

Resonant ocean current responses driven by coastal winds near the critical latitude

Sung Yong Kim¹ and Greg Crawford²

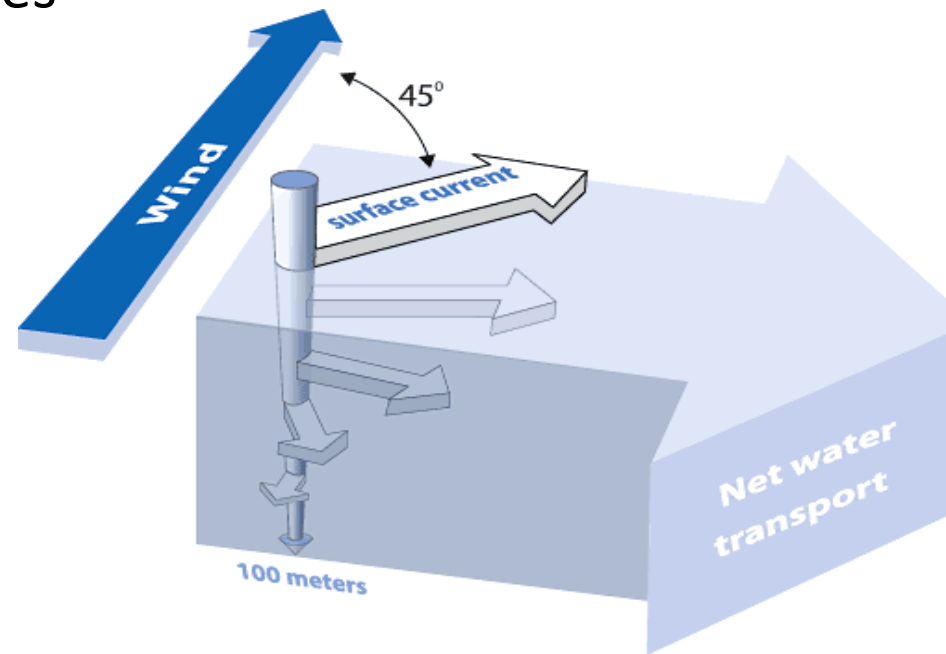
¹Division of Ocean Systems Engineering
School of Mechanical, Aerospace & Systems Engineering,
Korea Advanced Institute of Science and Technology (KAIST)
Republic of Korea
syongkim@kaist.ac.kr

²The Faculty of Science, University of Ontario Institute of
Technology, Oshawa, Ontario, Canada



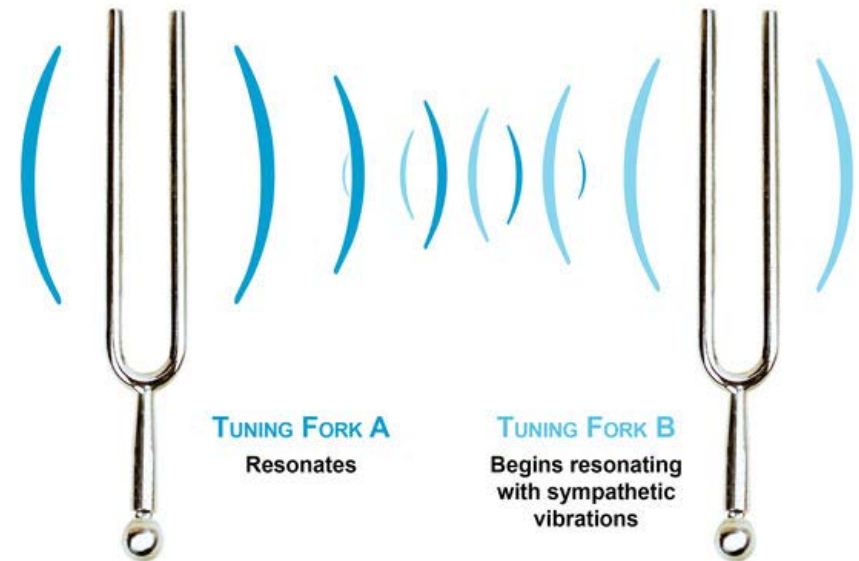
Resonant ocean current responses driven by coastal winds near the critical latitude

- Wind and current responses
 - Ekman theory..



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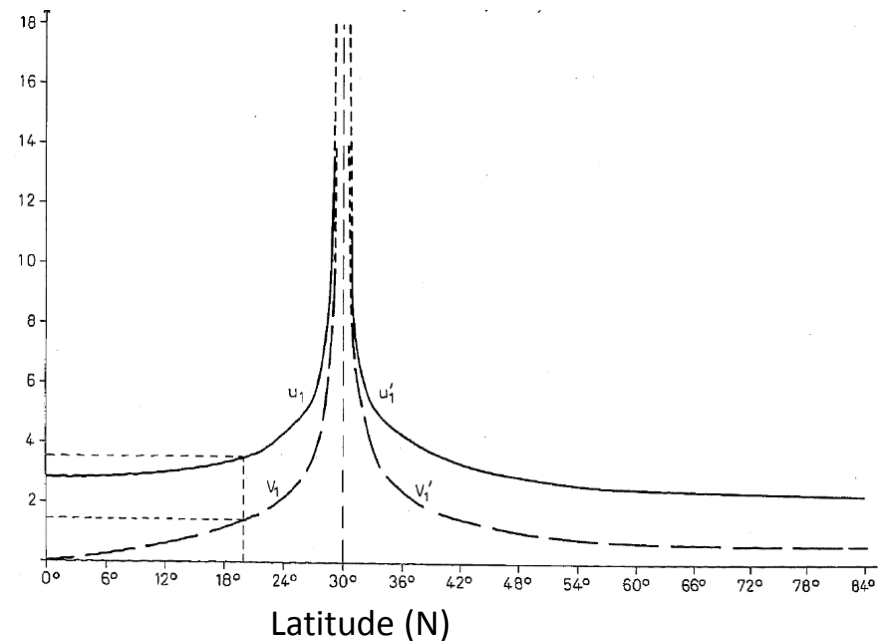
- Wind and current responses
 - Ekman theory..
- Resonance
 - Forcing-response in the frequency domain
 - Natural frequency – Coriolis frequency



$$f_c = 2 \sin(\text{latitude}) \text{ [cycles per day]}$$

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- Wind and current responses
 - Ekman theory..
- Resonance
 - Forcing-response in the frequency domain
 - Natural frequency – Coriolis frequency
- Critical latitude
 - Observations at different latitudes – wind and surface currents off the USWC



Shaffer, 1972; Ekman model

Resonant ocean current responses driven by coastal winds near the critical latitude

- At a given latitude, what would be the wind-current response in the frequency domain?
- At a given frequency, what would be the wind-current response as a function of latitude?

Wind-current responses in the freq. domain

$$\frac{\partial \mathbf{u}}{\partial t} + if_c \mathbf{u} + r\mathbf{u} = \frac{1}{\rho} \frac{\partial \boldsymbol{\tau}}{\partial z}$$

Ekman theory

Wind-current responses in the freq. domain

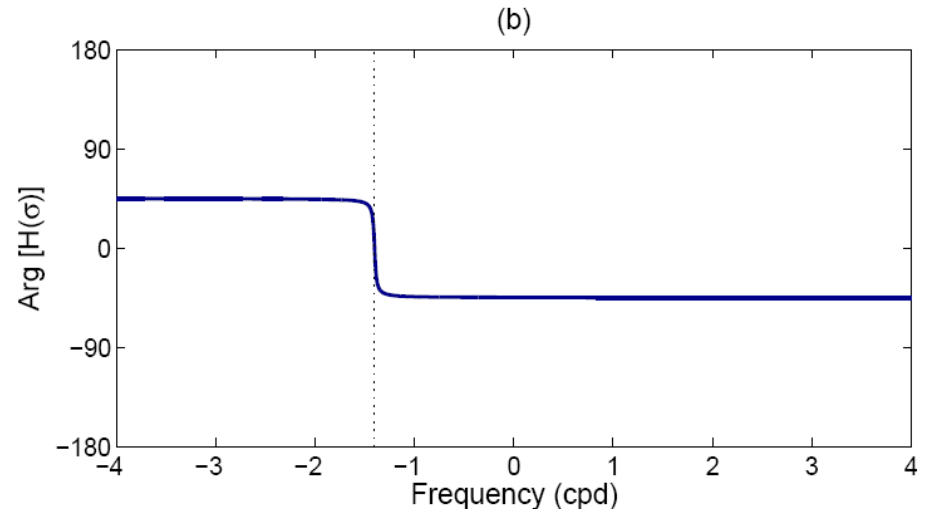
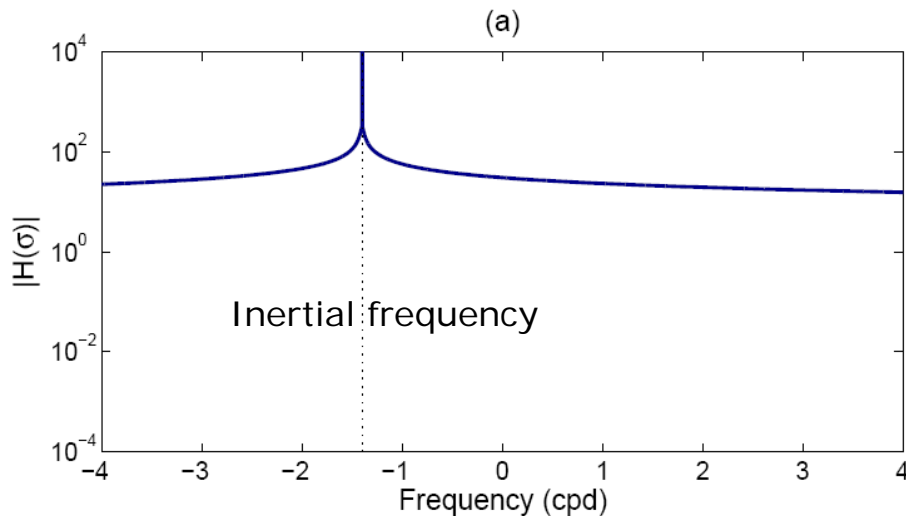
$$\frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \boldsymbol{\tau}}{\partial z}$$

Ekman theory

$$\mathbf{H}_E(z, \sigma) = \frac{\hat{\mathbf{u}}(z, \sigma)}{\hat{\boldsymbol{\tau}}(\sigma)} = \frac{e^{\lambda z}}{\lambda \rho \nu}$$

as a function of frequency

$$\lambda = \sqrt{[i(\sigma + f_c) + r] / \nu}$$



At a given latitude, the relationship between wind stress and surface currents is given as a transfer function in the frequency domain.

Wind-current responses in latitude

$$\frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \boldsymbol{\tau}}{\partial z}$$

Ekman theory

$$\mathbf{H}_E(z, \sigma) = \frac{\hat{\mathbf{u}}(z, \sigma)}{\hat{\boldsymbol{\tau}}(\sigma)} = \frac{e^{\lambda z}}{\lambda \rho \nu}$$

as a function of Coriolis freq. (latitude)

$$\lambda = \sqrt{[i(\sigma + f_c) + r] / \nu}$$

Wind-current responses in latitude

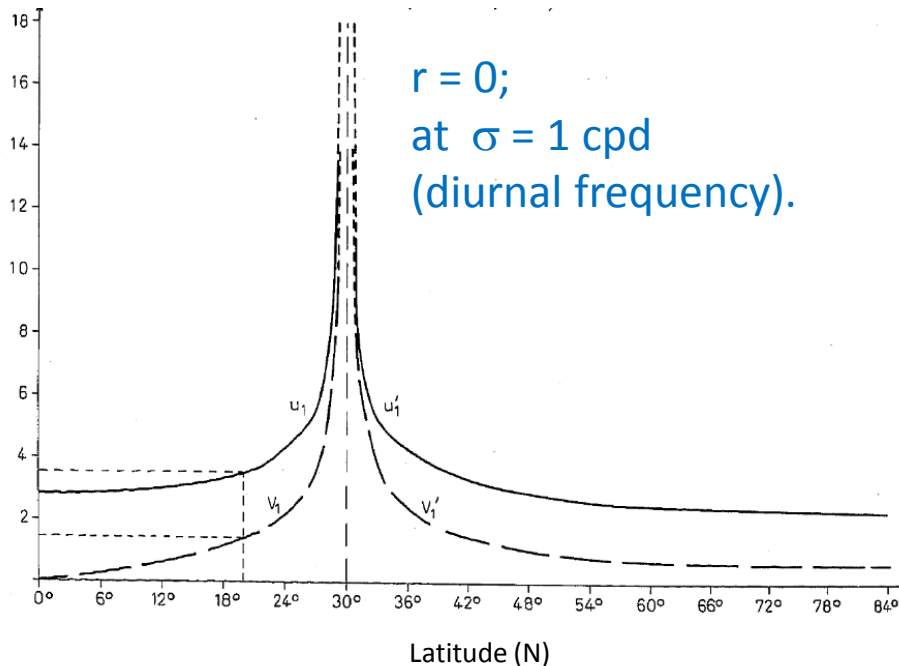
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Ekman theory

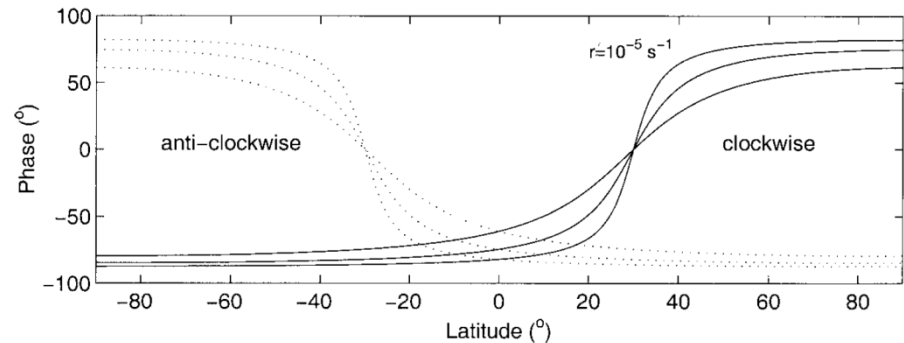
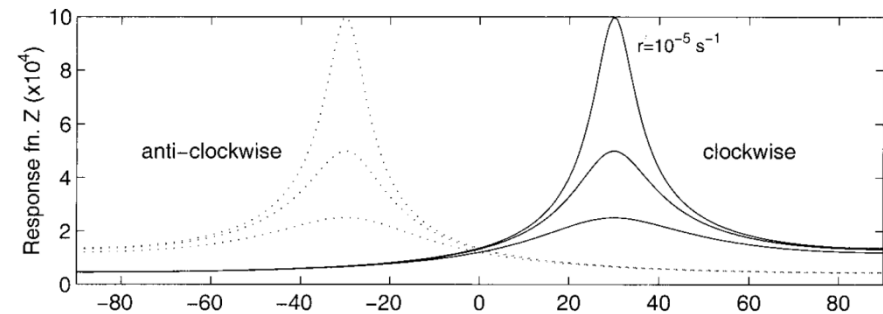
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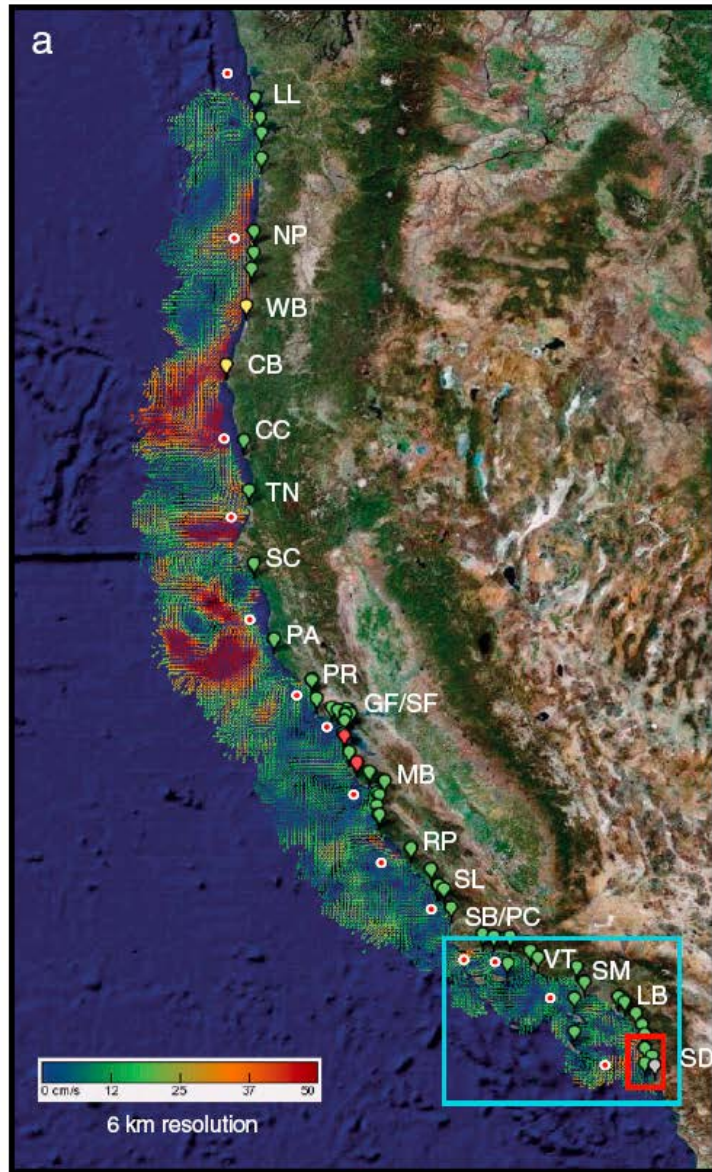
Shaffer, 1972; Ekman model



Simpson et al, JPO 2002 (Slab layer model)

Resonant latitude due to land/sea breeze: $\pm 30^\circ \text{N}$

Latitudinal coastal observations



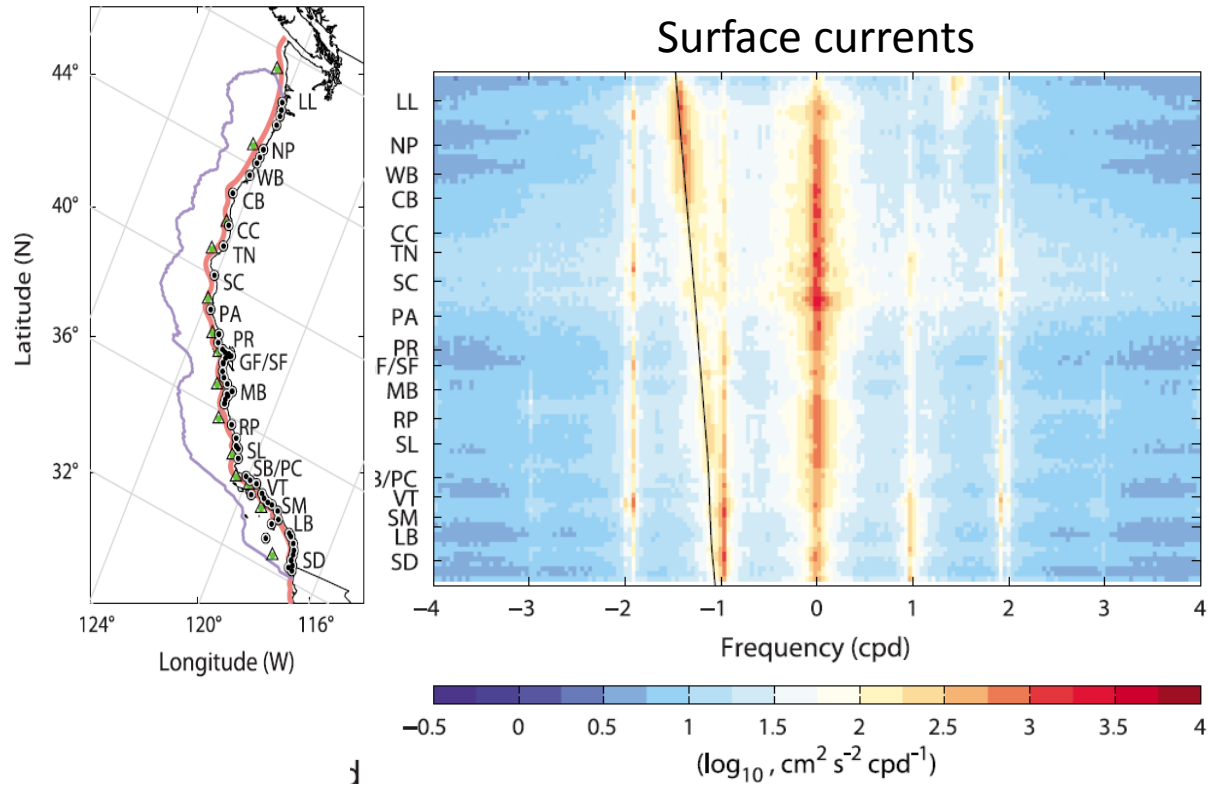
- US West Coast high-frequency radar network-derived surface currents and wind stress (red dots) at NDBC buoys.
- Latitudinal variation of 32°N to 47°N

$$\hat{\mathbf{u}}(z, \omega) = \mathbf{H}(z, \omega) \hat{\boldsymbol{\tau}}(\omega)$$

$$\mathbf{H}(z, \omega) = \left(\langle \hat{\mathbf{u}}(z, \omega) \hat{\boldsymbol{\tau}}^\dagger(\omega) \rangle \right) \left(\langle \hat{\boldsymbol{\tau}}(\omega) \hat{\boldsymbol{\tau}}^\dagger(\omega) \rangle + \mathbf{R}_a \right)^{-1}$$

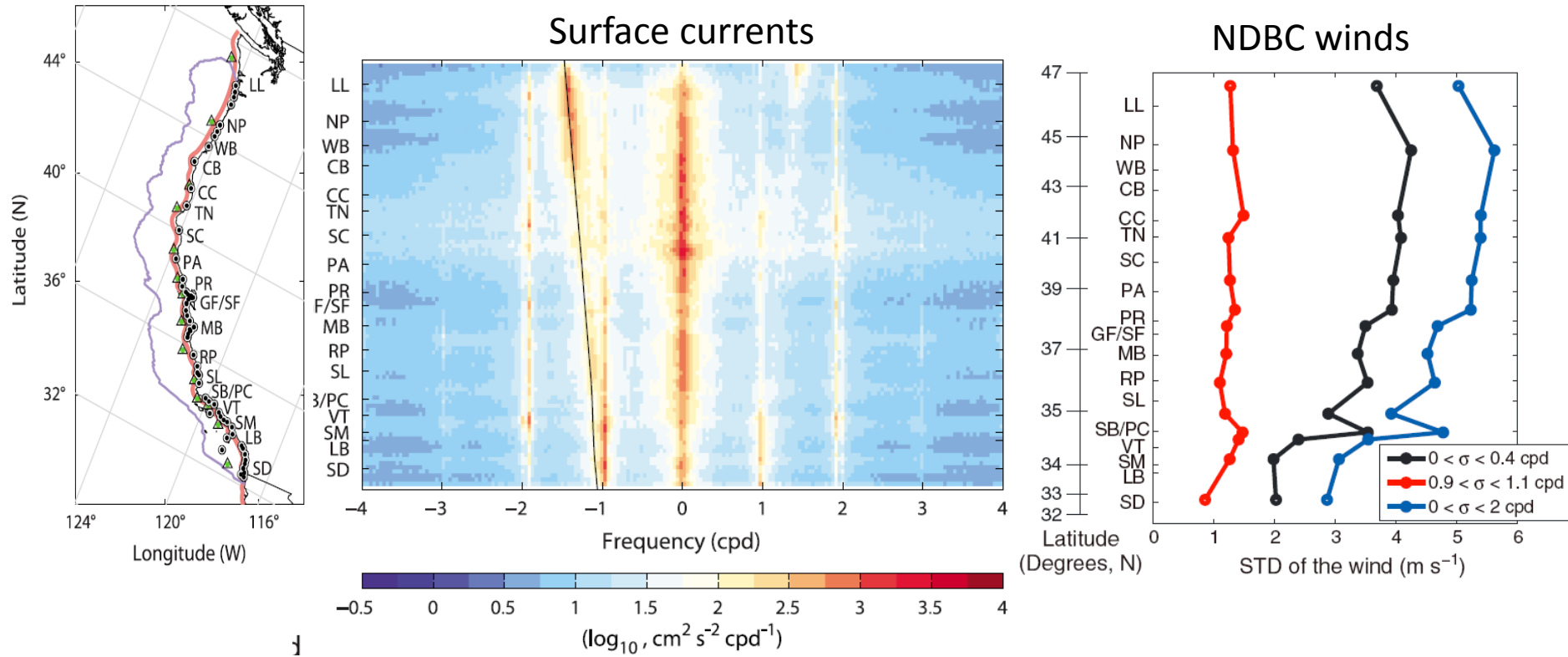
\mathbf{R}_a : Regularization matrix

Variability of surface currents and wind



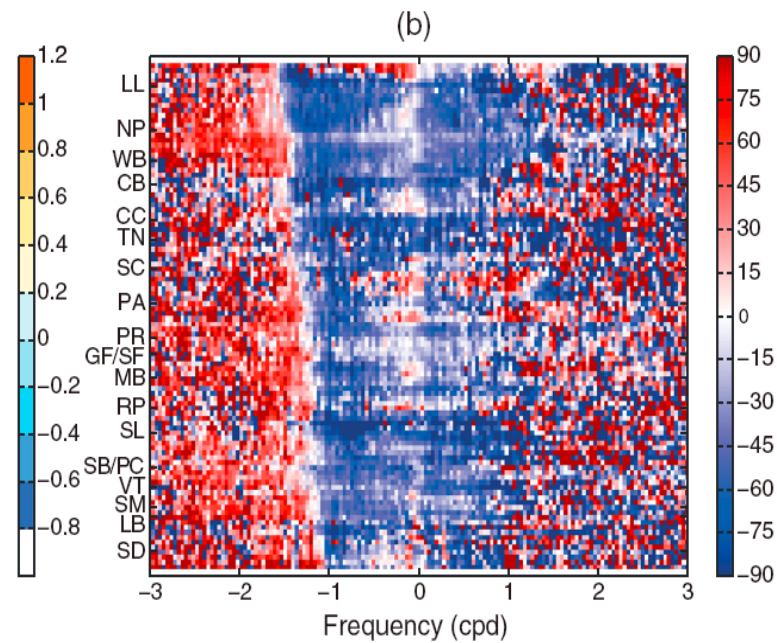
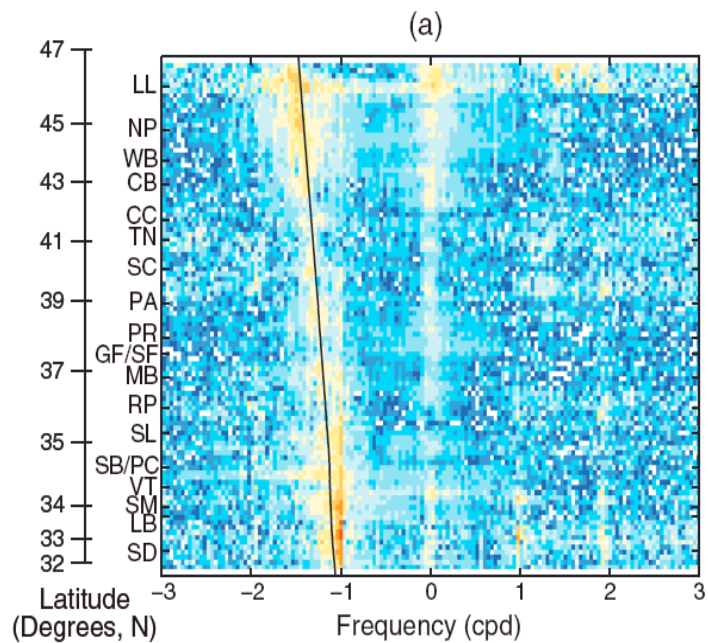
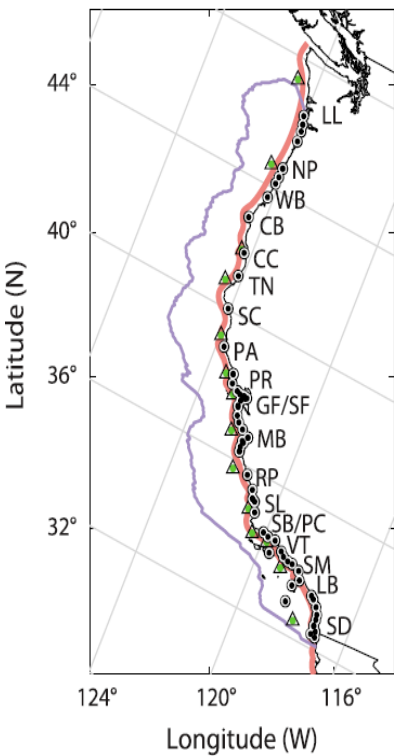
- Wind- and tide-coherent, low-frequency variance, and inertial variance

Variability of surface currents and wind



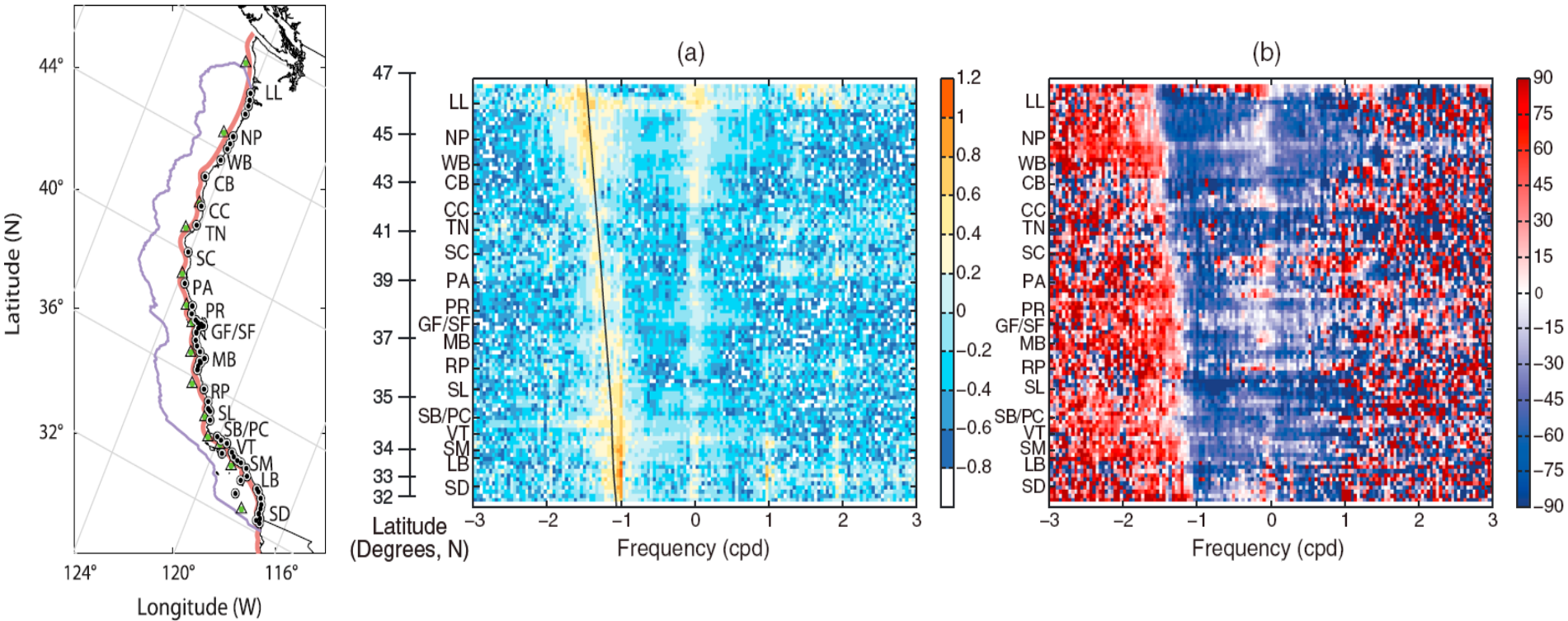
- Wind- and tide-coherent, low-frequency variance, and inertial variance
- Variance of the diurnal wind **does not vary that much in the along-shore direction**, but it is given as a function of distance from the shoreline (cross-shore direction).

Coast-wide wind transfer functions



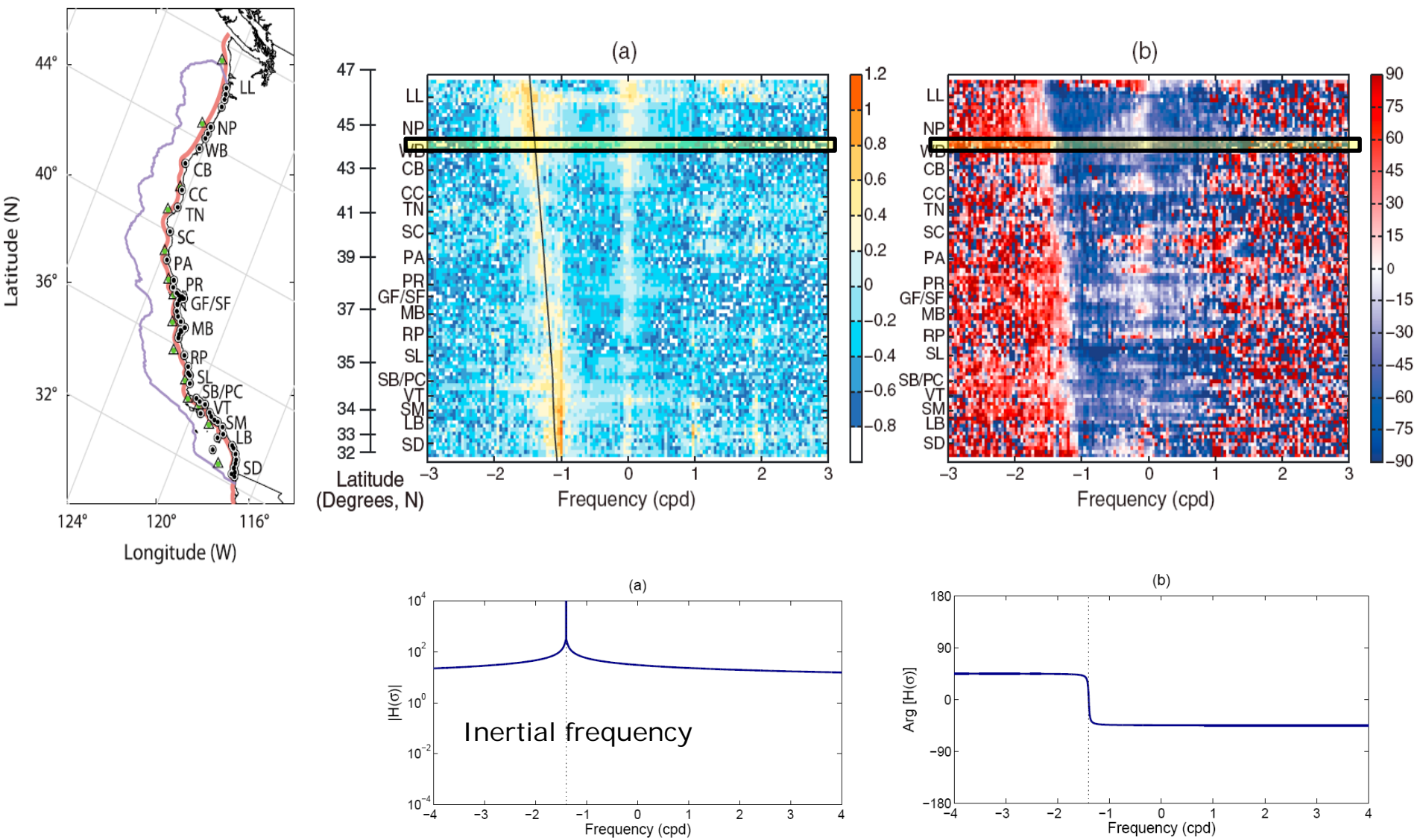
$$\hat{\mathbf{u}}(z, \omega) = \mathbf{H}(z, \omega) \hat{\boldsymbol{\tau}}(\omega)$$

Coast-wide wind transfer functions

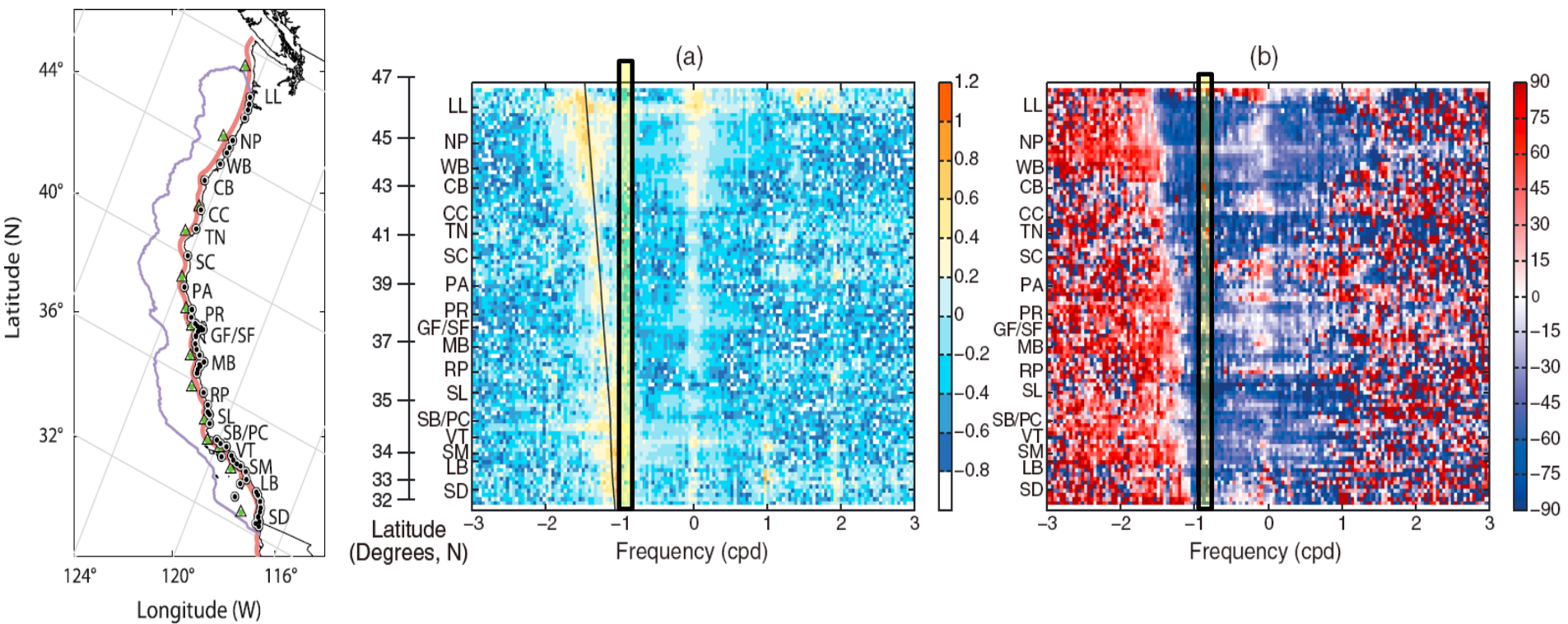


- **At a given latitude**, what would be the wind-current response in the frequency domain?
- **At a given frequency**, what would be the wind-current response as a function of latitude?

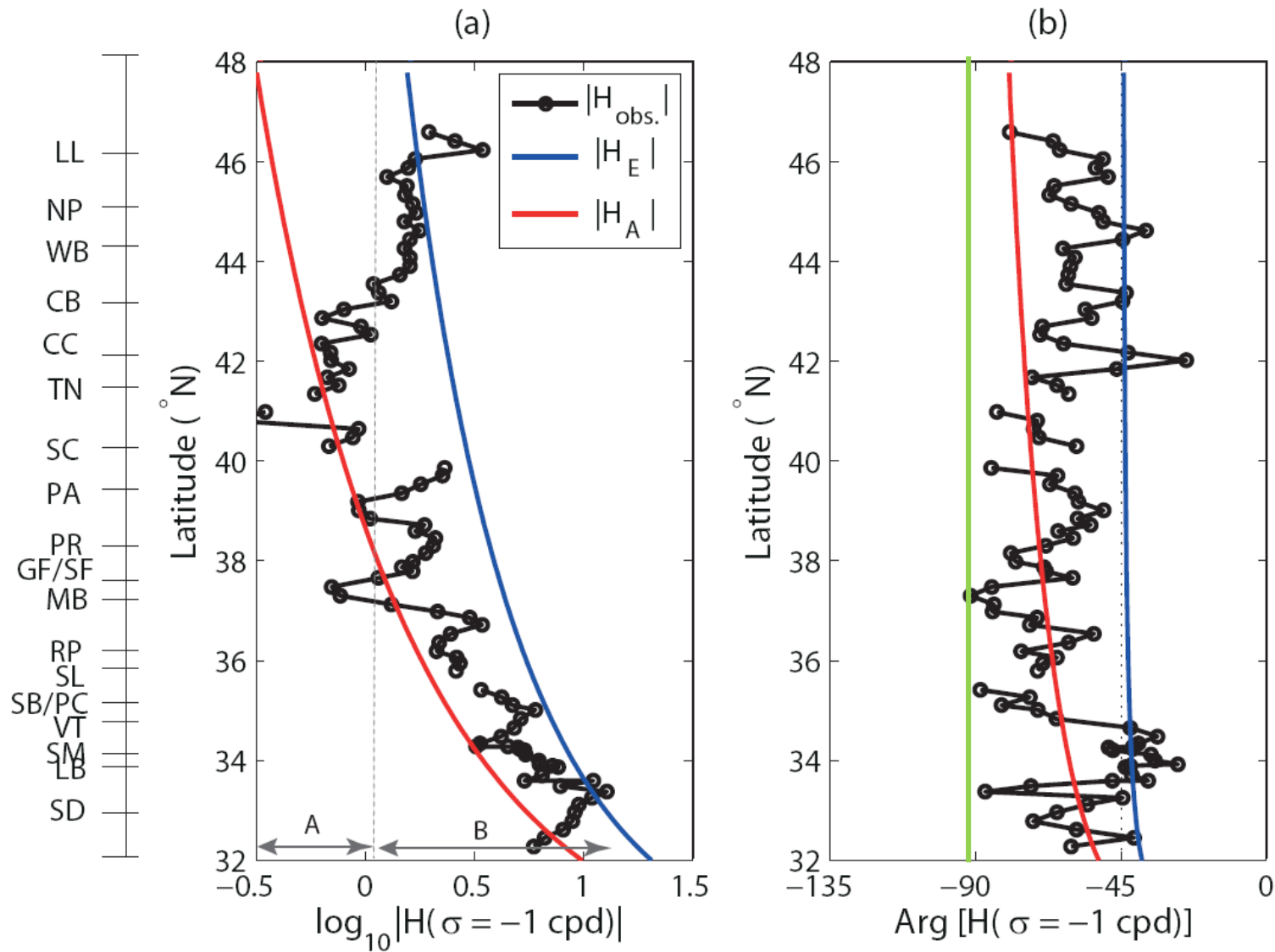
Coast-wide wind transfer functions



Coast-wide wind transfer functions



Resonant responses near the critical latitude



Slab layer model

$Z = 0$ (Ekman)

$Z = 0.35\delta_E$ (Near-surface avg. Ekman)

Resonant latitude due to land/sea breeze: $\pm 30^{\circ} \text{N}$

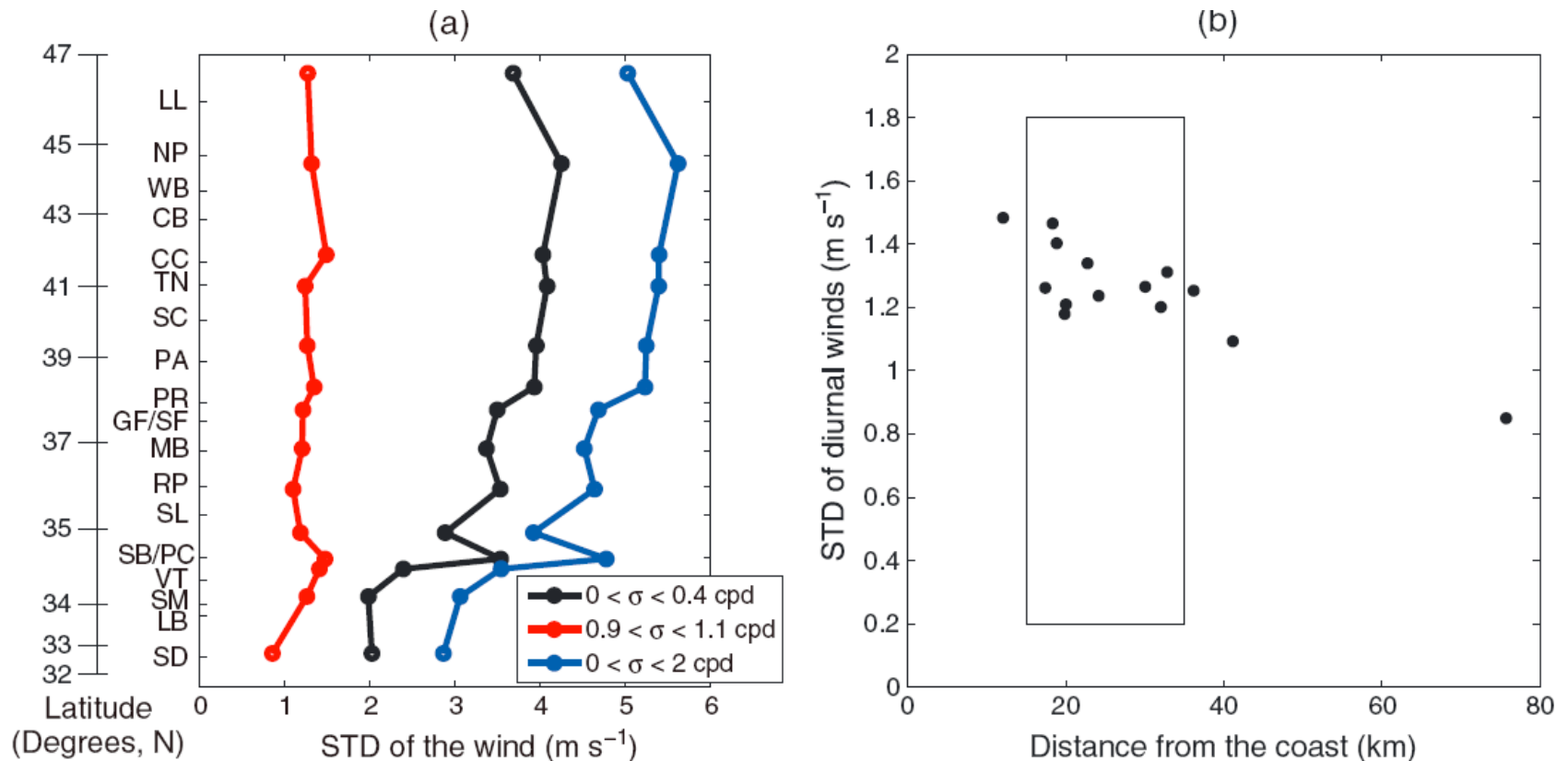
Summary

- Wind-current responses are examined in the frequency domain and latitude using analytic solutions of Ekman model (and slab layer and surface-averaged Ekman models) and observations off the US West Coast.
- The current responses are enhanced at the local inertial frequency.
- Resonant responses can be expected at the $\pm 30^\circ$ latitude in the diurnal land-sea breeze environment.
- Energetic mixing and potential internal motions near the critical latitude are expected.

BACKUP SLIDES

Wind variability

- Variance of the diurnal wind **does not vary that much in the along-shore direction**, but it is given as a function of distance from the shoreline (cross-shore direction).



Resonant responses at the critical latitude

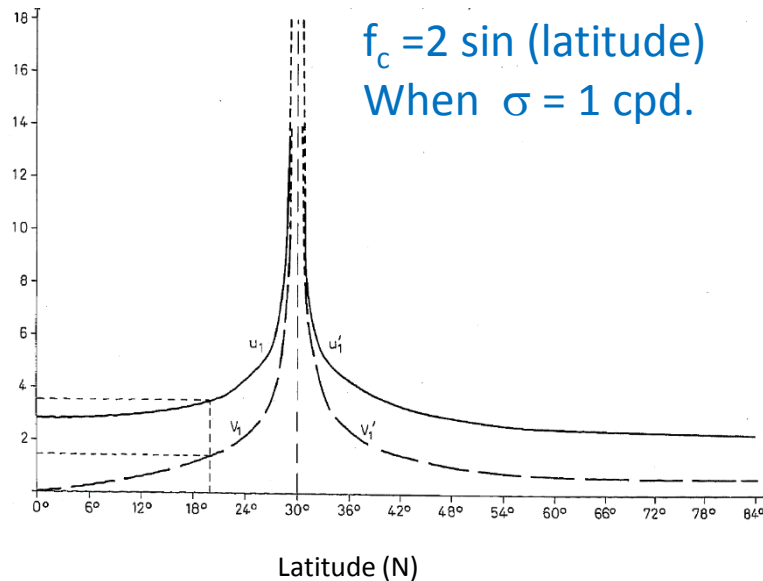
$$\frac{\partial \mathbf{u}}{\partial t} + if_c \mathbf{u} + r\mathbf{u} = \frac{1}{\rho} \frac{\partial \boldsymbol{\tau}}{\partial z}$$

$$\frac{\partial \mathbf{u}}{\partial t} + if_c \mathbf{u} + r\mathbf{u} = \frac{\boldsymbol{\tau}^W}{\rho h}$$

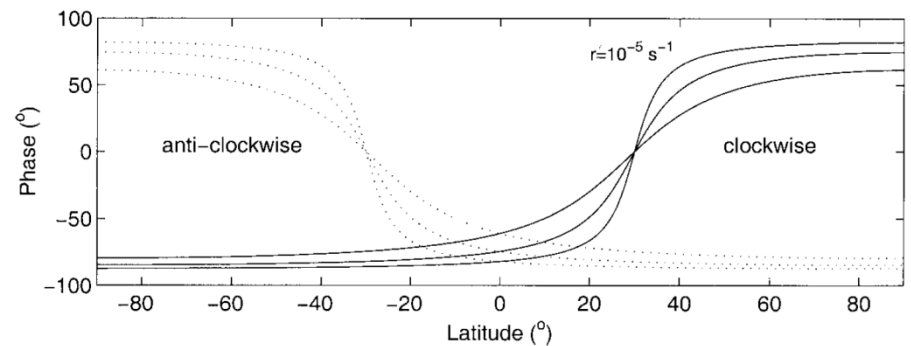
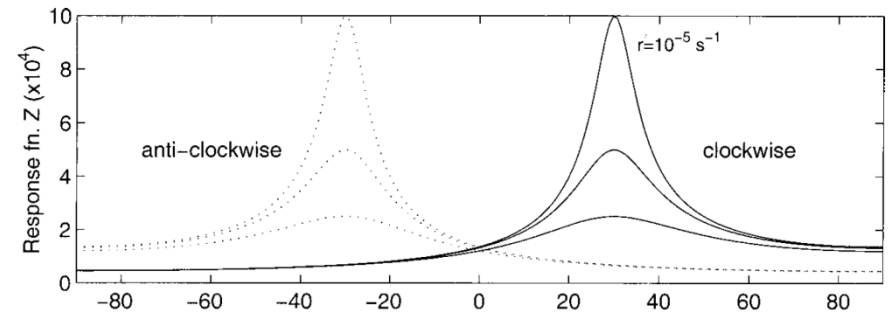
$$\mathbf{H}_E(z, \sigma) = \frac{\hat{\mathbf{u}}(z, \sigma)}{\hat{\boldsymbol{\tau}}(\sigma)} = \frac{e^{\lambda z}}{\lambda \rho \nu}$$

$$\mathbf{H}_S(\sigma) = \frac{\hat{\mathbf{u}}(\sigma)}{\hat{\boldsymbol{\tau}}(\sigma)} = \frac{1}{\rho h [i(\sigma + f_c) + r]}$$

$$\lambda = \sqrt{[i(\sigma + f_c) + r] / \nu}$$



Shaffer, 1972; Ekman model



Simpson et al, JPO 2002 (Slab layer model)

Resonant latitude due to land/sea breeze: $\pm 30^\circ \text{N}$

Resonant responses at the critical latitude

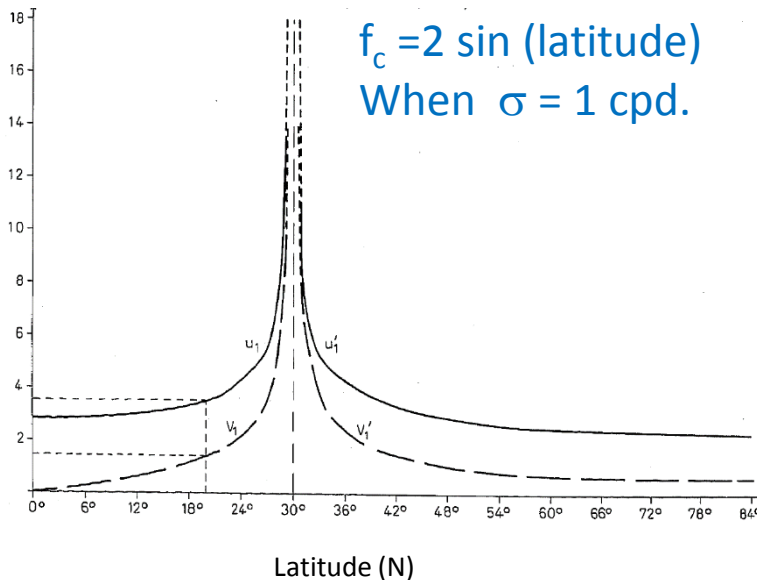
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$$\mathbf{H}_E(z, \sigma) = \frac{\hat{\mathbf{u}}(z, \sigma)}{\hat{\boldsymbol{\tau}}(\sigma)} = \frac{e^{\lambda z}}{\lambda \rho \nu}$$

$$\lambda = \sqrt{[i(\sigma + f_c) + r] / \nu}$$

$$\begin{aligned} \mathbf{H}_A(\sigma) &= \frac{1}{z^*} \int_0^{z^*} \mathbf{H}_E(z, \sigma) dz, \\ &= \frac{1}{z^*} \frac{1}{\lambda^2 \rho \nu} (e^{\lambda z^*} - 1), \end{aligned}$$

Near-surface averaged Ekman model can be appropriate to explain the HFR-derived surface currents.



Shaffer, 1972; Ekman model

Resonant latitude due to land/sea breeze: $\pm 30^\circ \text{N}$