Relevance of submesoscale surface currents from high-resolution sea surface heights in a coastal region

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Courtesy: Alex Kurapov and P. Mike Kosro (OSU)

Submesoscale indicates $O(1\text{-}10)$ km in space and less than several days up to hours in time. (Kim et al, JGR 2011, Kim and Crawford, GRL 2014)
The wavenumber domain energy spectra of the inertial gravity waves and submesoscale processes may have similar spectral decay slopes.

Balanced and unbalanced motions should be considered separately. A transition scale is suggested to divide them.

We may need to develop a proxy of sea surface heights (SSHs) as a tool to extend a high-resolution surface current map into a high-resolution SSH map [as a complement of shore-based tide gauges] and to extract a high-resolution surface current map from a high-resolution SSH map [aligned with upcoming satellite missions, e.g., SWOT, COMPIRA].

We suggest the stream function as a proxy, which satisfy the balanced motions (at least mesoscale; need to evaluate it for submesoscale).
Knowns, unknowns, and suggestions

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IGWs and submesoscale processes

- Frequency-wavenumber-domain energy spectra of Inertia gravity waves (IGWs) and submesoscale processes can be overlapped.

\[ \sigma_j^2 = \frac{N^2 k_h^2 + f_c^2 m_j^2}{k_h^2 + m_j^2} + U \cdot k_h, \]
• ROMS-simulated (hourly, 2km)
• HFR-derived surface currents (hourly, 6 km)
• AVISO optimally interpolated SSHs and geostrophic currents (daily, 25km)
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- We suggest the stream function as a proxy, which satisfy the balanced motions (at least mesoscale; need to evaluate it for submesoscale).
Energy spectra in the cross-shore direction

- Increased amplitudes onshore and enhanced NI amplitudes offshore
- Near-inertial variance (~1.4 cpd) and tidal currents (K1 and M2)
- ROMS show consistent variance distribution to observation in space and time.
Variance distribution of surface currents (ROMS)

\[
\alpha(\sigma) = \frac{1}{S} \int_{|\sigma_0|}^{|\sigma|} \langle \mathbf{u}_g^2(\sigma') \rangle d\sigma',
\]

\[
\beta(\sigma) = \frac{1}{S} \int_{|\sigma_0|}^{|\sigma|} \langle \mathbf{u}_{ag}^2(\sigma') \rangle d\sigma',
\]

\[
\gamma(\sigma) = \frac{1}{S} \int_{|\sigma_0|}^{|\sigma|} \langle \mathbf{u}_g^2(\sigma') \rangle + \langle \mathbf{u}_{ag}^2(\sigma') \rangle d\sigma'.
\]

Table 1: Fraction of variance of the geostrophic currents (\(\mathbf{u}_g\)) and Rossby number (\(\zeta_g\)), the ageostrophic surface currents (\(\mathbf{u}_{ag}\)) and Rossby number (\(\zeta_{ag}\)), and the total surface currents (\(\mathbf{u}\)) and Rossby number (\(\zeta\)) in the primary frequency bands: low-frequency (0 < \(|\sigma_L| \leq 0.36\) cpd), diurnal (0.95 cpd \(\leq |\sigma_D| \leq 1.05\) cpd), near-inertial (1.21 cpd \(\leq |\sigma_{NI}| \leq 1.57\) cpd), semi-diurnal (1.90 cpd \(\leq |\sigma_S| \leq 2.05\) cpd) frequency bands, the rest of frequencies (\(|\sigma_E|\)), and the entire frequencies (0 < \(|\sigma_A| \leq 12\) cpd).

<table>
<thead>
<tr>
<th>(\sigma_L)</th>
<th>(\sigma_D)</th>
<th>(\sigma_{NI})</th>
<th>(\sigma_S)</th>
<th>(\sigma_E)</th>
<th>(\sigma_A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\langle \mathbf{u}_g^2 \rangle)</td>
<td>44.2</td>
<td>0.5</td>
<td>0.3</td>
<td>5.2</td>
<td>4.0</td>
</tr>
<tr>
<td>(\langle \mathbf{u}_{ag}^2 \rangle)</td>
<td>11.6</td>
<td>1.0</td>
<td>18.1</td>
<td>6.7</td>
<td>8.4</td>
</tr>
<tr>
<td>(\langle \mathbf{u}^2 \rangle)</td>
<td>55.8</td>
<td>1.5</td>
<td>18.4</td>
<td>11.9</td>
<td>12.4</td>
</tr>
</tbody>
</table>
Degree of ageotrophy and geotrophic unbalance

\[
\frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial t} + u \cdot \nabla u - f_c v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u, \right]
\]

\[
\frac{\partial}{\partial y} \left[ \frac{\partial v}{\partial t} + u \cdot \nabla v + f_c u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v, \right]
\]

\[
\frac{\partial \delta}{\partial t} + \nabla_h \cdot (u \cdot \nabla u) = f_c \zeta - g \nabla_h^2 \eta, \quad p = \rho g \eta
\]

\[
\delta = \nabla_h \cdot u = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y'},
\]

\[
\zeta = \nabla_h \times u = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x},
\]

\[
\nabla_h = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y'} \hat{j},
\]

\[
\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k},
\]

excluding high order terms \[\nabla_h \cdot (\nabla^2 u)\]
Degree of ageotrophy and geotrophic unbalance

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\frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla u - f_c v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u, \right]
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\]

\[
\kappa = \frac{\left\langle (f_c \zeta - g \nabla_h^2 \eta)^2 \right\rangle}{2 \left[ \left\langle (f_c \zeta)^2 \right\rangle + \left\langle (g \nabla_h^2 \eta)^2 \right\rangle \right]} = \kappa(\sigma),
\]

where \( \kappa \to 0 \) and \( \kappa \to 1 \) denote the dominance of geostrophy and ageostrophy, respectively. Based on Cauchy’s inequality

Cauchy’s inequality \((a^2 + b^2)(c^2 + d^2) \geq (ac + bd)^2\).
Degree of ageotrophy and geotrophic unbalance

\[
\frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla u - f_c v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \nabla^2 u, \right] \\
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where $\chi \to -1$ and $\chi \to 1$ denote the dominance of the Coriolis force ($f_c \zeta$) and pressure gradients ($g \nabla_h^2 \eta$), respectively.

Cauchy’s inequality

\[
(a^2 + b^2)(c^2 + d^2) \geq (ac + bd)^2.
\]
Degree of ageotrophy and geotrophic unbalance

The degree of ageostrophy

\[ \kappa = \frac{\langle (f_c \zeta - g \nabla_h^2 \eta)^2 \rangle}{2 \left[ \langle (f_c \zeta)^2 \rangle + \langle (g \nabla_h^2 \eta)^2 \rangle \right]} = \kappa(\sigma) \]

The degree of geostrophic unbalance

\[ \chi = \frac{-\langle (f_c \zeta)^2 \rangle + \langle (g \nabla_h^2 \eta)^2 \rangle}{\langle (f_c \zeta)^2 \rangle + \langle (g \nabla_h^2 \eta)^2 \rangle} = \chi(\sigma) \]

where \( \kappa \to 0 \) and \( \kappa \to 1 \) denote the dominance of geostrophy and ageostrophy, respectively (by Cauchy’s inequality). Where \( \chi \to -1 \) and \( \chi \to 1 \) denote the dominance of the Coriolis force \((f_c \zeta)\) and pressure gradients \((g \nabla_h^2 \eta)\), respectively.
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Estimates of stream functions and velocity potentials

\[ \mathbf{u} = \mathbf{u}_\psi + \mathbf{u}_\phi = \mathbf{k} \times \nabla_h \psi + \nabla_h \phi, \]

\[ \hat{\mathbf{Y}} = \text{cov}_{dm}^+ \text{cov}_{dd}^{-1} \mathbf{u} \]

\[ (\text{cov}_{dd})_{jk} = \langle u_j u_k^\dagger \rangle + \delta_{jk} \gamma^2, \]

\[ (\text{cov}_{dm})_{ji} = \langle u_j \mathbf{Y}_i^\dagger \rangle, \]

\[ = \begin{bmatrix} \langle u_\psi \psi^\dagger \rangle & \langle u_\phi \phi^\dagger \rangle \\ \langle v_\psi \psi^\dagger \rangle & \langle v_\phi \phi^\dagger \rangle \end{bmatrix}_{ji} \]

\[ \hat{\mathbf{Y}} = [\hat{\psi} \ \hat{\phi}]^\dagger \]
Velocity fields corresponding to stream function are estimated at different decorrelation scales.

SSHs do not contain near-inertial variance.

SSHs have sharp peaks at the tidal frequencies.

Higher coherence in the low-frequency band.

Higher coherence in NI can be explained by the advection of near-inertial currents as the non-propagating components or non-divergent horizontal near-inertial currents.

Coherence between stream functions and SSHs:

$$c(\sigma) = \frac{\langle \eta'(\sigma) \hat{\psi}'(\sigma) \rangle^*}{\sqrt{\langle \eta'^2(\sigma) \rangle \langle \hat{\psi}'^2(\sigma) \rangle}}$$
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