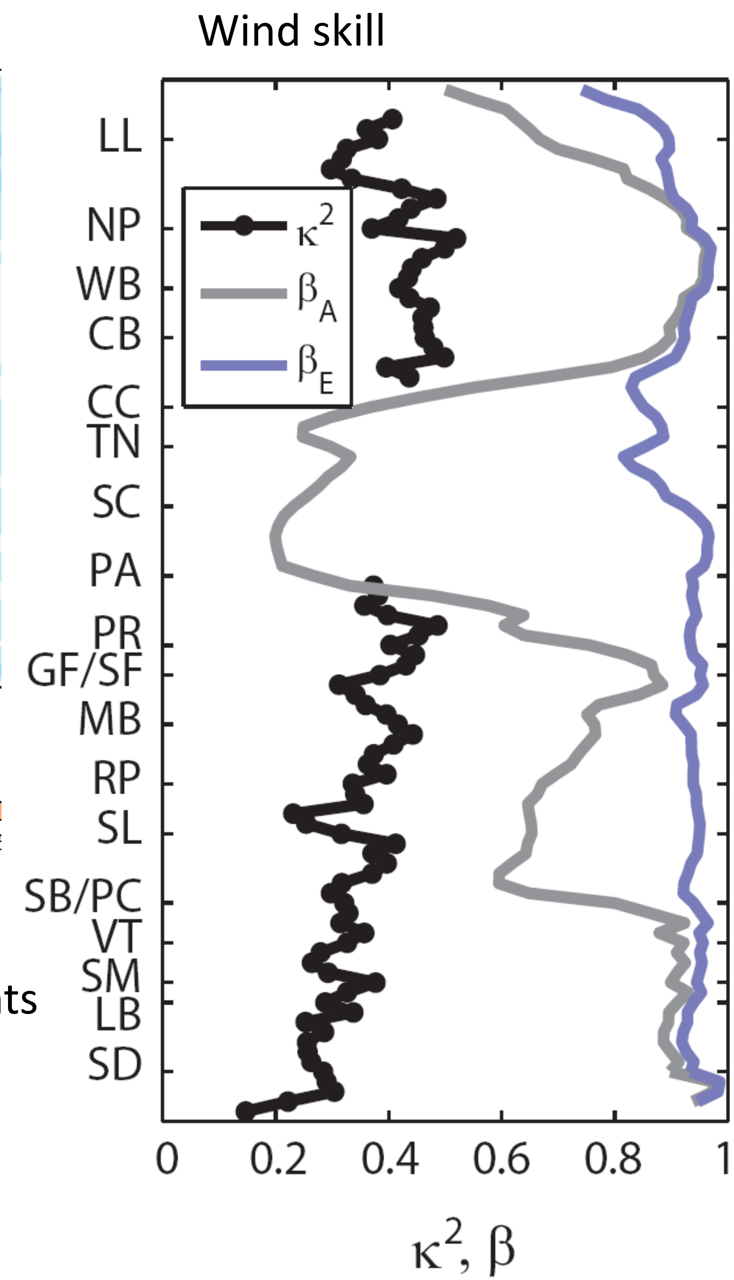
Variance of surface currents (alongshore view)



- Variance coherent with tides, wind, low frequency signals, and Coriolis force.
- Regional noise levels

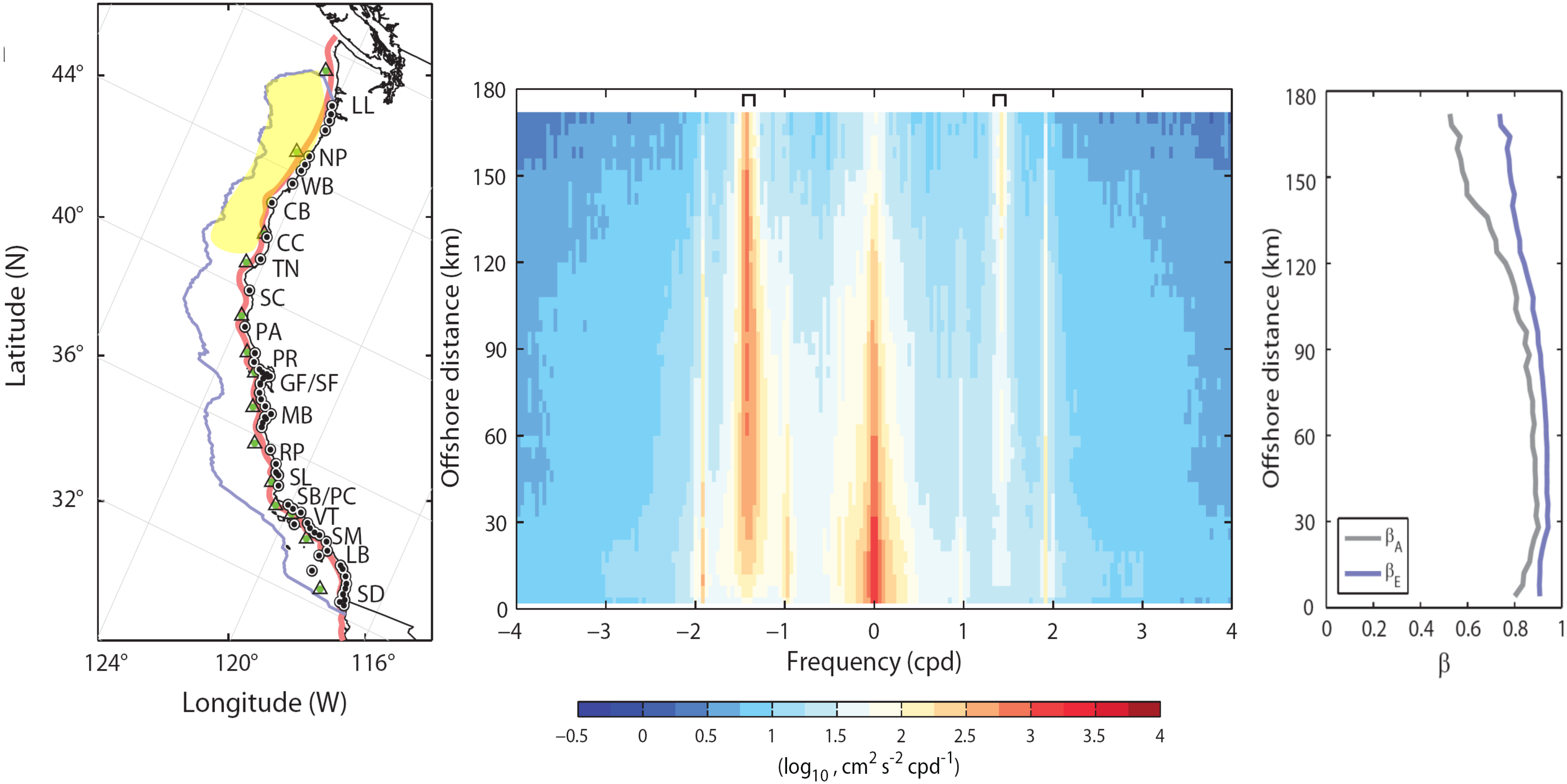
Variance of surface currents (alongshore view)





- Wind skill explains how much variance of surface currents can be explained by wind (20 – 40%).
- Skill is aligned with the local wind variance.

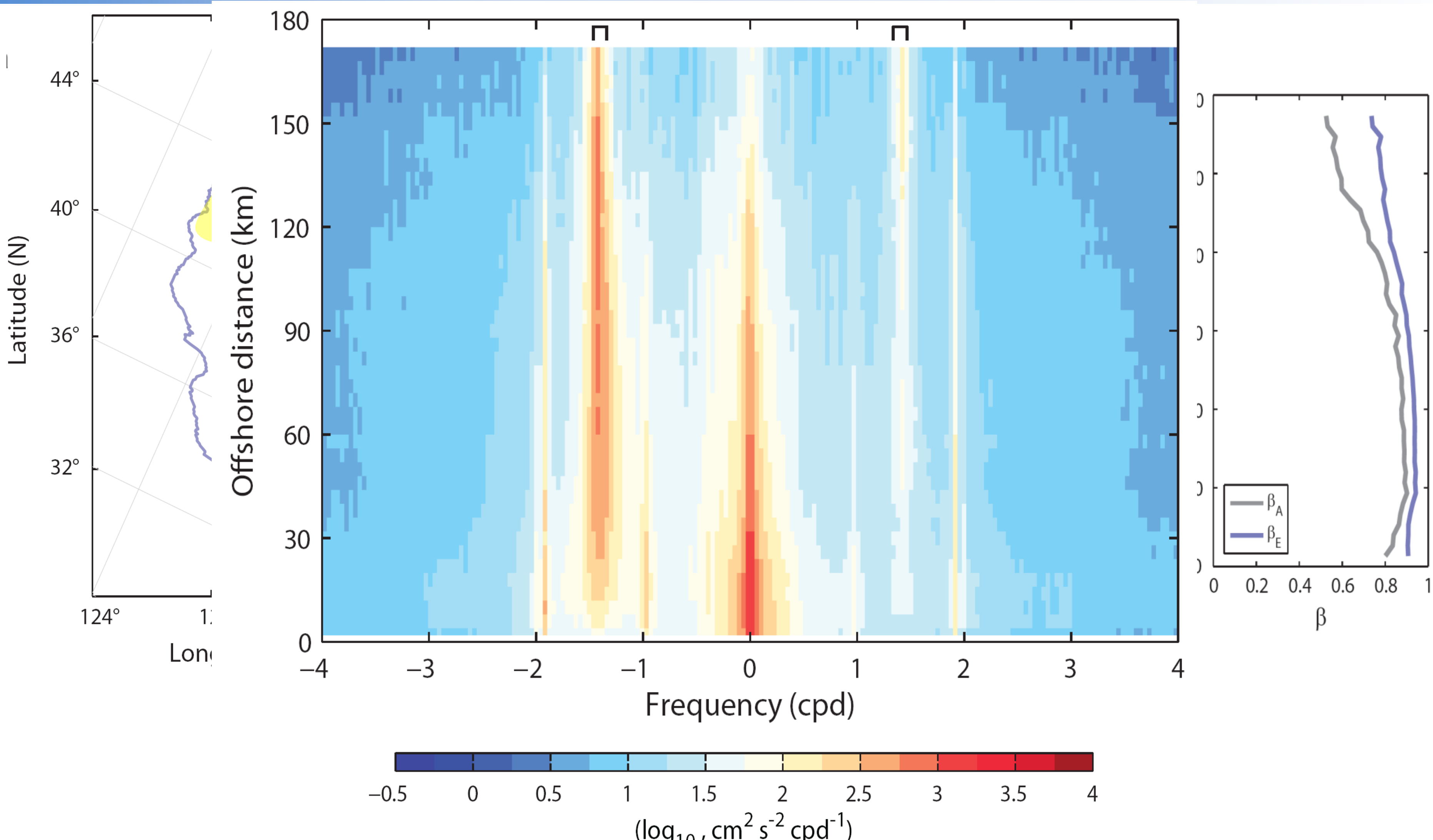
Variance of surface currents (cross-shore view)



- Cross-shore variation of tide-, wind-, low frequency-forced energy
- Low frequency pressure setup against the coast
- Inertial variance gets narrow offshore
- Variance of tide-coherent currents decrease with offshore distance

(Kim *et al*, JGR 2011)

Variance of surface currents (cross-shore view)



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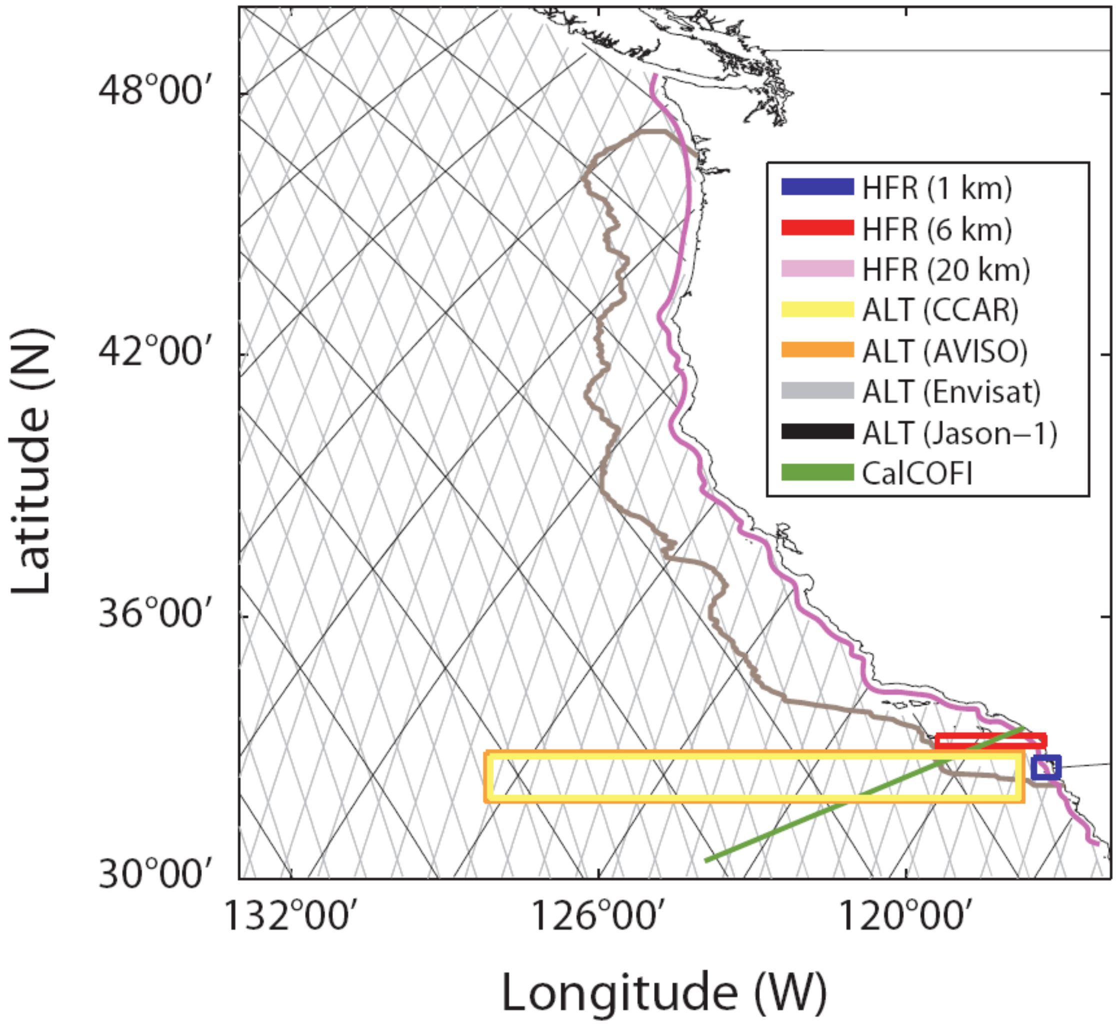
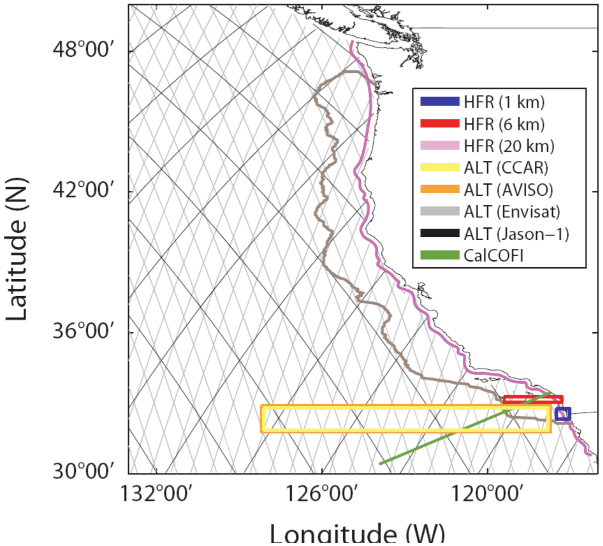
(Kim *et al*, JGR 2011)

Sampling domain in computation of energy spectra



- HFR surface currents (1, 6, and 20 km resolution) off southern California and on coastline axis (USWC)
- Gridded ALT products (CCAR and AVISO) and along-track altimeter (ALT; Envisat/Jason-1) on NE Pacific
- CalCOFI shipboard ADCP (Line 90)
- SoCAL was chosen because it contains relatively minimum ageostrophic components.

Energy spectra in the wavenumber domain (1D)

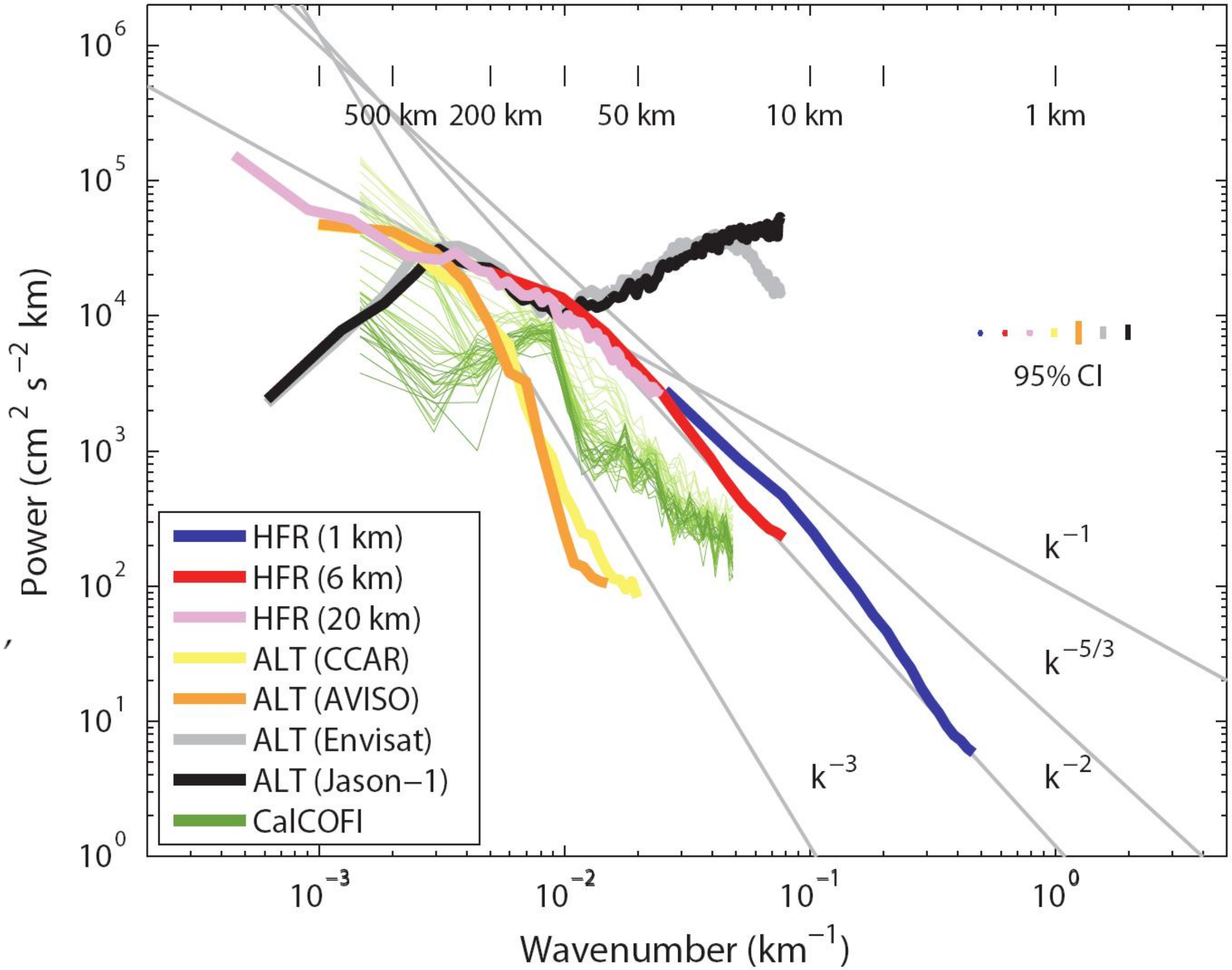


$$S_{\mathbf{u}_\perp}(k_\parallel) = \left(\frac{g}{f_c}\right)^2 \left(2\pi k_\parallel\right)^2 S_{\boldsymbol{\eta}_\parallel}(k_\parallel),$$

Power spectrum of cross-track geostropic currents from along-track SSHAs

K^{-2} power law related to sub-mesoscale.

Robust estimate on k-2 spectra with data in other regions.



Two kinds of ALT data: Envisat and Jason-1
HFR data with three resolutions:
1 km and 6 km data are sampled from SoCAL,
because minimum ageostropic components are expected.
20 km data are from the coastline axis.

Conversion between covariance and power spectra

$$E_\circ(k_x, k_y) = \mathcal{F}(d)^\dagger \mathcal{F}(d),$$
$$= \frac{1}{\Delta k_x}\frac{1}{\Delta k_y}\left|\frac{1}{NM}\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} d(x_n, y_m) e^{-ik_x x_n - ik_y y_m}\right|^2$$

$$E_\bullet(\alpha_x, \alpha_y) = |\mathcal{F}(c)|,$$
$$= \frac{1}{\Delta\alpha_x}\frac{1}{\Delta\alpha_y}\left|\frac{1}{N^* M^*}\sum_{n=N^-}^{N^+}\sum_{m=M^-}^{M^+} c(n\Delta x, m\Delta y) e^{-i\alpha_x n\Delta x - i\alpha_y m\Delta y}\right|$$

Wiener-Khinchin theorem

where

$$c(n\Delta x, m\Delta y) = \langle d(x,y,t) d(x+n\Delta x, y+m\Delta y, t)^\ddagger\rangle,$$
$$= \frac{1}{L} d(x,y,t) d(x+n\Delta x, y+m\Delta y, t)^\ddagger,$$

and k_x and α_x, respectively, and k_y and α_y are wave-numbers in the x and y directions. † and ‡ denote the complex conjugate and matrix transpose, respectively. L is the number of time records, N and M are the number of space in x and y directions ($N^* = 2N-1$ and $M^* = 2M-1$; X^+ and X^- indicate $-X+1$ and $X-1$).

Examples:

$$c(x) = e^{-\frac{x^2}{\lambda^2}},$$
$$E(k) = \sqrt{\pi}\lambda e^{-\frac{1}{4}k^2\lambda^2}$$

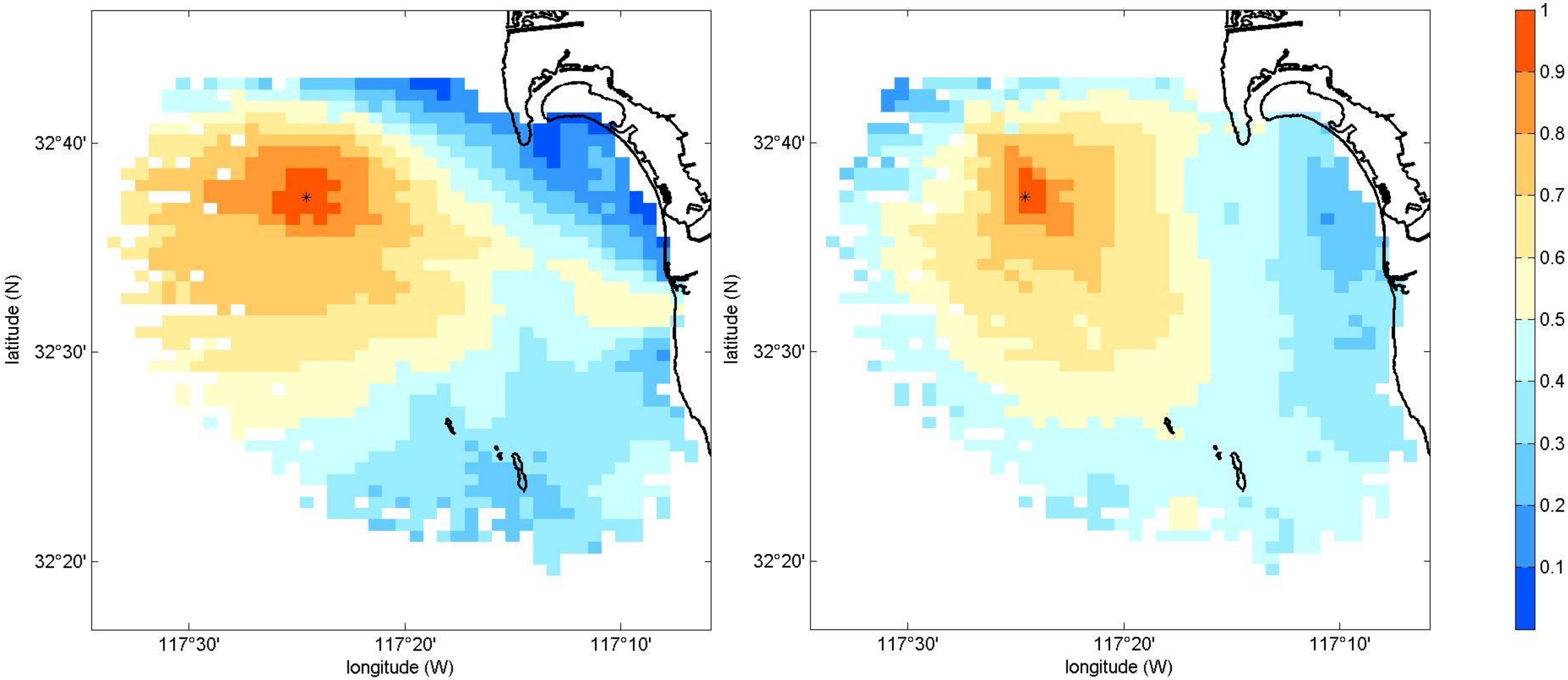
Gaussian covariance ↔ Gaussian wavenumber spectra

$$c(x) = e^{-\frac{|x|}{\lambda}},$$
$$E(k) = \frac{2\lambda}{1+k^2\lambda^2}$$

Exponential covariance ↔ (approximate) k-2 wavenumber spectra

Spatial correlations of surface currents

$\rho_{uu}(\mathbf{x}, \mathbf{y}, x', y')$ **offshore** $\rho_{vv}(\mathbf{x}, \mathbf{y}, x', y')$

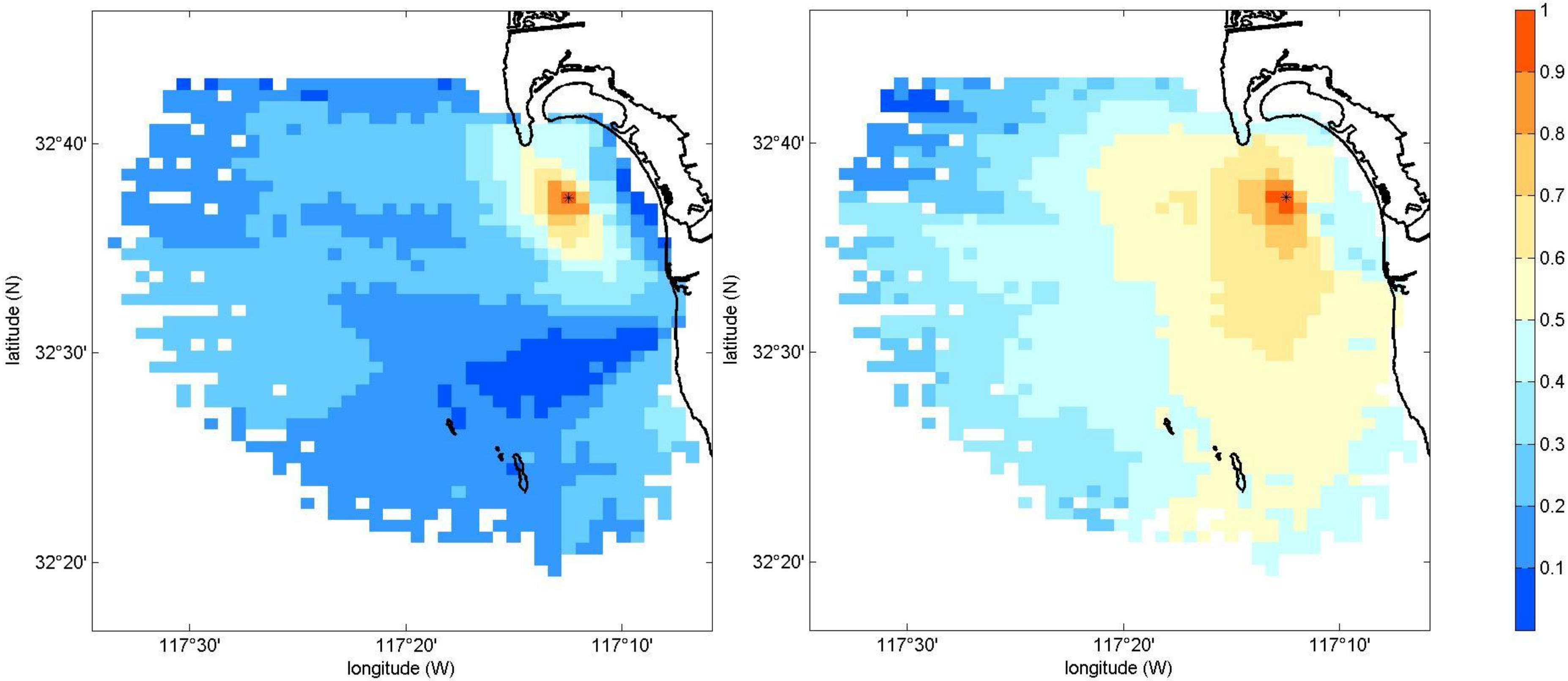


- Correlations of the 2-year vector current data

(Kim *et al*. JGR, 2007)

Spatial correlations of surface currents

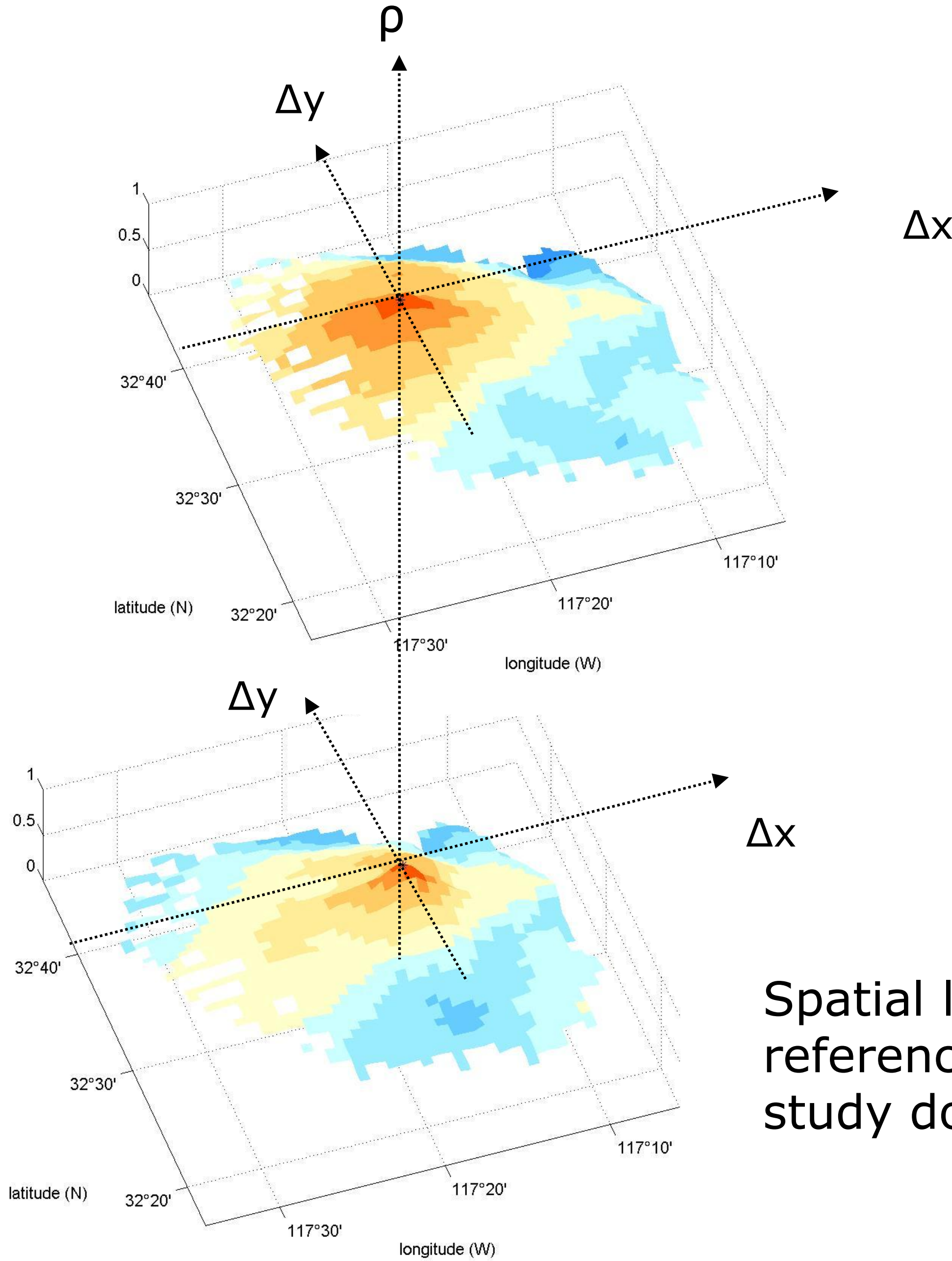
$\rho_{uu}(\mathbf{x}, \mathbf{y}, x', y')$ **nearshore** $\rho_{vv}(\mathbf{x}, \mathbf{y}, x', y')$



- De-correlation scale is the function of space (x, y).

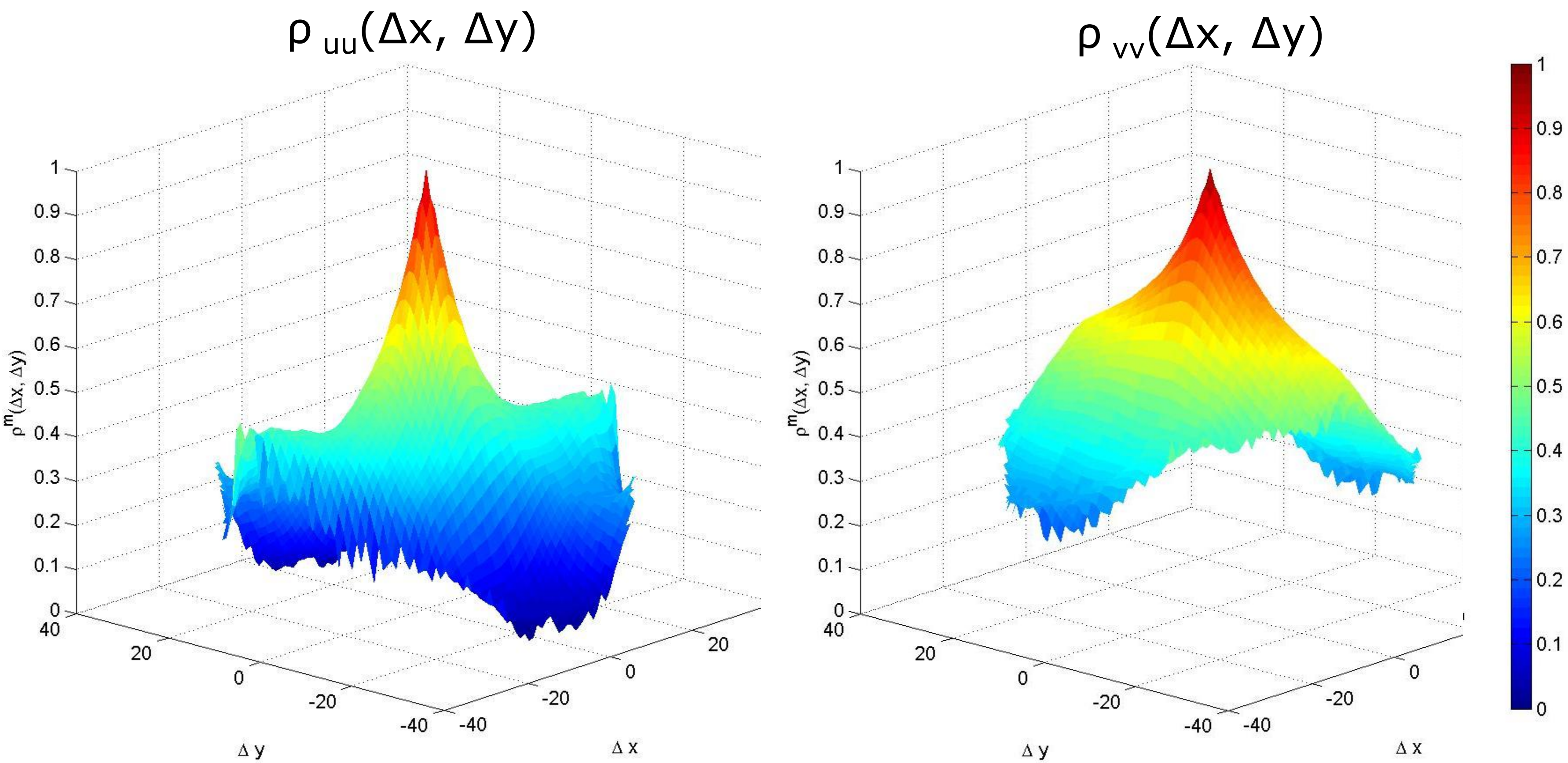
(Kim *et al*. JGR, 2007)

Spatial averaging of correlations



Spatial lag average over all reference grid points in the study domain

Spatially averaged correlations of surface currents

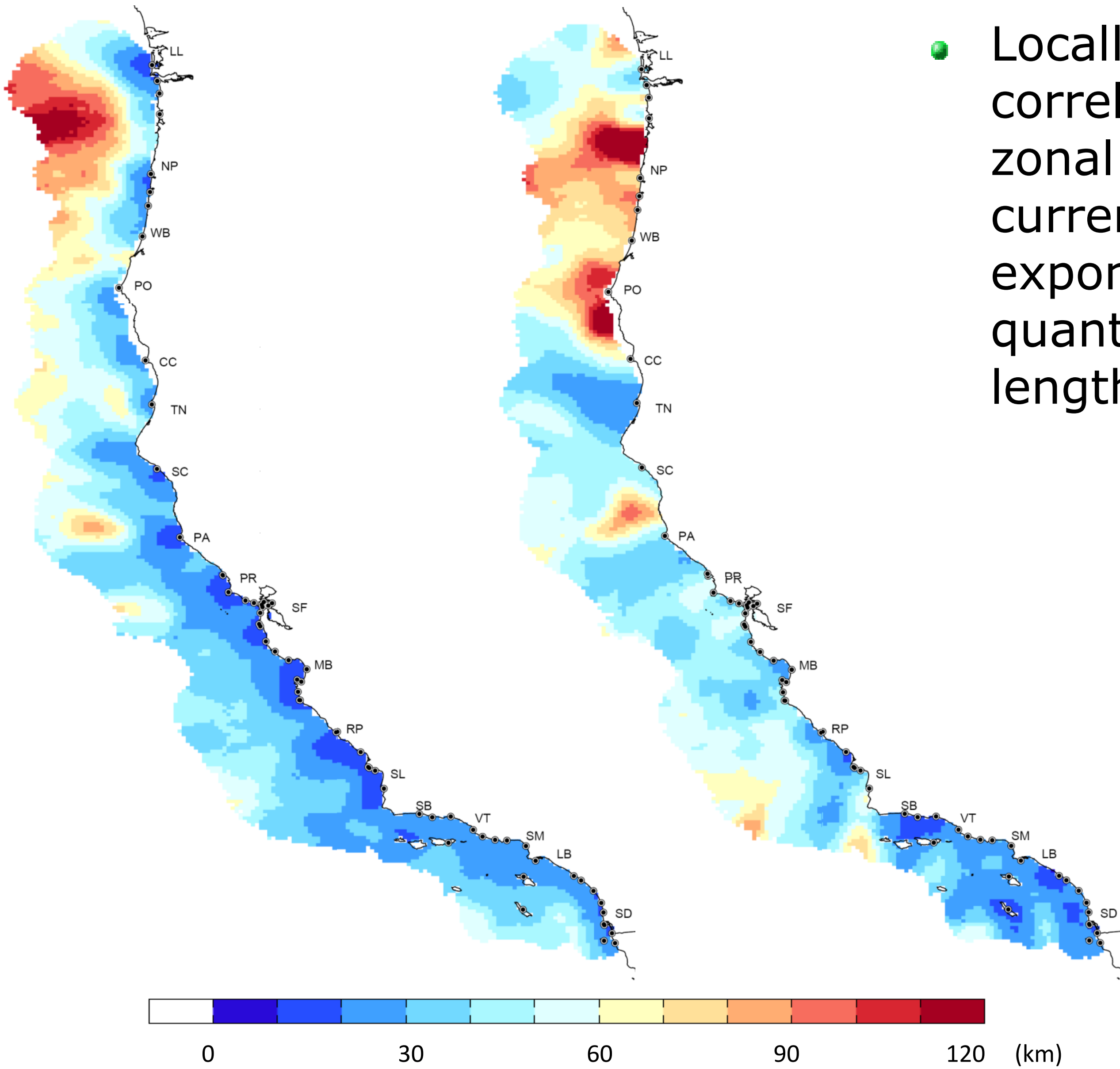


- Spatially composite correlation over the study domain
- Exponential shape (not Gaussian)

(Kim *et al*. JGR, 2007)

Decorrelation length scales

U (zonal current)   V (meridional current)



- Locally averaged spatial correlation functions of zonal and meridional currents are fitted with exponential functions to quantify decorrelation length scales.

Summary

- Energy spectra at mesoscale and sub-mesoscale are examined with altimeter-, high-frequency radar-, shipboard ADCP-derived (coastal) currents.
- The operational HFR network provides the detailed aspects of coastal surface circulation and ocean dynamics at a resolution (km in space and hourly in time) containing responses to the low frequency, tides, wind forcing, and Earth rotation.
- Due to the noise at 100 km scale in altimeter observations, studies on energy spectra and flux below that scale can be explored with sub-mesoscale observations.
- The spatial covariance appears as an anisotropic exponential shape with decorrelation length scales of 20 km nearshore and 100 km offshore parallel to the shoreline, consistent with approximate k-2 decay behavior.

Thank you!

Sung Yong Kim syongkim@mpl.ucsd.edu