

Surface horizontal diffusivity estimated from submesoscale observations of surface currents and passive tracers

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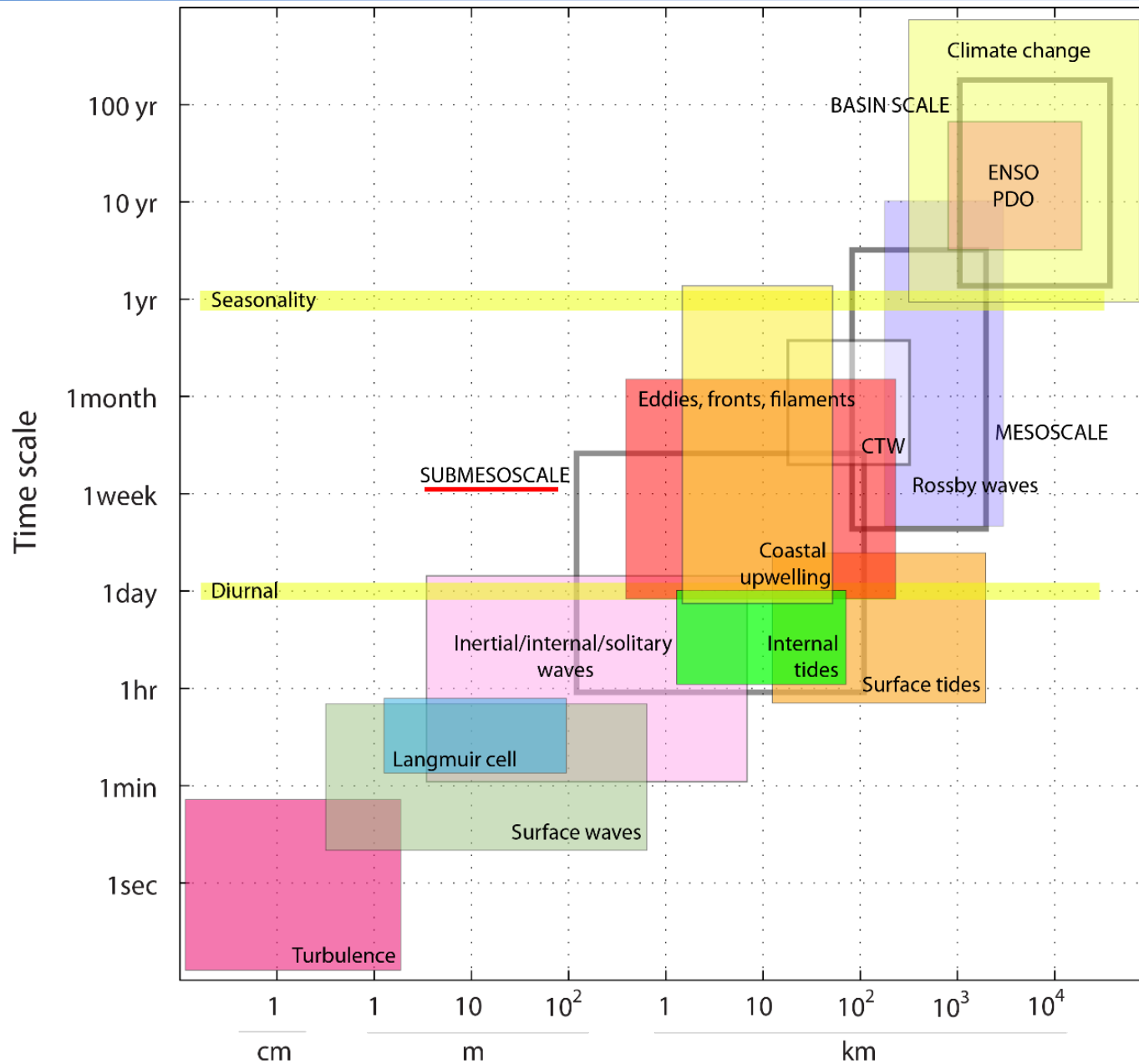
Motivation

- In tracking of water-borne materials (pollutants, search-rescue, larvae, etc), understanding of their advection and diffusion matter.
- Particularly, the physical processes at submesoscale (less than one hour in time and 100 km in space) have got attentions in ocean science community.
- Available observational resources of coastal radar-derived surface currents (km, hourly) and GOCI imagery (km, hourly) can be used to examine the advection-diffusion issue.
- We try to quantify the diffusion based on observations, which can be applicable to tracking water-borne materials with cautions in handling the observational noise and errors....

Outline

- Introduction to oceanic submesoscale processes
- Advection-diffusion equations
- Simulations with idealized currents and density maps
- Conclusion

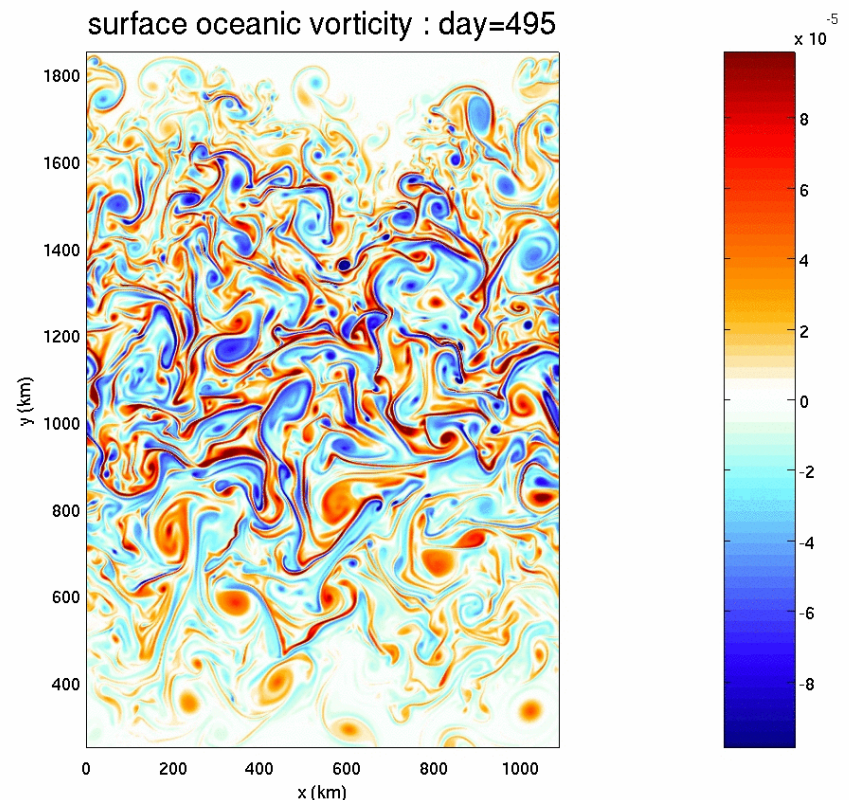
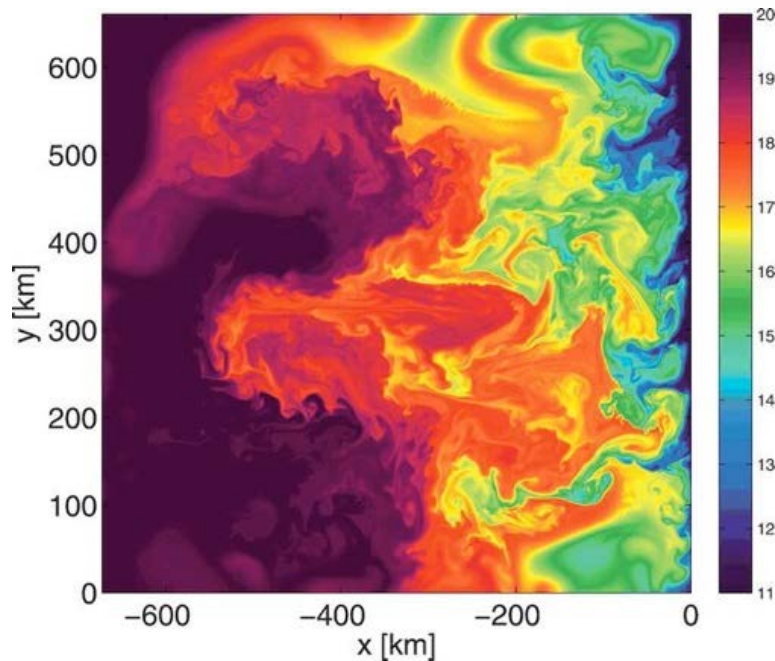
Oceanic processes in time and spatial scales



(Chelton 2001, Dickey *et al*, RG 2006; Kim 2015)

Submesoscale processes

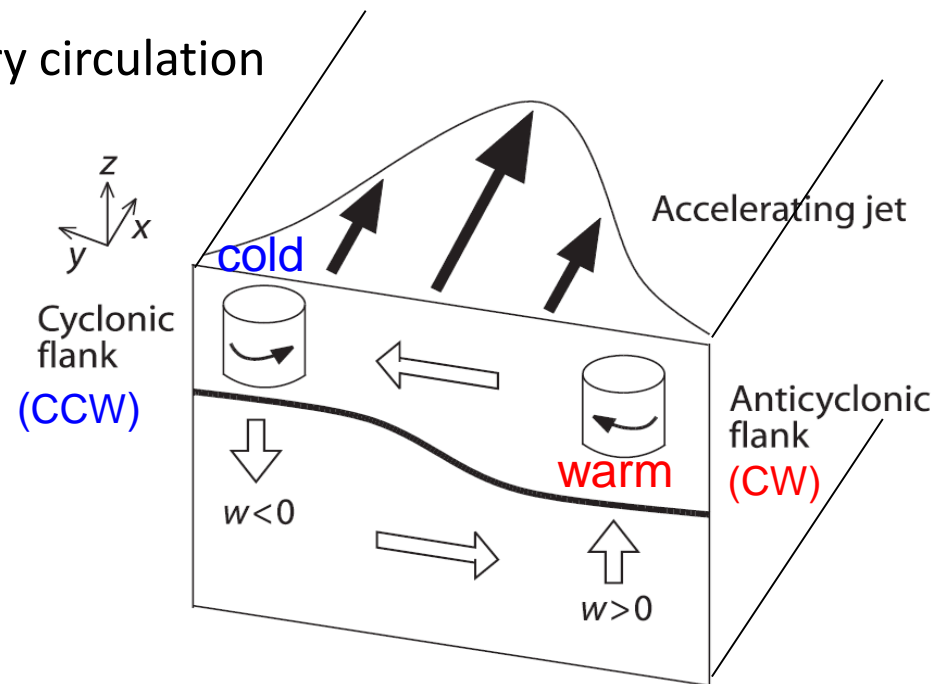
- $O(1)$ Rossby number [$Ro = \zeta/f$]
- A horizontal scale smaller than the first baroclinic Rossby deformation radius; $O(1-10)$ km
- Frequently observed as fronts, eddies, and filaments



(Courtesy of X. Capet and P. Klein)

Submesoscale processes

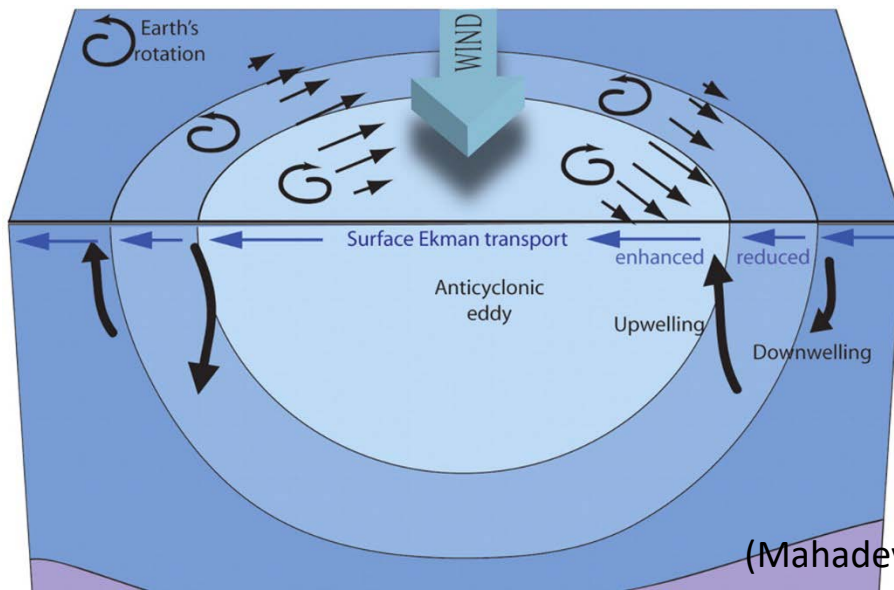
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- Contribute to the vertical transport of oceanic tracers, mass, and buoyancy and rectify the mixed-layer structure and upper-ocean stratification
 - e.g., vertical frontal scale secondary circulation



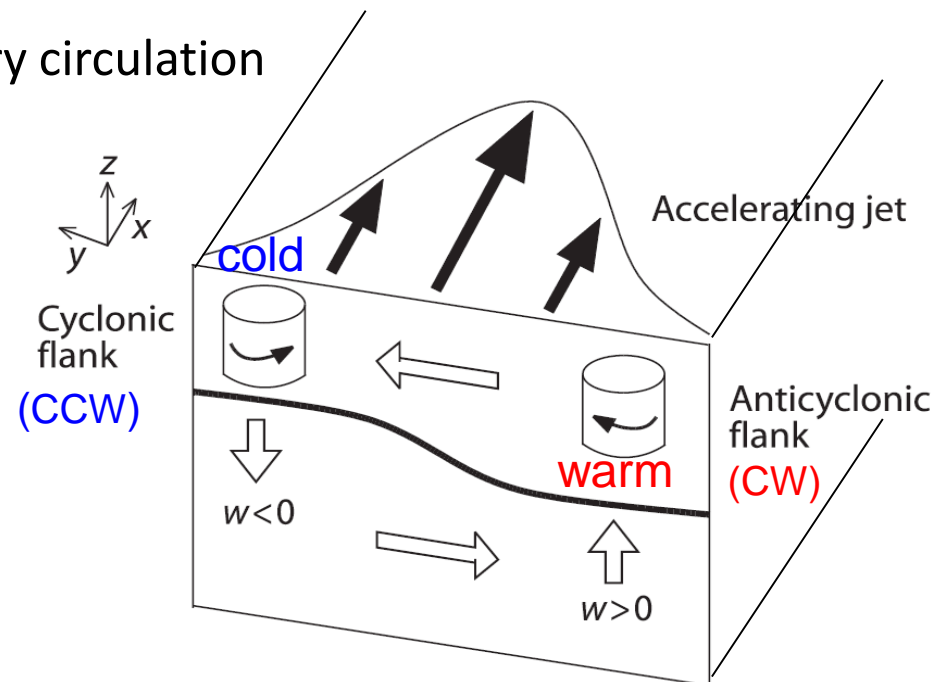
(Williams and Follows, 2003)

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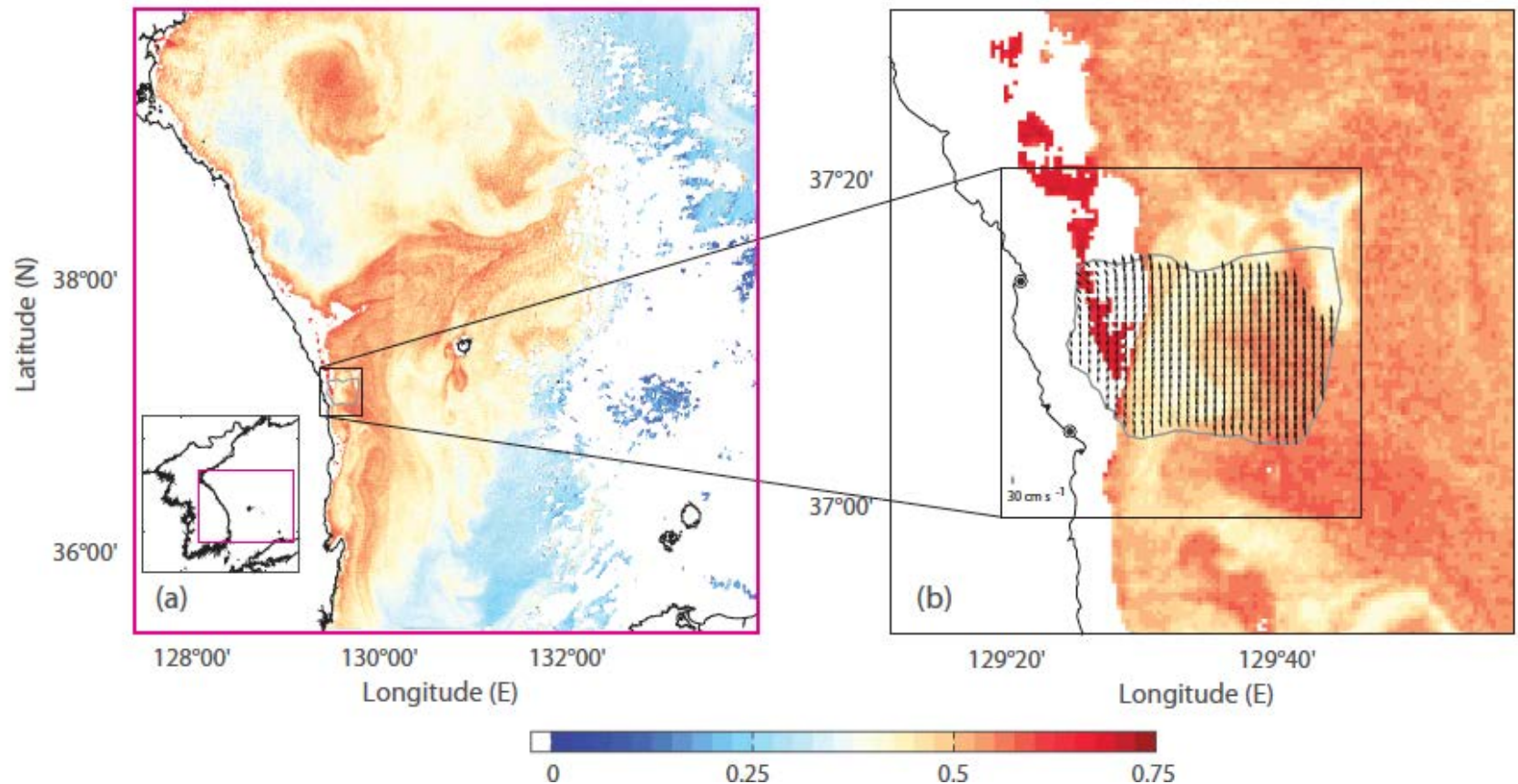
(Mahadevan et al, Science, 2008)



(Williams and Follows, 2003)

Submesoscale process studies

- Have been benefited from primarily idealized numerical models and theoretical frameworks because they require the use of high-resolution observations of **less than one hour in time and $O(1-10)$ km in space.**



Advection-diffusion equations (2D)

$$\frac{\partial C(\mathbf{x}, t)}{\partial t} + \mathbf{u}(\mathbf{x}, t) \frac{\partial C(\mathbf{x}, t)}{\partial \mathbf{x}} = \kappa \nabla^2 C(\mathbf{x}, t)$$

where

- $C(\mathbf{x}, t)$: Concentration of harmful algae and pollutants. In-situ observations, satellite-derived maps of passive tracers (e.g., Chlorophyll; TSS; CDOM) [e.g., GOCI, AVHRR products]
- $\mathbf{u}(\mathbf{x}, t)$: (Geostrophic) current field [e.g., HFR surface current maps; AVISO geostrophic currents]
- $\kappa(\mathbf{x}, t)$: Diffusion coefficients [Unknowns]

Known current fields and concentration maps

- A spectral model is built based on **spectra of observed surface currents.**

$$u(x, y, t) = \sum_{m=-M^*}^{M^*} \sum_{n=-N^*}^{N^*} \sum_{s=-S^*}^{S^*} \hat{A}_{mns} \cos \vartheta_{mns} + \hat{B}_{mns} \sin \vartheta_{mns}, \quad (\text{A1})$$

$$v(x, y, t) = \sum_{m=-M^*}^{M^*} \sum_{n=-N^*}^{N^*} \sum_{s=-S^*}^{S^*} \hat{C}_{mns} \cos \vartheta_{mns} + \hat{D}_{mns} \sin \vartheta_{mns},$$

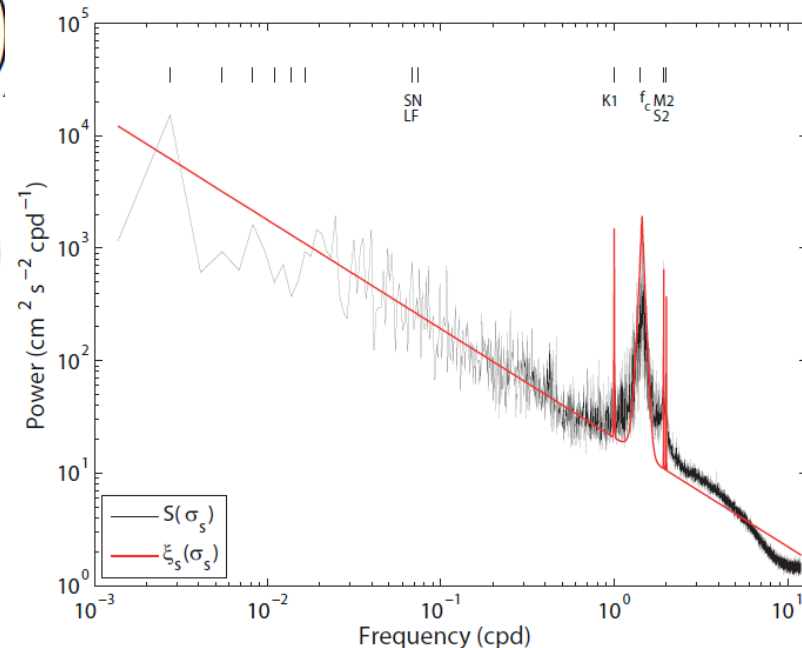
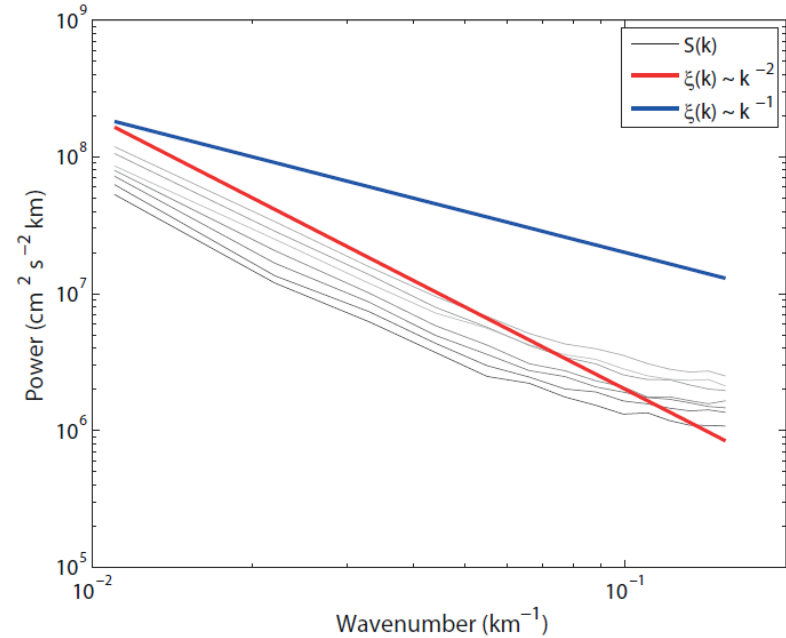
$$\vartheta_{mns} = 2\pi (k_m x + l_n y - \sigma_s t) = 2\pi \left(\frac{m}{L_x} x + \frac{n}{L_y} y - \sigma_s t \right)$$

$$\hat{A}_{mns} = (\zeta_{mn})^{\frac{1}{2}} \zeta_s \mathbf{N}(0, 1),$$

$$\zeta(k_m, l_n) = \pi \lambda_x \lambda_y \exp \left(-\pi^2 k_m^2 \lambda_x^2 - \pi^2 l_n^2 \lambda_y^2 \right)$$

$$\zeta(k_m, l_n) = \frac{4\lambda_x \lambda_y}{(1 + 4\pi^2 k_m^2 \lambda_x^2 + 4\pi^2 l_n^2 \lambda_y^2)^{3/2}},$$

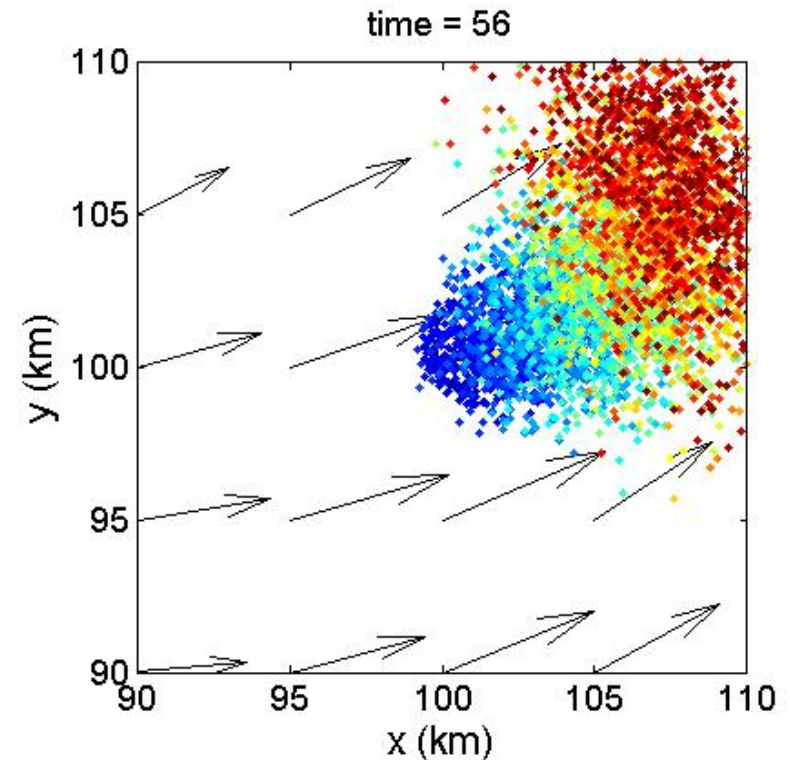
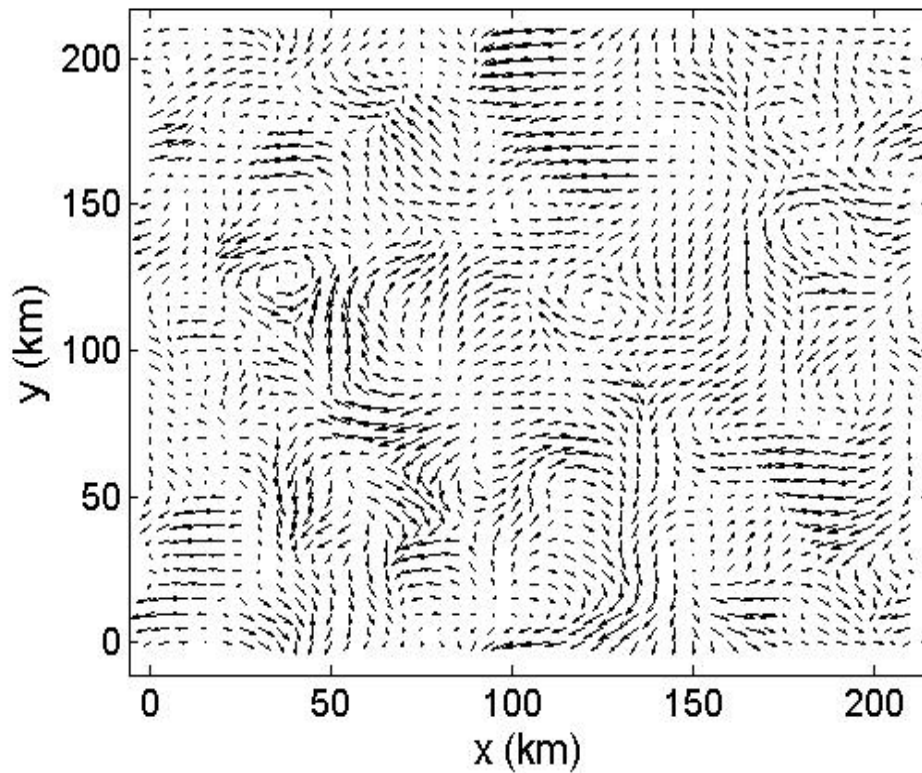
$$\zeta_s(\sigma_s) = A \sigma_s^{-\alpha} + \sum_{n=1}^N B_n \exp \left(-\frac{|\sigma_s - \nu_n|}{(\lambda_t)_n} \right),$$



Known current fields and concentration maps

- A spectral model, built based on spectra of observed surface currents, generates current fields in space and time.
- From particle tracking using a random walk scheme, we generate the concentration map of tracer.

$$x(t) = \int_{t_0}^t (u(t') + \varepsilon^u) dt' + x(t_0) \approx \sum_k (u(t_k) + \varepsilon_k^u) \Delta t + x(t_0)$$

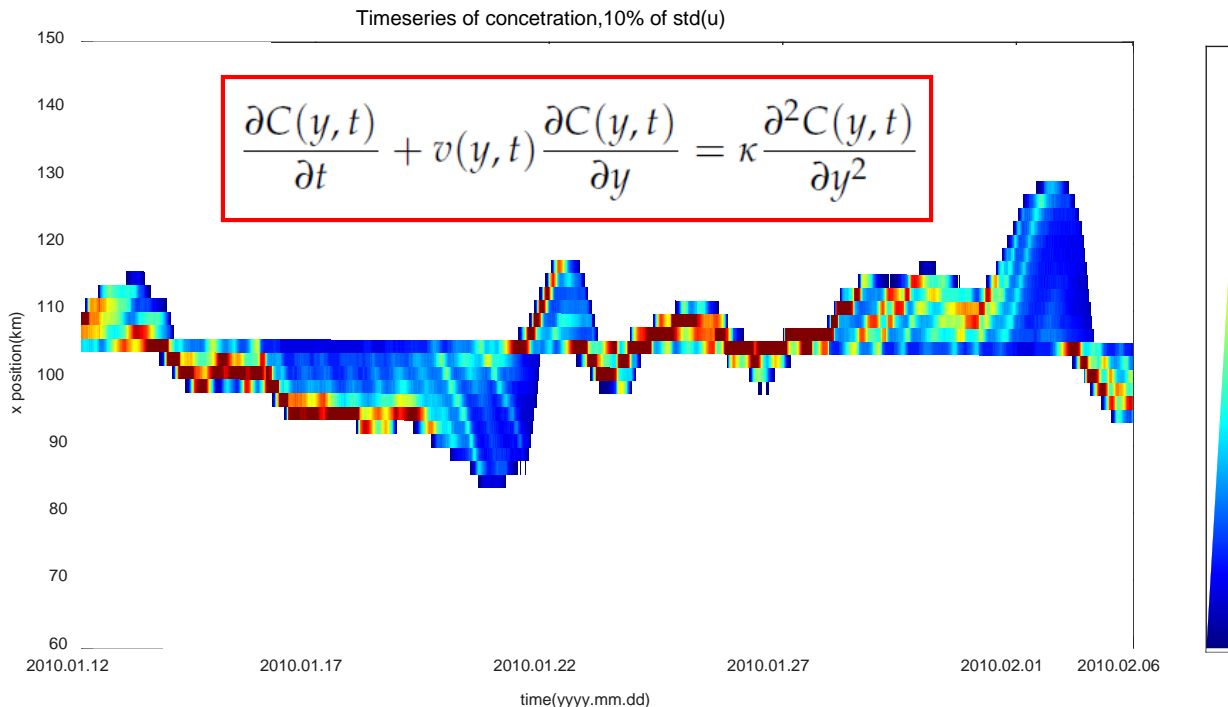


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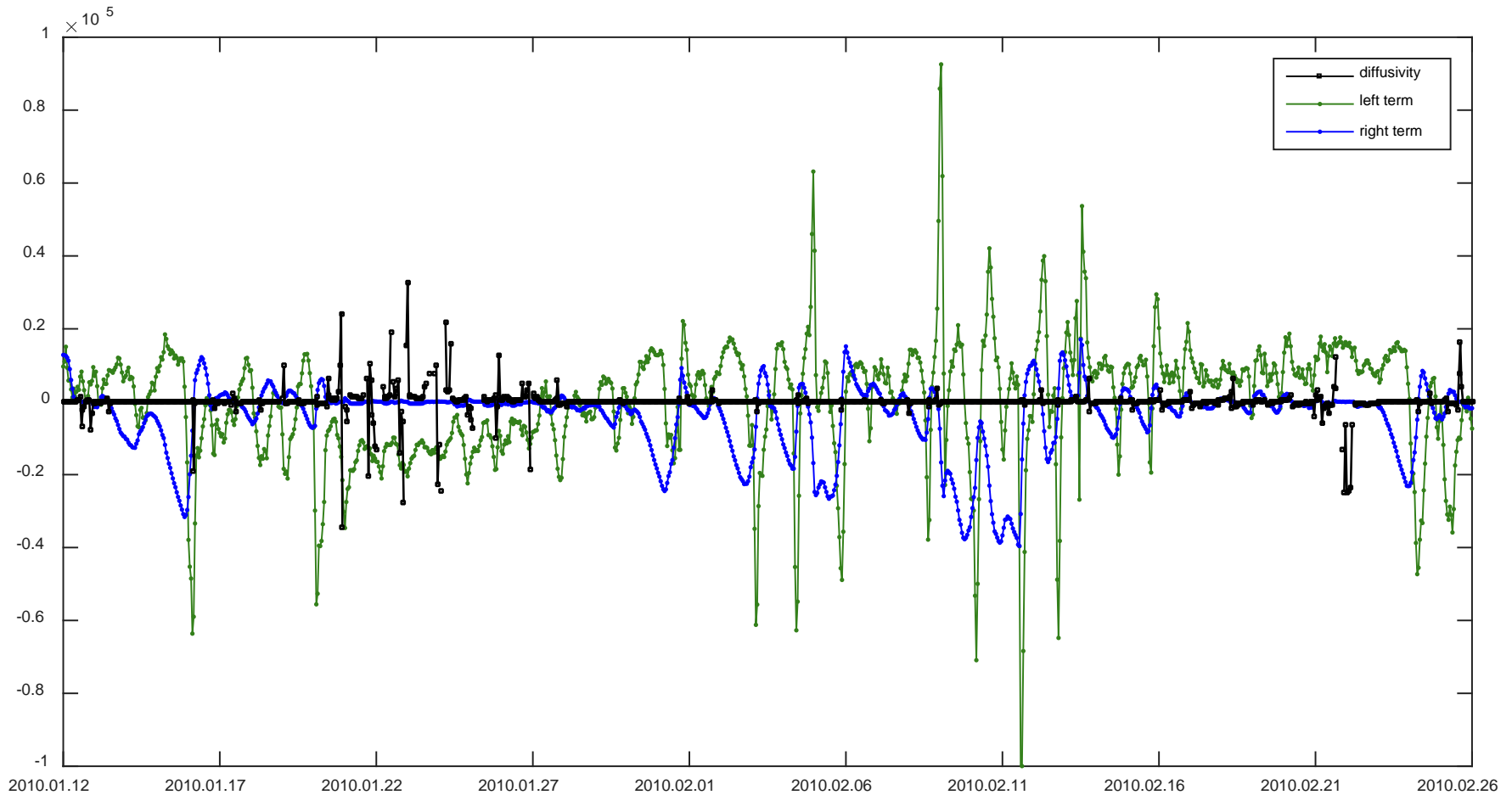
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- Find a relationship between a random parameter (ε) and diffusion coefficient (κ) in the equation (2D->1D).

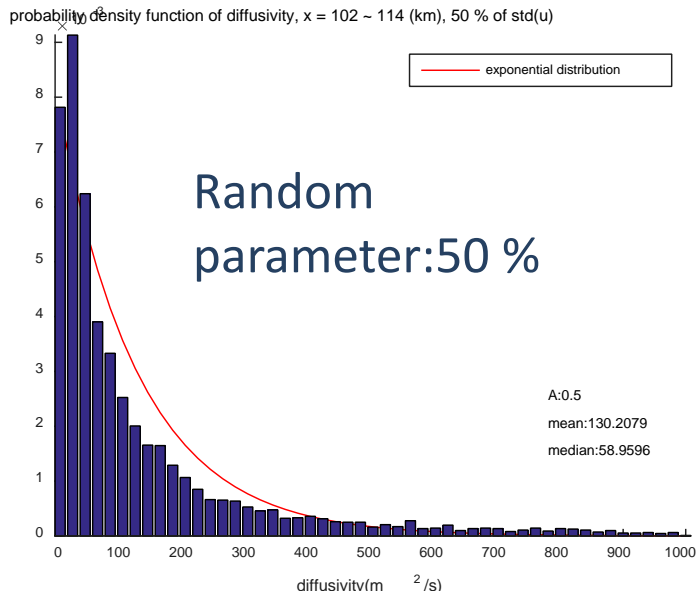
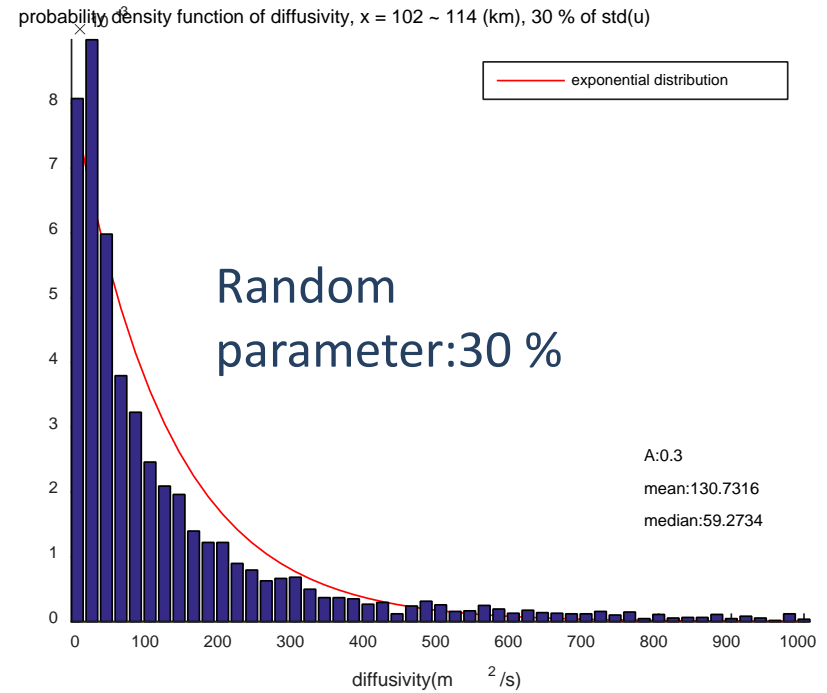
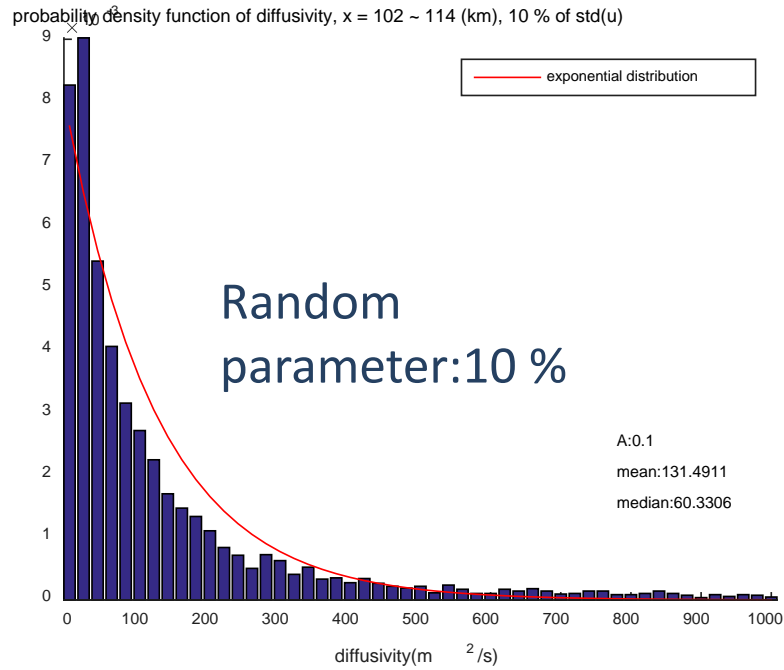


An example of time series in AD equations

$$\frac{\partial C(y, t)}{\partial t} + v(y, t) \frac{\partial C(y, t)}{\partial y} = \kappa \frac{\partial^2 C(y, t)}{\partial y^2}$$

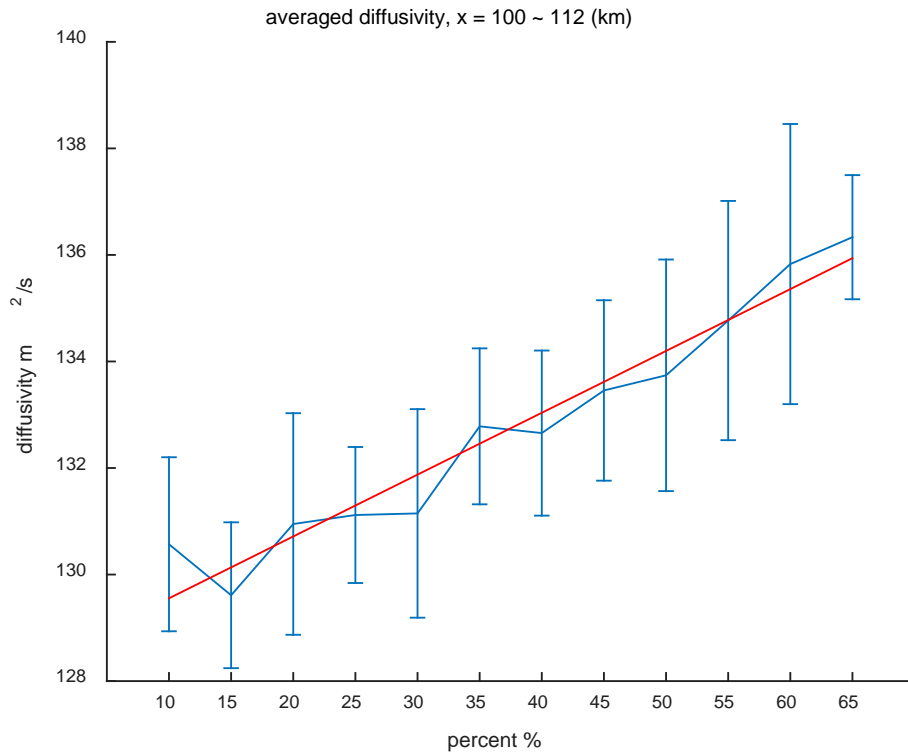


Distribution of diffusion coefficients



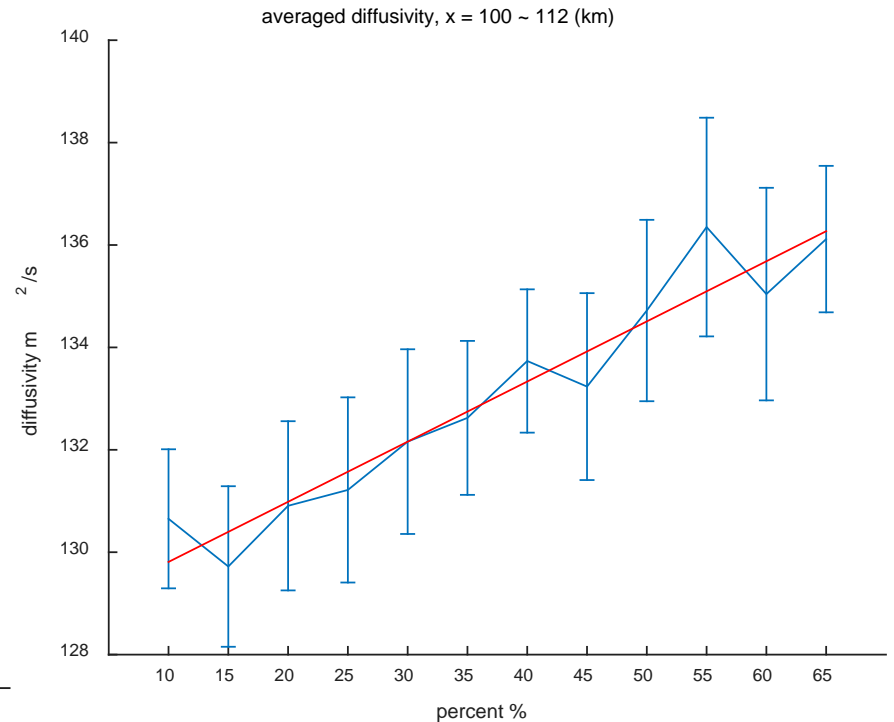
- 1) particle 20, life span: 50h
- 2) abnormally large diffusivity is removed (>1000)

Random parameter and diffusion coefficient



$$b = 0.1161a + 128.3934$$

residual = 1.6680



$$b = 0.1174 a + 128.6356$$

residual = 2.0005

a: random parameter
b: diffusivity

Conclusion

- Given observations of surface currents and GOCI Chlorophyll maps, diffusivity at submesoscale can be investigated
- Using idealized current fields and concentration maps, diffusion coefficients are estimated based on advection-diffusion equations (1D).
- Simulations using random walk scheme and estimate diffusion coefficients are quantified.
- Diffusion coefficients estimated from observations can be implemented with a random walk model for tracking water-borne contaminants.

Thank you for your attention!

