Mapping coastal wind field using wind-current transfer function analysis

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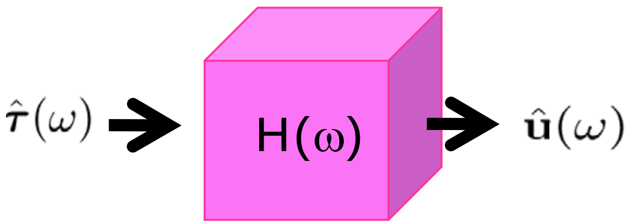
What we are exploring….

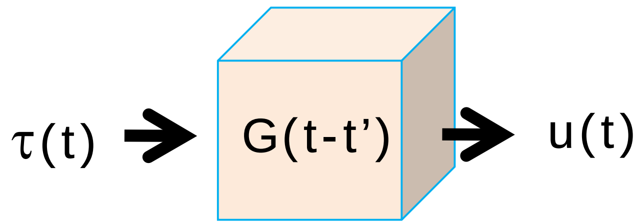
- Environmental parameterization using dynamical model and statistical model
  - Wind transfer function analysis
  - Isotropic vs. anisotropic responses
  - Linearized momentum equations and derivation of transfer function
  - Observations of high-frequency radar-derived surface currents and costal wind
- Wind field mapping using transfer function analysis (in progress)
- Summary

Wind transfer/response functions

- A statistical framework to represent the link between wind and currents in the frequency and time domains.
- Isotropic and anisotropic analyses/models.



Transfer function



Response function

$$\hat{\mathbf{u}}(z,\omega) = \mathbf{H}(z,\omega)\hat{\boldsymbol{\tau}}(\omega)$$

$$\mathbf{u}(z,t) = \int_{t'} \mathbf{G}(z,t-t')\boldsymbol{\tau}(t')\,\mathrm{d}t',$$

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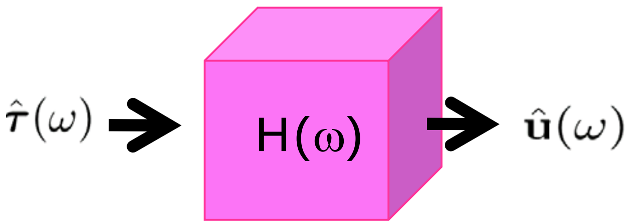

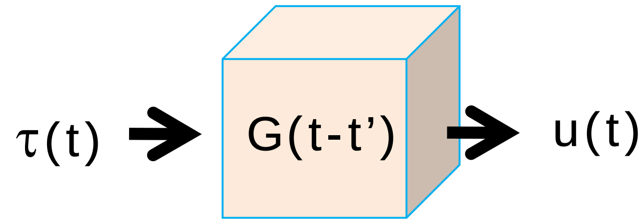

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$$\mathbf{u}(z,t) = \int_{t'} \mathbf{G}(z,t-t')\boldsymbol{\tau}(t')\,\mathrm{d}t',$$

$$\mathbf{H}(z,\omega) = \left(\langle \hat{\mathbf{u}}(z,\omega)\,\hat{\boldsymbol{\tau}}^{\dagger}(\omega)\rangle\right)\left(\langle \hat{\boldsymbol{\tau}}(\omega)\,\hat{\boldsymbol{\tau}}^{\dagger}(\omega)\rangle + \mathbf{R_a}\right)^{-1}$$

$\mathbf{R_a}$: Regularization matrix

$$\mathbf{G}(z,t) = \left(\langle \mathbf{u}(z,t)\,\boldsymbol{\tau}_N^{\dagger}(t)\rangle\right)\left(\langle \boldsymbol{\tau}_N(t)\,\boldsymbol{\tau}_N^{\dagger}(t)\rangle + \mathbf{R_b}\right)^{-1}$$

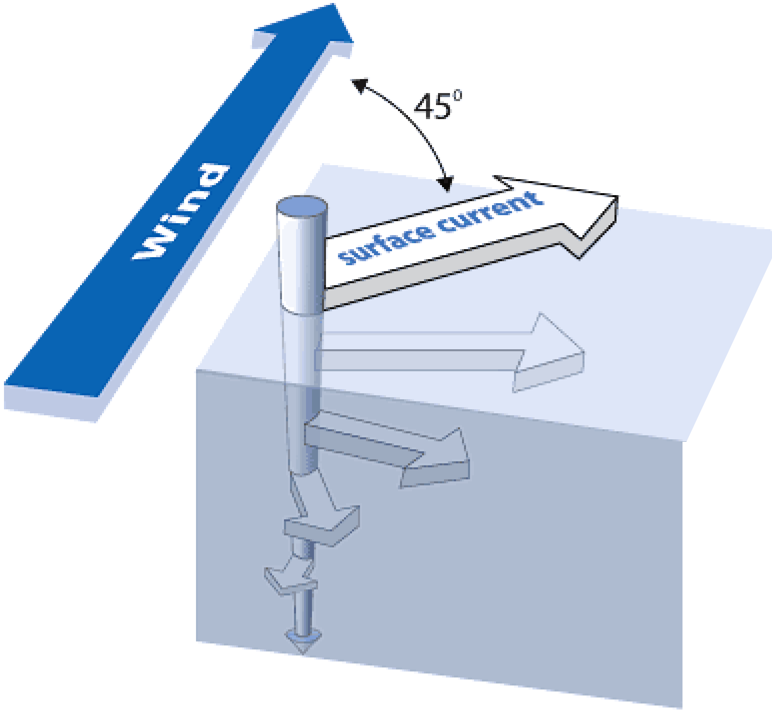
$\boldsymbol{\tau}_N$: *N*-hour advanced time lagged wind stress

$\mathbf{R_b}$: Regularization matrix

Isotropic and anisotropic responses

- "In the Northern Hemisphere, the wind-driven current at the very surface will be directed 45° to the right of the velocity of the wind." (Phys. Oceanogr. 101)

(Ekman, 1905)



http://oceanservice.noaa.gov/education/kits/currents/

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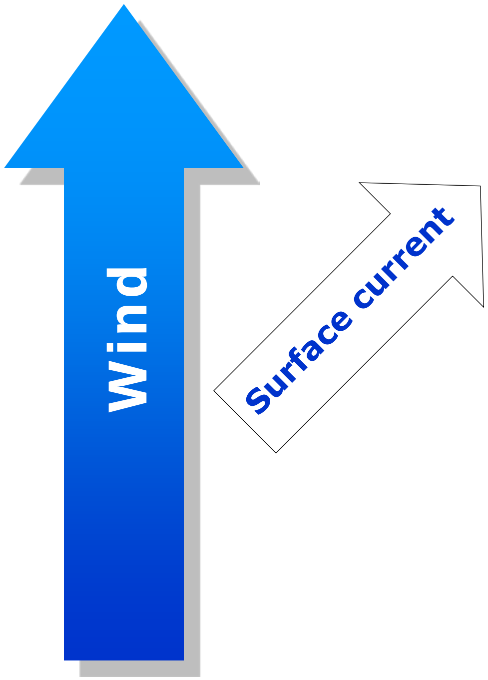

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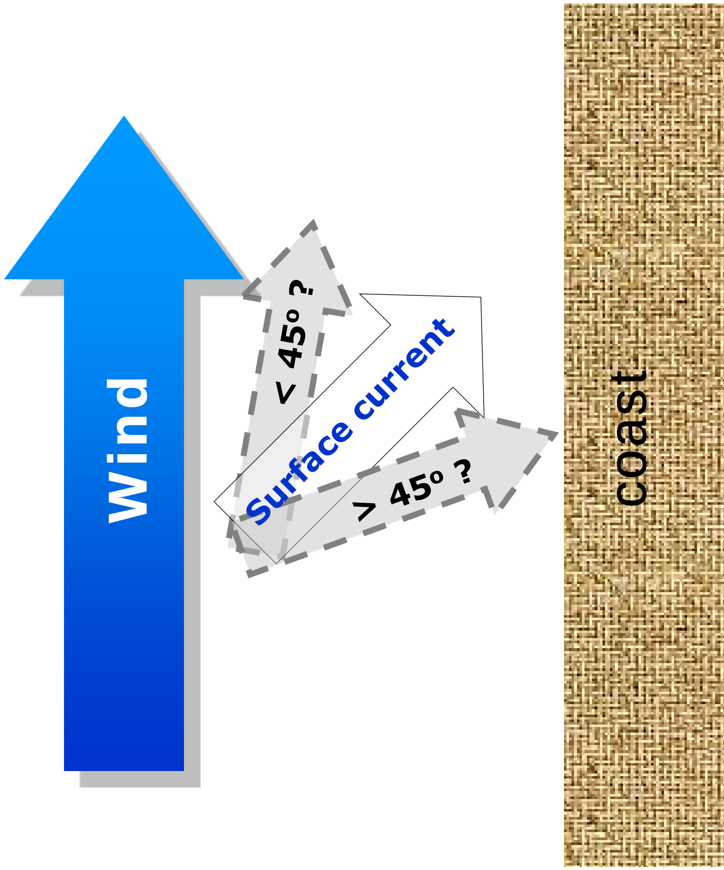

Isotropic and anisotropic responses

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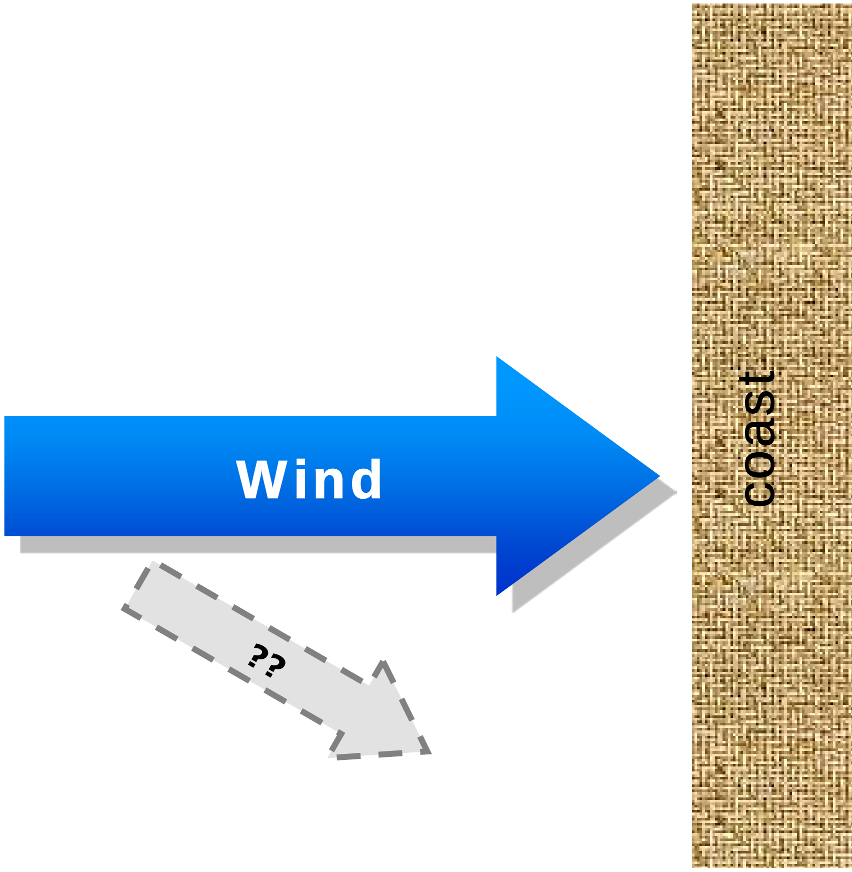
  "The wind-driven current will depend on the relative depth and on the angle between the wind-direction and the coastline." (Ekman, 1905)



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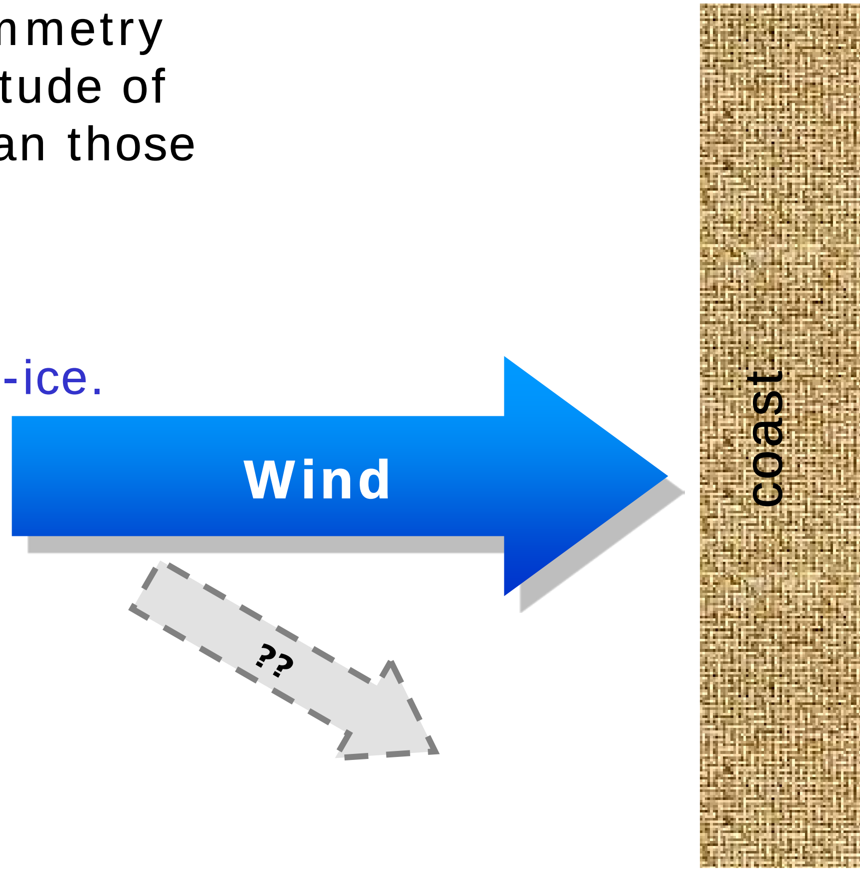
- "Stratification causes a response asymmetry wherein the offshore scale and magnitude of the upwelling responses are larger than those for the downwelling responses."
  (Weisberg, JGR 2001)

- Anisotropic wind responses of the sea-ice.
  (Overland and Pease, JGR 1988)

- Wind-current transfer function.
  (Gonella, DSR 1972;
  Weller *et al*, JGR 1981)



Isotropic model

$$\frac{\partial u}{\partial t} - f_c v = \frac{1}{\rho}\frac{1}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)$$
$$\frac{\partial v}{\partial t} + f_c u = \frac{1}{\rho}\frac{1}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)$$

$\mathbf{u} = u + iv$ $\boldsymbol{\tau} = \tau_x + i\tau_y$: isotropic assumption

Then, Fourier transform

$$\lambda^2 \hat{\mathbf{u}}(z,\omega) = \frac{\partial^2 \hat{\mathbf{u}}(z,\omega)}{\partial z^2},$$

where $\lambda = \sqrt{i(\omega + f_c)/\nu}$,

ν = Depth independent eddy viscosity

With BCs (finite or infinite depth)

$$\left.\frac{\partial \hat{\mathbf{u}}(z,\omega)}{\partial z}\right|_{z=0} = \frac{\hat{\boldsymbol{\tau}}(\omega)}{\rho\nu}. \qquad \hat{\mathbf{u}}(z,\omega)|_{z=-\infty} = 0,$$

$$\mathbf{H}(z,\omega) = \frac{\hat{\mathbf{u}}(z,\omega)}{\hat{\boldsymbol{\tau}}(\omega)} = \frac{e^{-\lambda z}}{\lambda\rho\nu},$$

(Gonella, DSR 1972)

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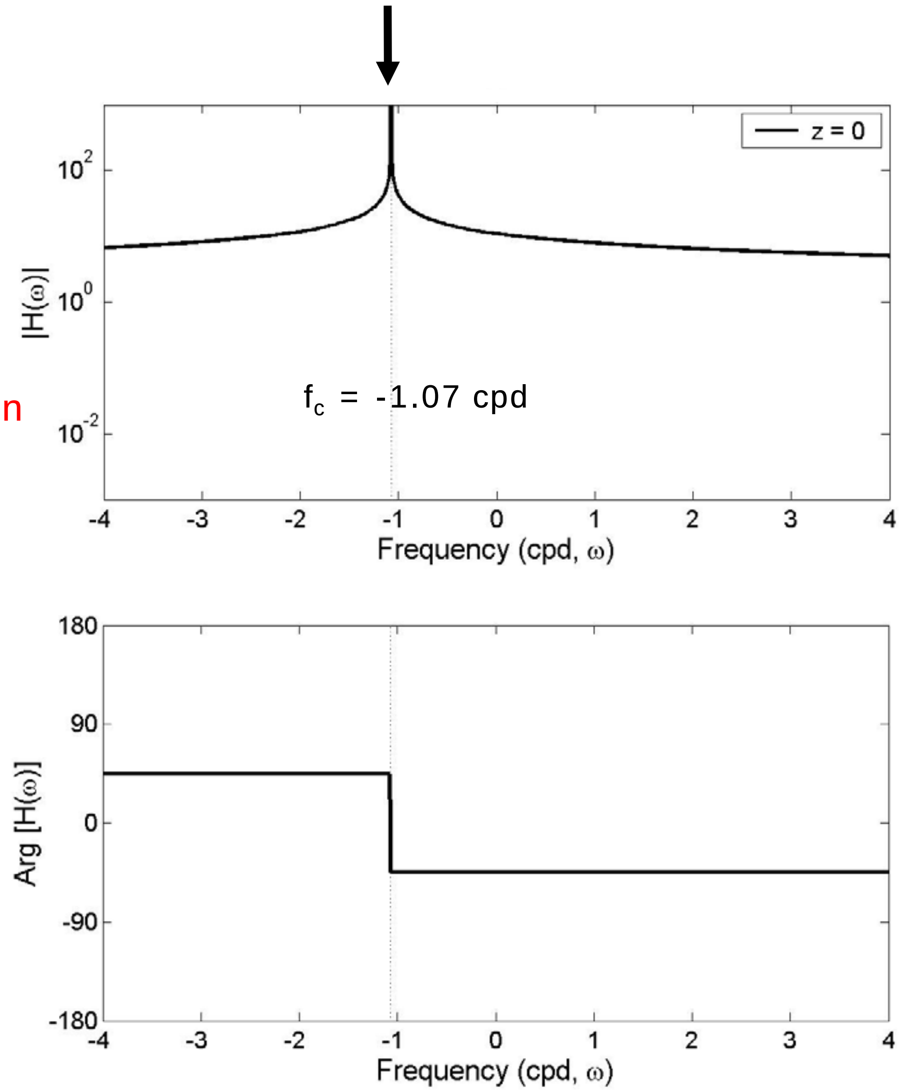
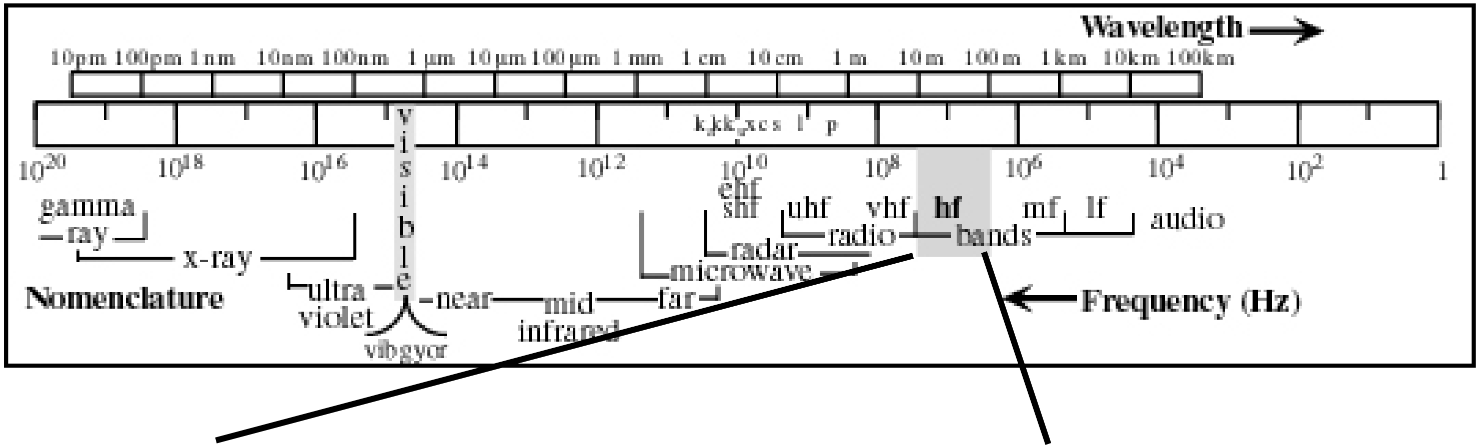


Radio signals used in high-frequency radar



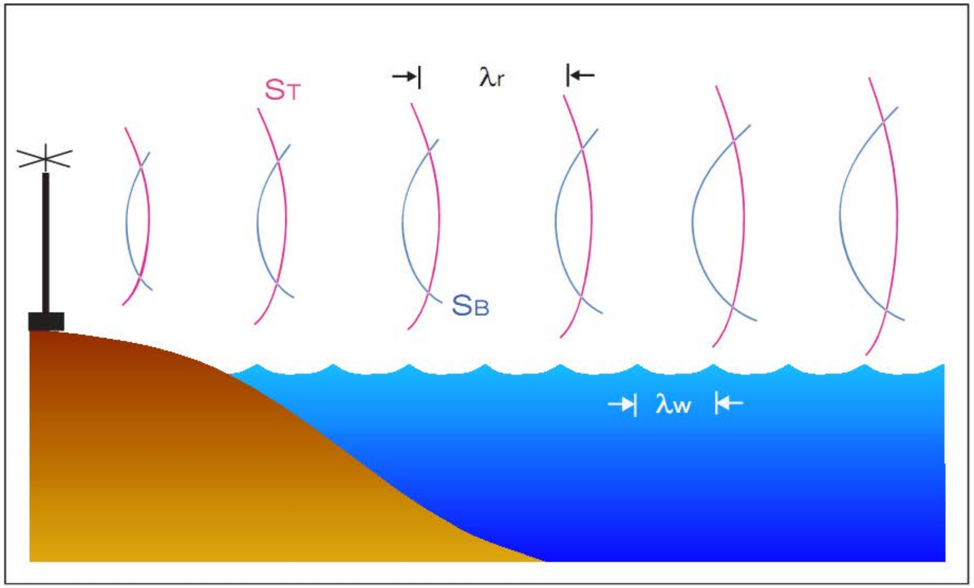
3-30 MHz (between AM radio and TV)

Wavelength (λ_r) : 10 ~ 100 (m)

Bragg backscattering

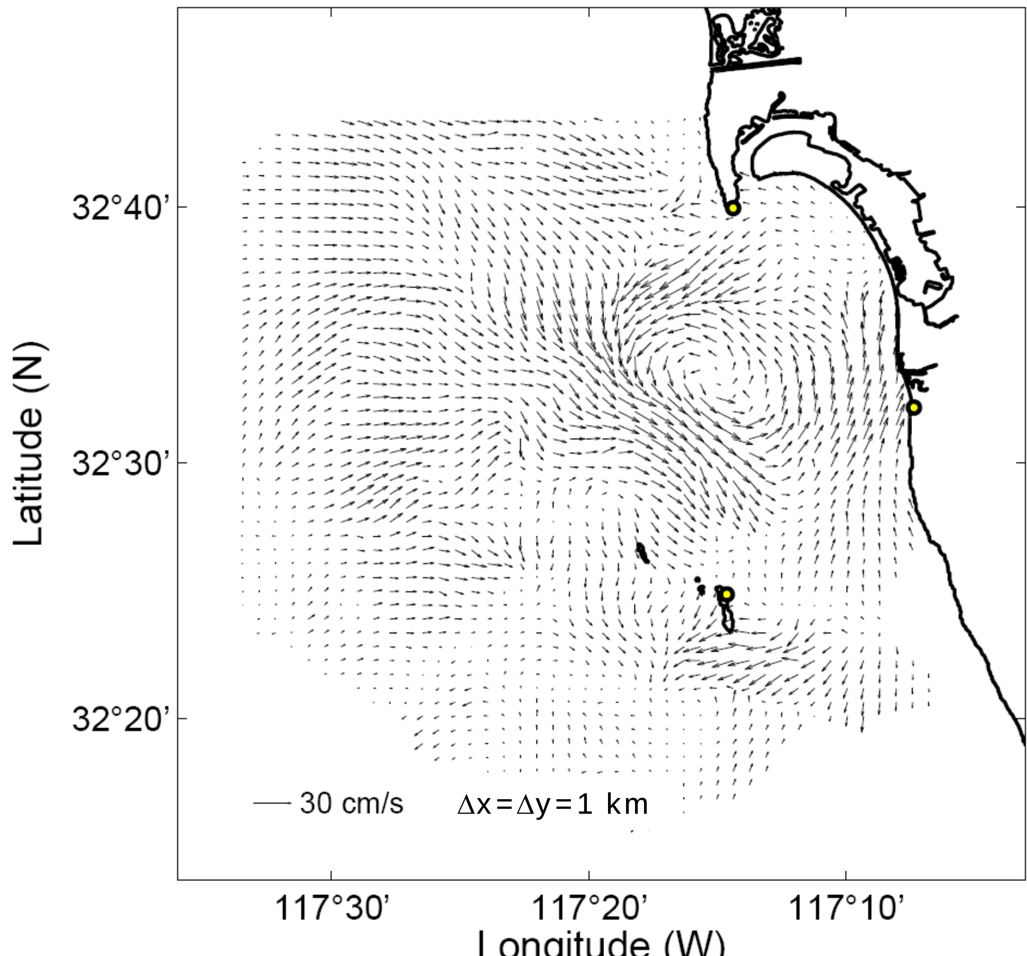
When the radar signals are backscattered in phase,

$$\lambda_w = \lambda_r / 2$$

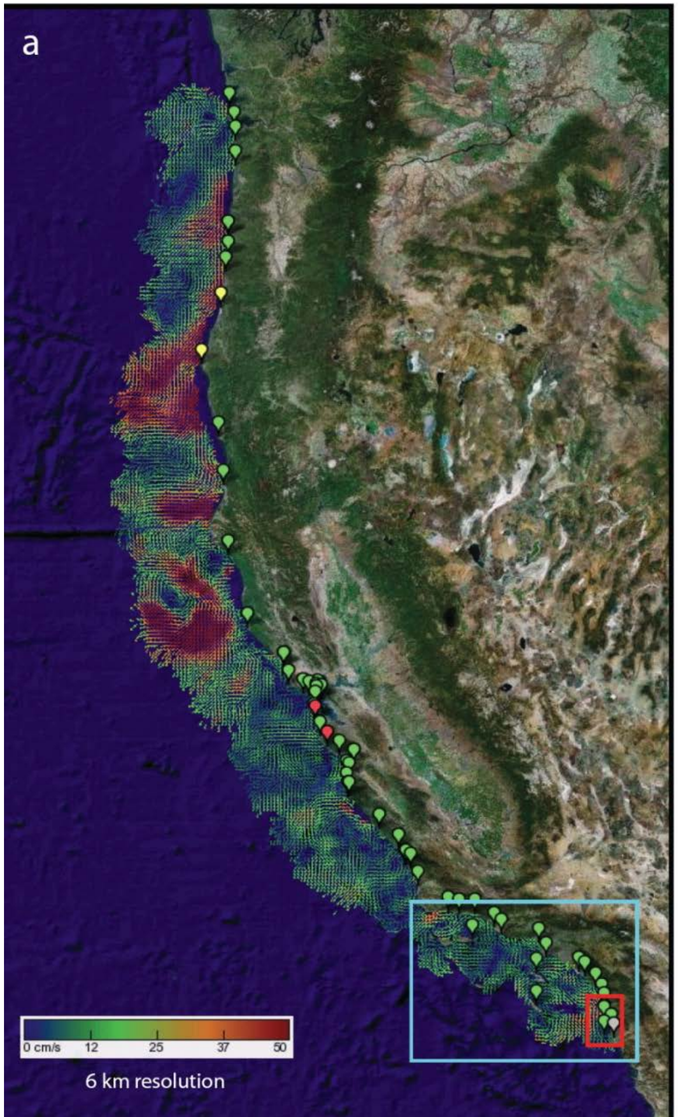


High-frequency (HF) radar

University of Hamburg, Germany

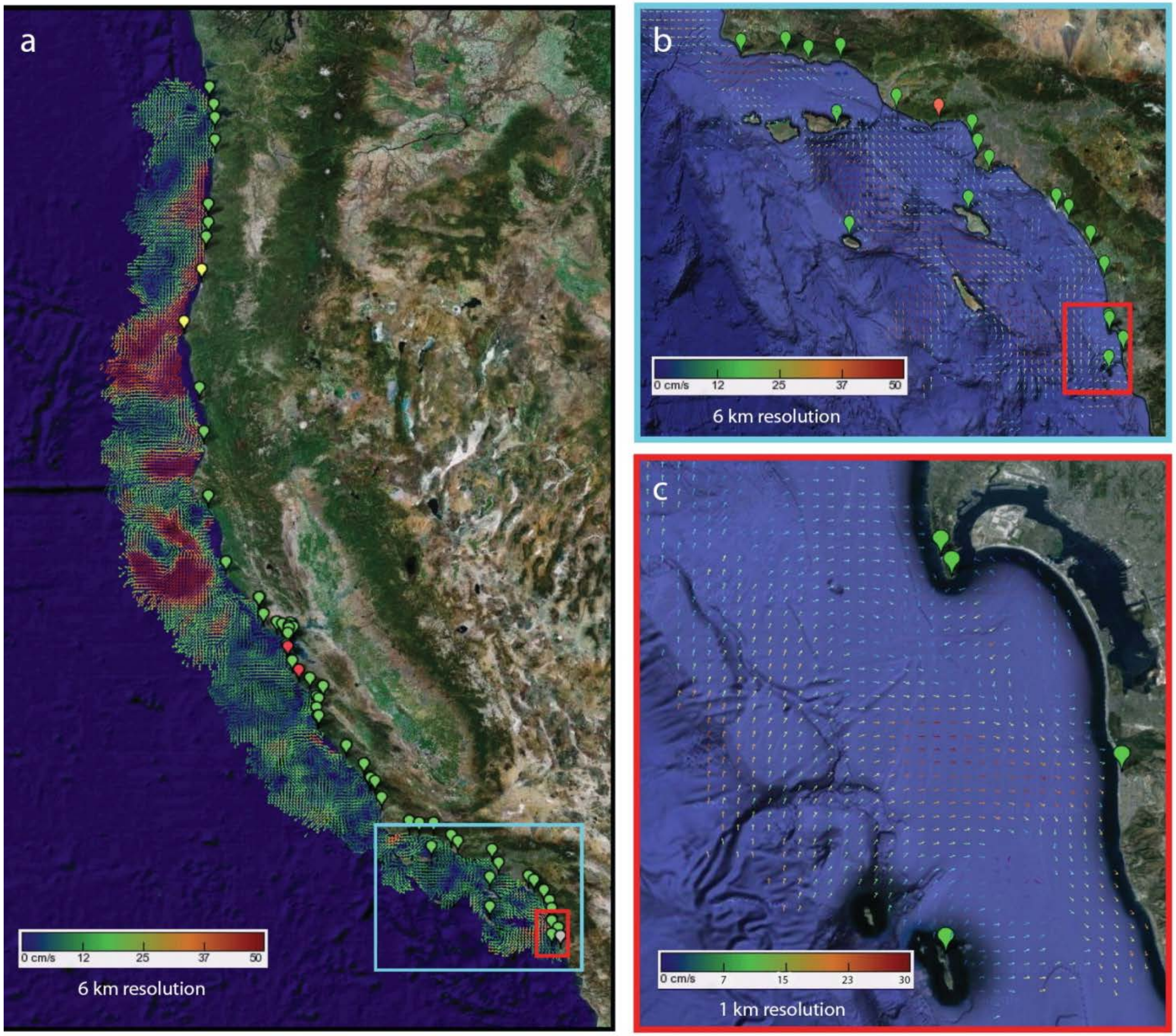


High-frequency coastal radar-derived surface currents off the U.S. West Coast



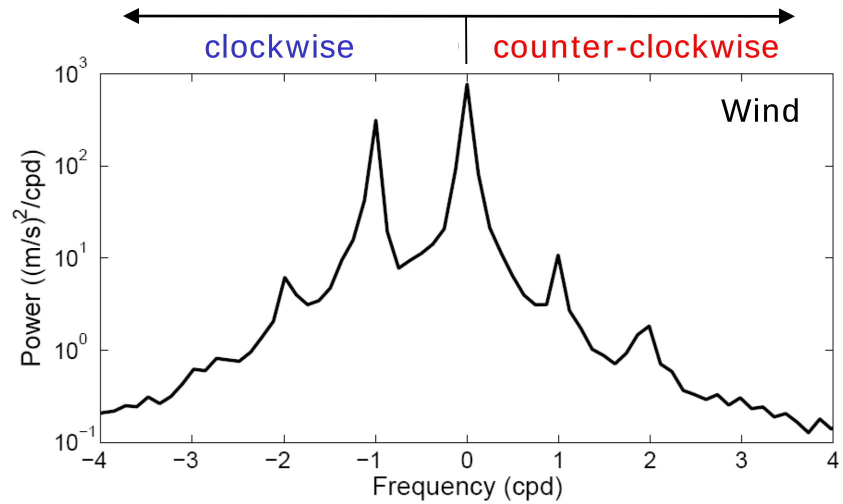
- A network of high-frequency radars (HFRs) along the coast over 2500 km of US West Coast provides km resolution and hourly surface current maps which cover about 150 km offshore from shoreline.
- Due to low signal-to-noise ratio of satellite remote sensing near coastal regions and high cost of transporting the wind energy to end users (e.g., cable), an approach to find hot spots of wind energy in coastal areas is proposed.

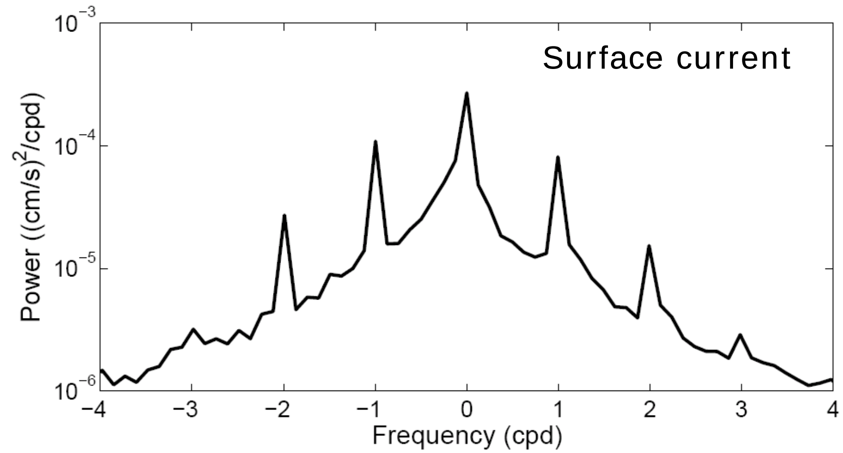
High-frequency coastal radar-derived surface currents off the U.S. West Coast (cascade maps)



Data analysis

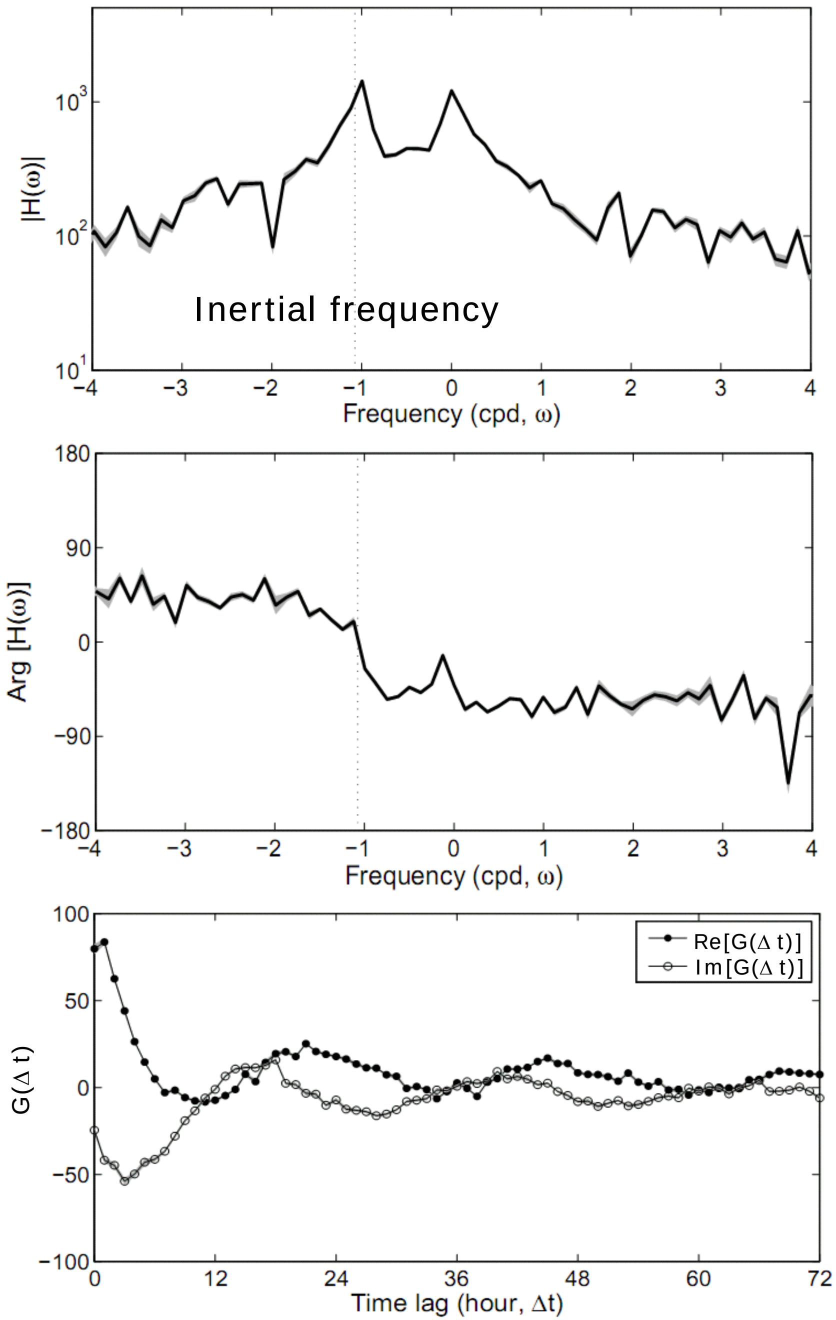
- Two-year records of hourly spatially averaged surface current and hourly wind observations near Tijuana River are used.
- Diurnal wind and its harmonics.
- Clockwise dominance.
- Major tides (K1, P1, O1, M2, and S2) in the surface current are removed for the WIRF estimate.





90 subsamples

Wind transfer function and response function



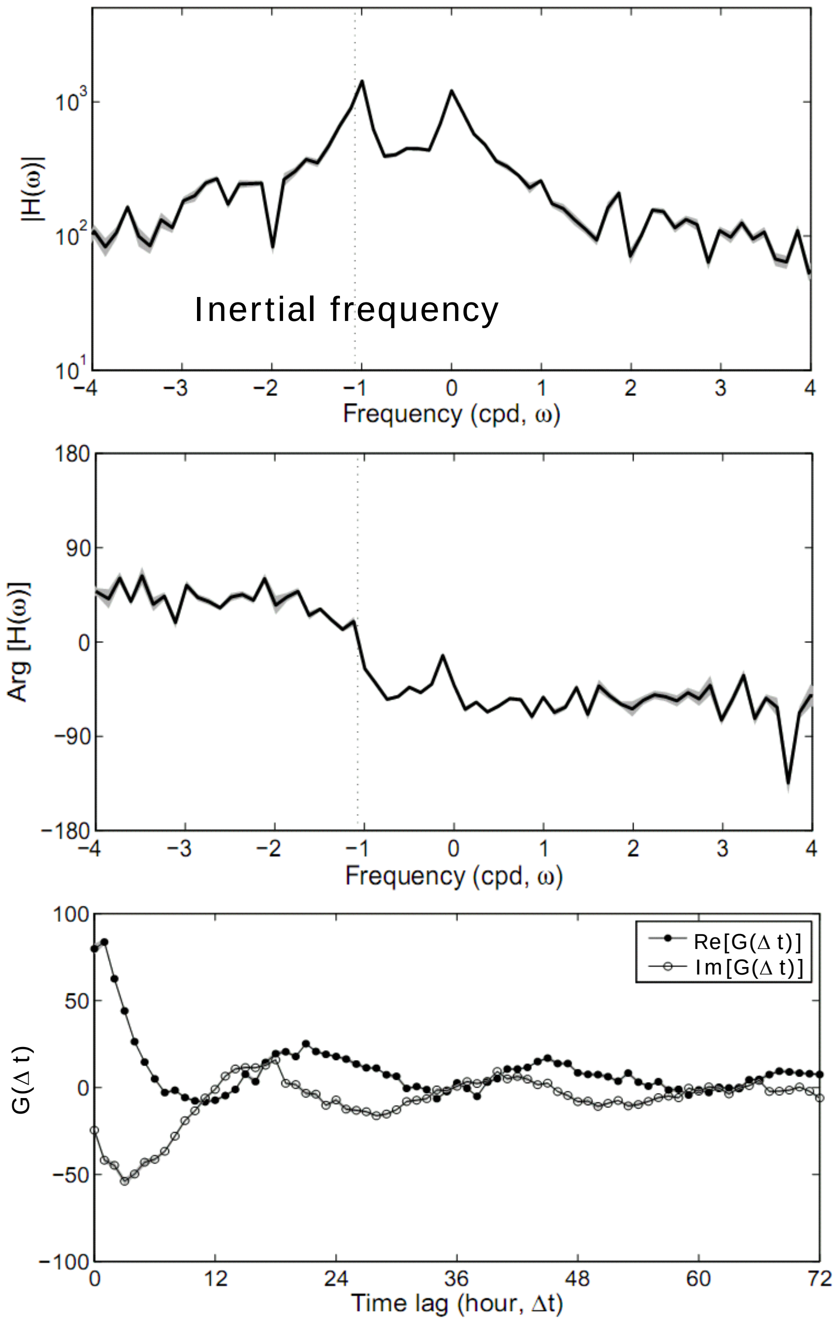
$$\hat{\mathbf{u}}(z,\omega) = \mathbf{H}(z,\omega)\hat{\boldsymbol{\tau}}(\omega)$$

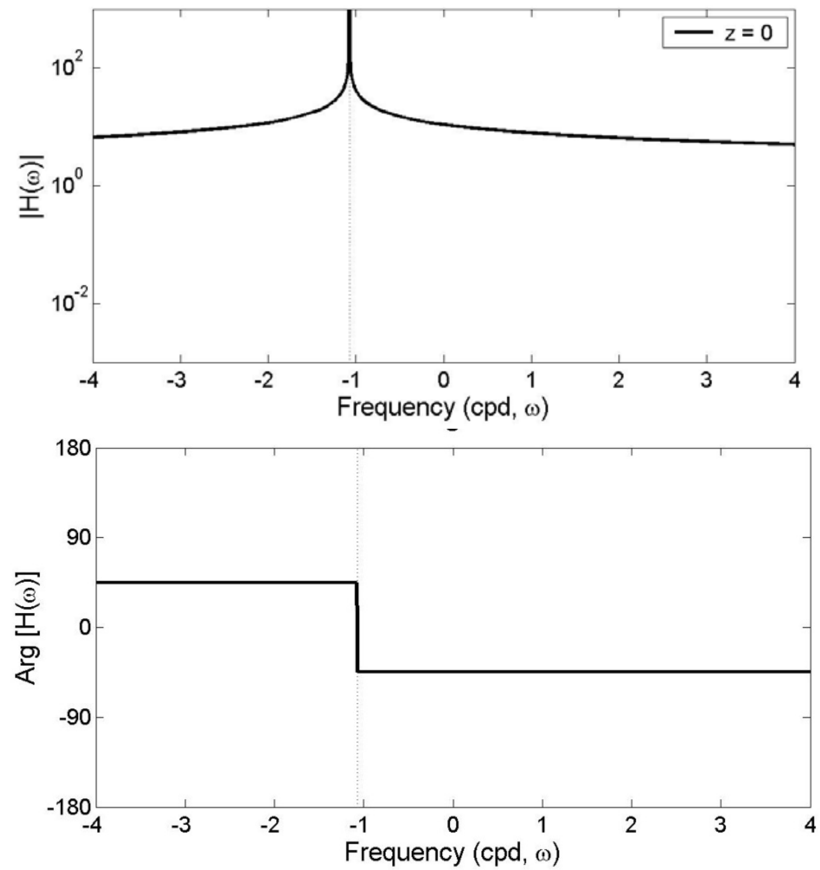
$$\mathbf{u}(z,t) = \int_{t'} \mathbf{G}(z, t-t')\boldsymbol{\tau}(t')\,\mathrm{d}t',$$

$$\boldsymbol{\tau}(t') = \delta(t'-\alpha)\ (\alpha > 0)$$

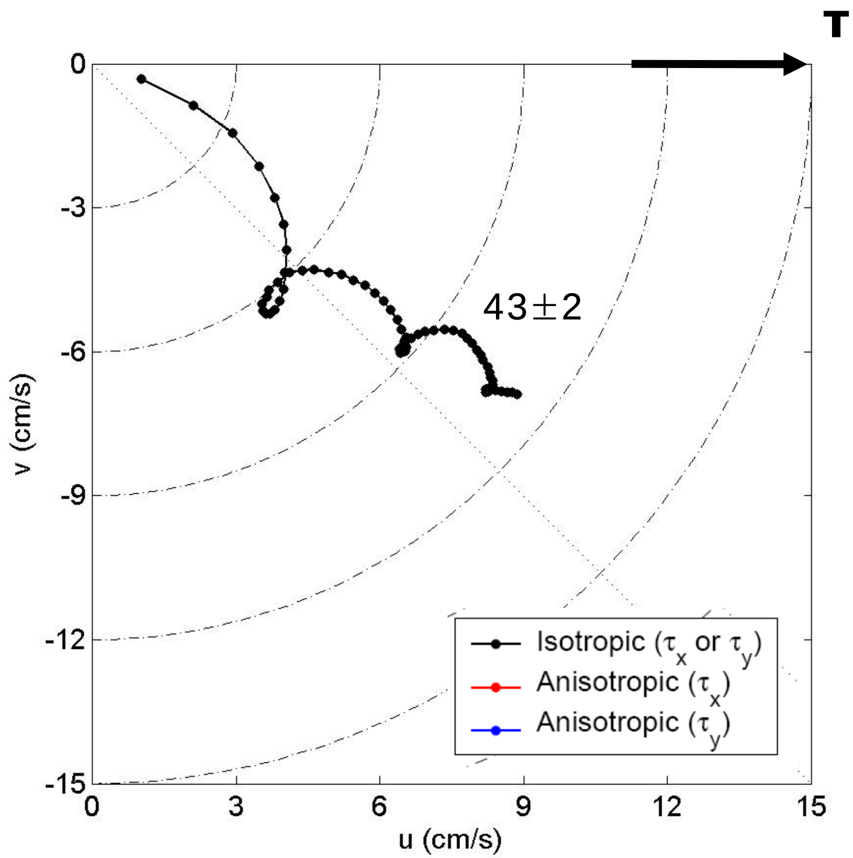
$$\mathbf{u}(z,t) = \mathbf{G}(z, t-\alpha)$$

Wind transfer function and response function





Steady wind response (Isotropic response function)



$$\mathbf{u}(t_N) = \sum_{j=0}^{N} \mathbf{G}(t_j)\,\boldsymbol{\tau}(t_j)$$

- Transfer function is inversely Fourier transformed into the impulse response function (temporal weighting), which decays with the inertial period.
- Cumulative time integration of the response function with constant wind speed for three days.
- Typical wind speed @ SD: 3 ms^{-1}
- Isotropic response: 43 ± 2^o

Anisotropic model

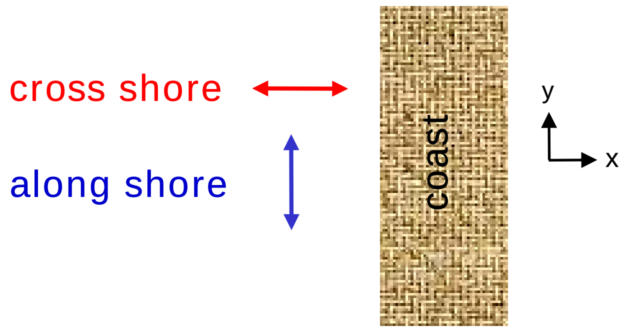
$$\frac{\partial u}{\partial t} - f_c v + A_x = \frac{1}{\rho}\frac{1}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)$$
$$\frac{\partial v}{\partial t} + f_c u + A_y = \frac{1}{\rho}\frac{1}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)$$

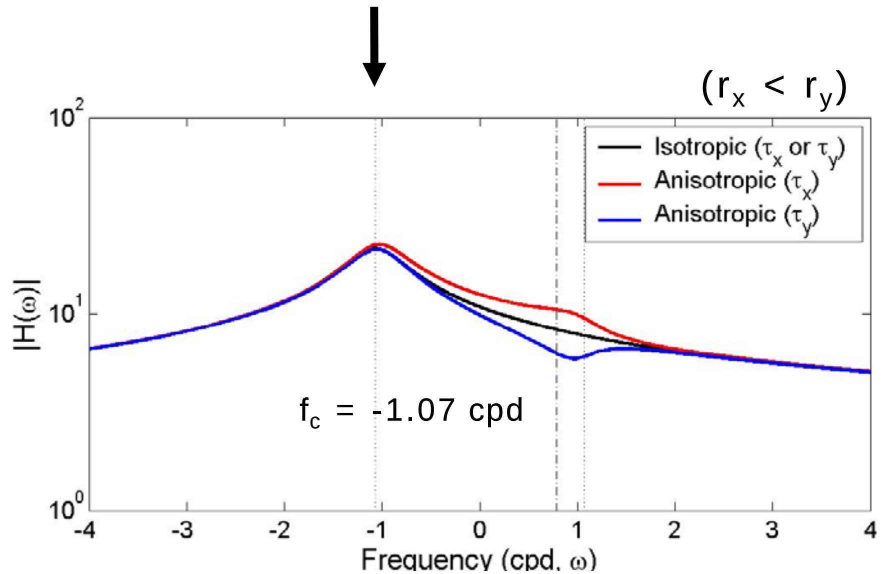
where $A_x = r_x\, \boldsymbol{\delta}(t) * u$

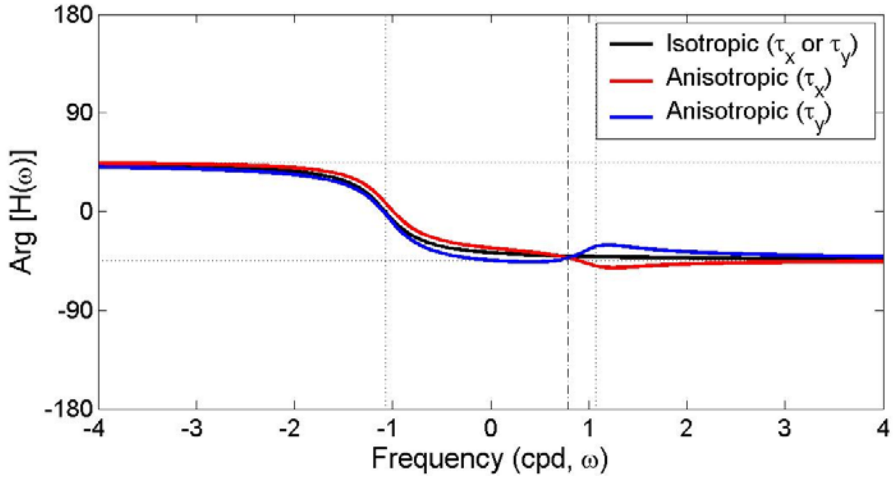
$A_y = r_y\, \boldsymbol{\delta}(t) * v$

*: time domain convolution

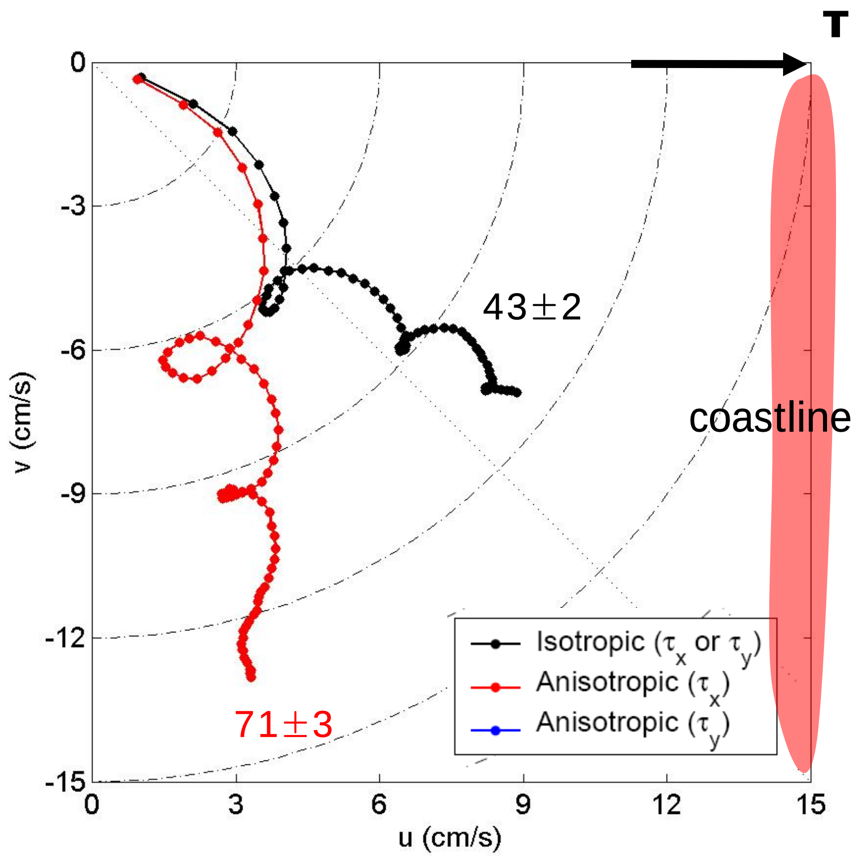
Fourth order PDE is solved with BCs using Ferrai-Cardan method for the quartic characteristic equation.







Steady wind response (Anisotropic response function)



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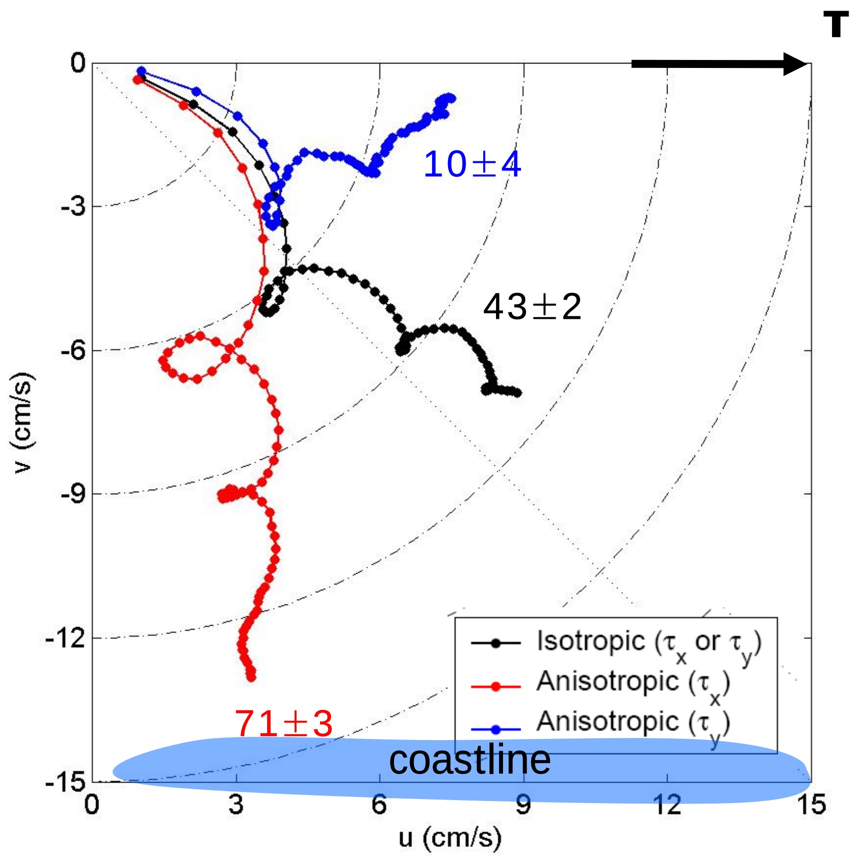
- Isotropic response: 43±2°
- Anisotropic responses follows the coastline: 71±3° (cross shore),

Steady wind response (Anisotropic response function)



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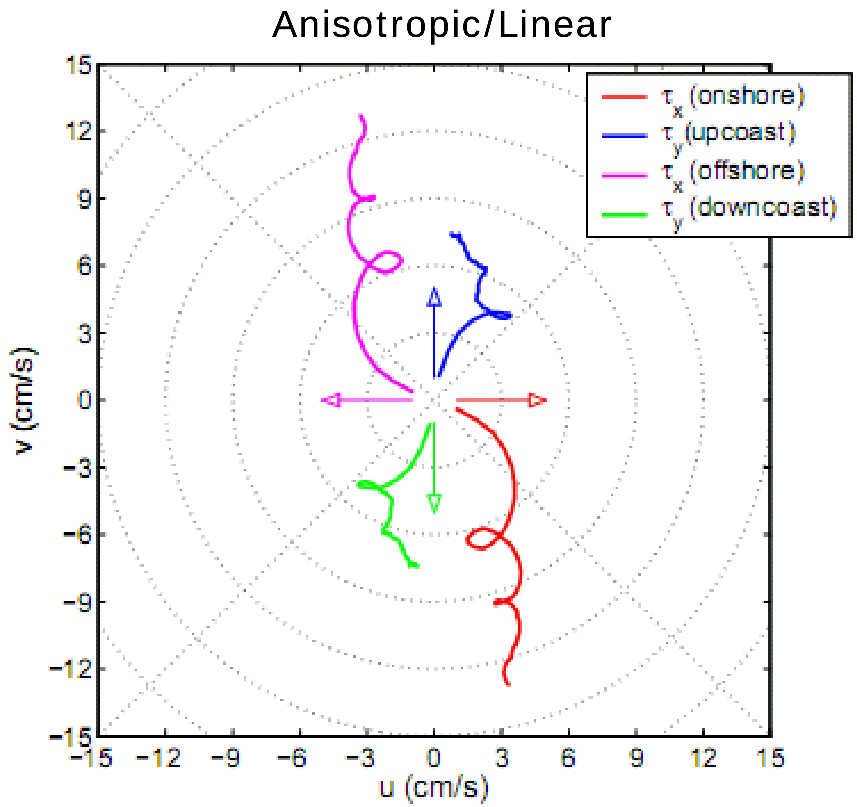
- Cumulative time integration of the response function with constant wind speed for three days.
- Typical wind speed @ SD: 3 ms^{-1}
- Isotropic response: 43 ± 2^o
- Anisotropic responses follows the coastline:
  71 ± 3^o (cross shore),
  10 ± 4^o (alongshore)
- Magnitude of wind-driven surface currents:
  3-5% of wind speed

Steady wind response



- Onshore and offshore wind responses are mirrored due to linear response function (upcoast and downcoast wind cases as well).

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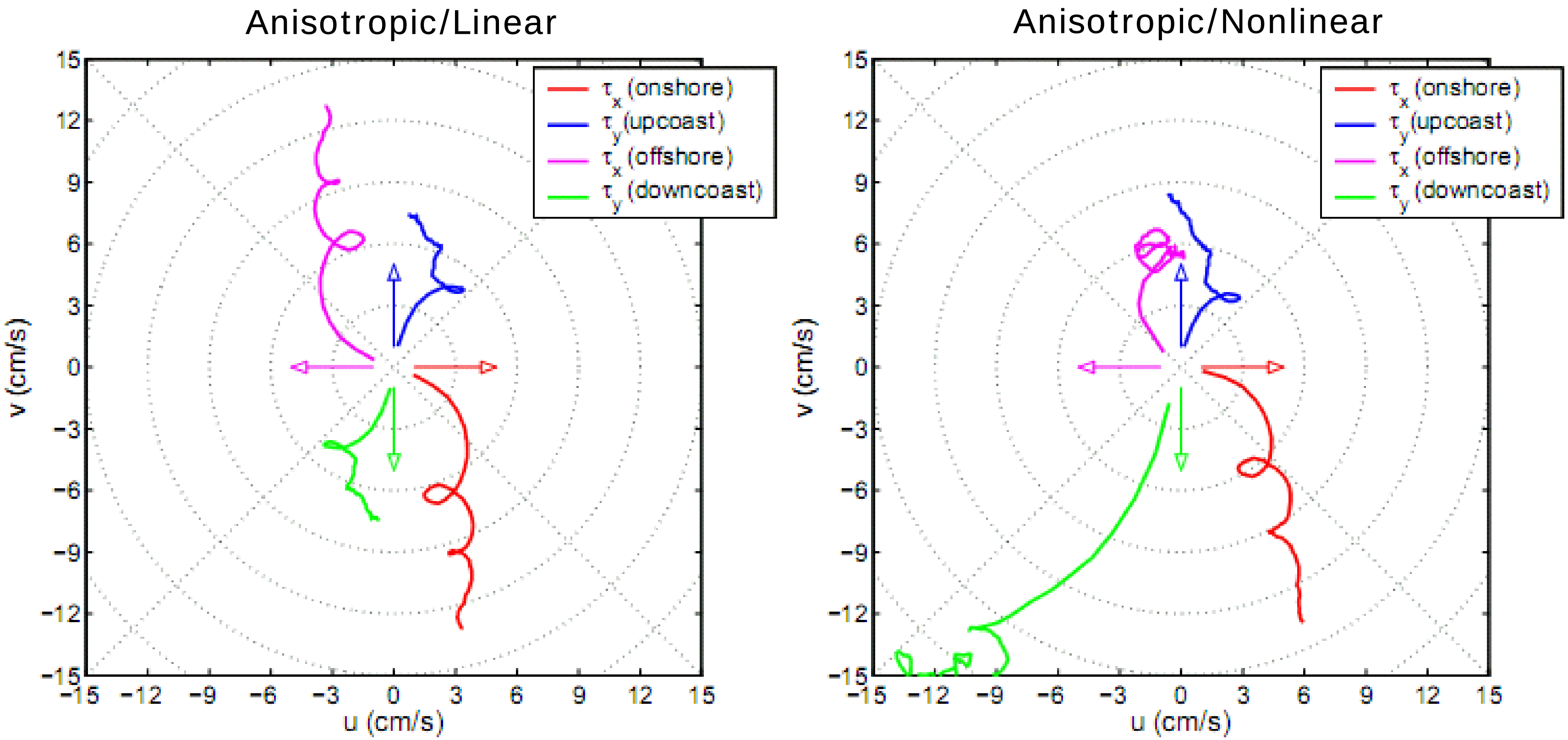
Steady wind response



- Onshore and offshore wind responses are mirrored due to linear response function (upcoast and downcoast wind cases as well).
- Nonlinear term breaks the symmetry.

$$\mathbf{u}(t_N) = \sum_{j=0}^{N} \mathbf{G}(t_j)\,\boldsymbol{\tau}(t_j)$$

$$\mathbf{u}(t_N) = \sum_{j=0}^{N} \underbrace{\mathbf{G_1}(t_j)\,\boldsymbol{\tau}(t_j)}_{\text{Linear}} + \underbrace{\mathbf{G_2}(t_j)\,|\boldsymbol{\tau}(t_j)|}_{\text{Nonlinear}}$$

Summary

- Environmental parameterization using wind-current transfer function provides statistical framework, consistent with analytic solutions derived from linearized momentum equations.
- Isotropic and anisotropic responses near the coast can be applicable to wind-driven surface transport model in the coastal regions.
- More results will be updated in ICOWES 2013.