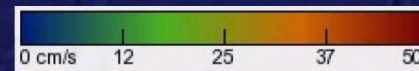


# Characterization of oceanic mesoscale and sub-mesoscale energy spectra

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## Collaborators:

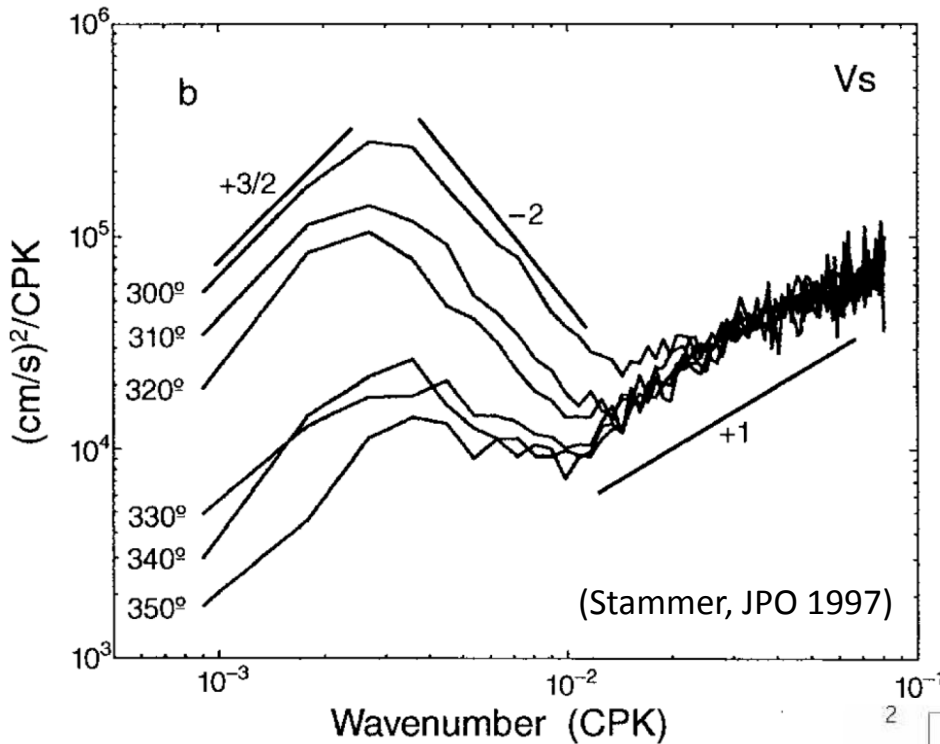
Bruce Cornuelle, Burt Jones, Libe Washburn, Mark Moline,  
Jeffrey Paduan, Toby Garfield, John L. Largier, Greg Crawford  
P. Michael Kosro, and Xavier Capet



# Outline

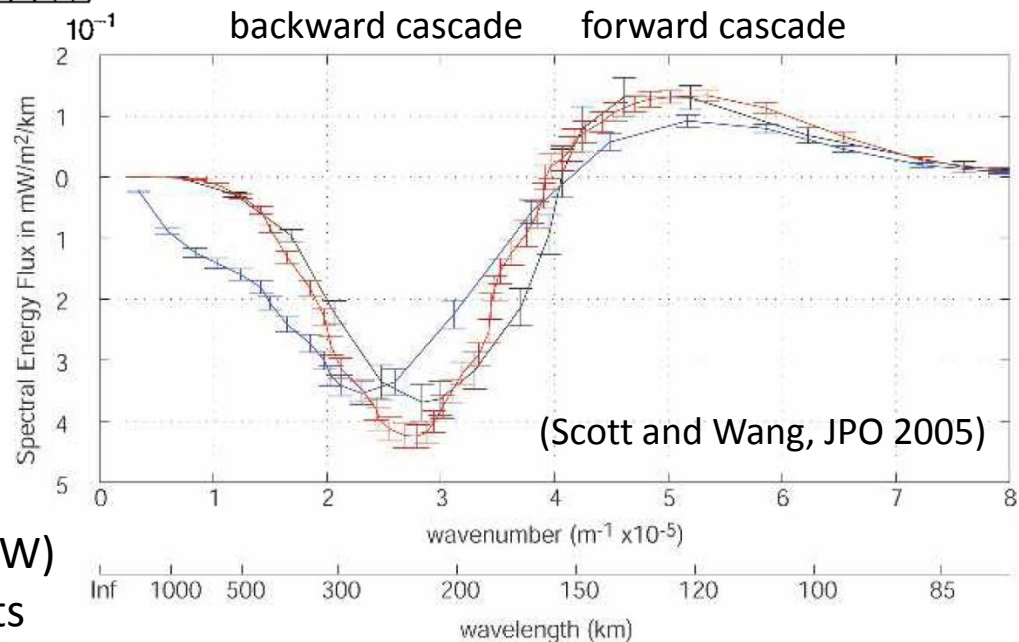
- Motivation
  - Oceanic processes in time and spatial scales
  - Wavenumber energy spectra derived from satellite altimeters
- Overview
  - Coastal surface current measurements using high-frequency radar (HFR)
  - Spectral contents in coastal surface currents off the U.S. West Coast
- Comparison of energy spectra
  - Energy spectra in wavenumber (1D) and frequency domain
  - Energy flux estimates from O(1) km HFR surface currents and sub-mesoscale model results
- Summary

# Motivation

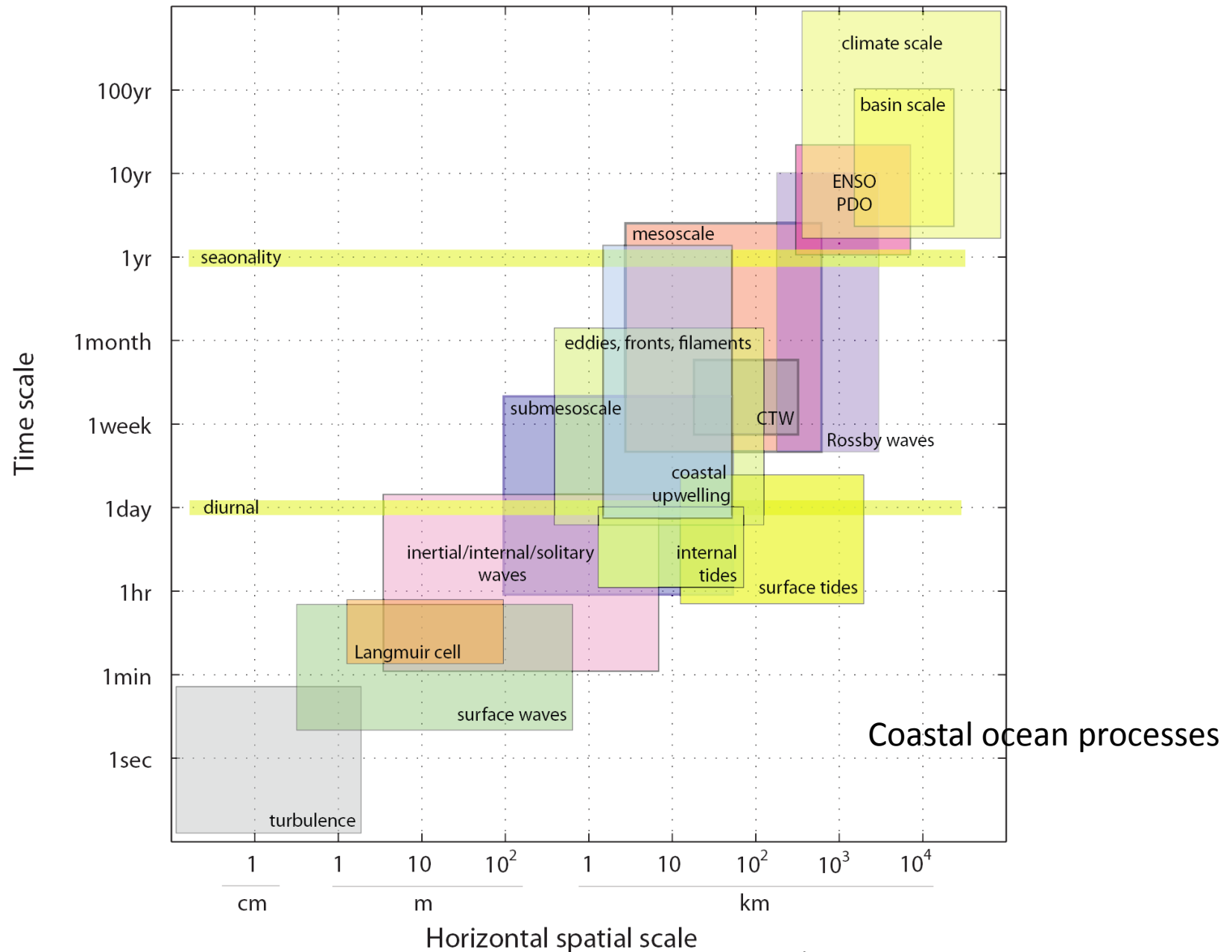


- What can be the slope of energy spectra and kinetic energy flux below 100 km scale?

Kinetic energy flux in ACC region (57S, 120W)  
 Optimally interpolated 1/3° AVISO products

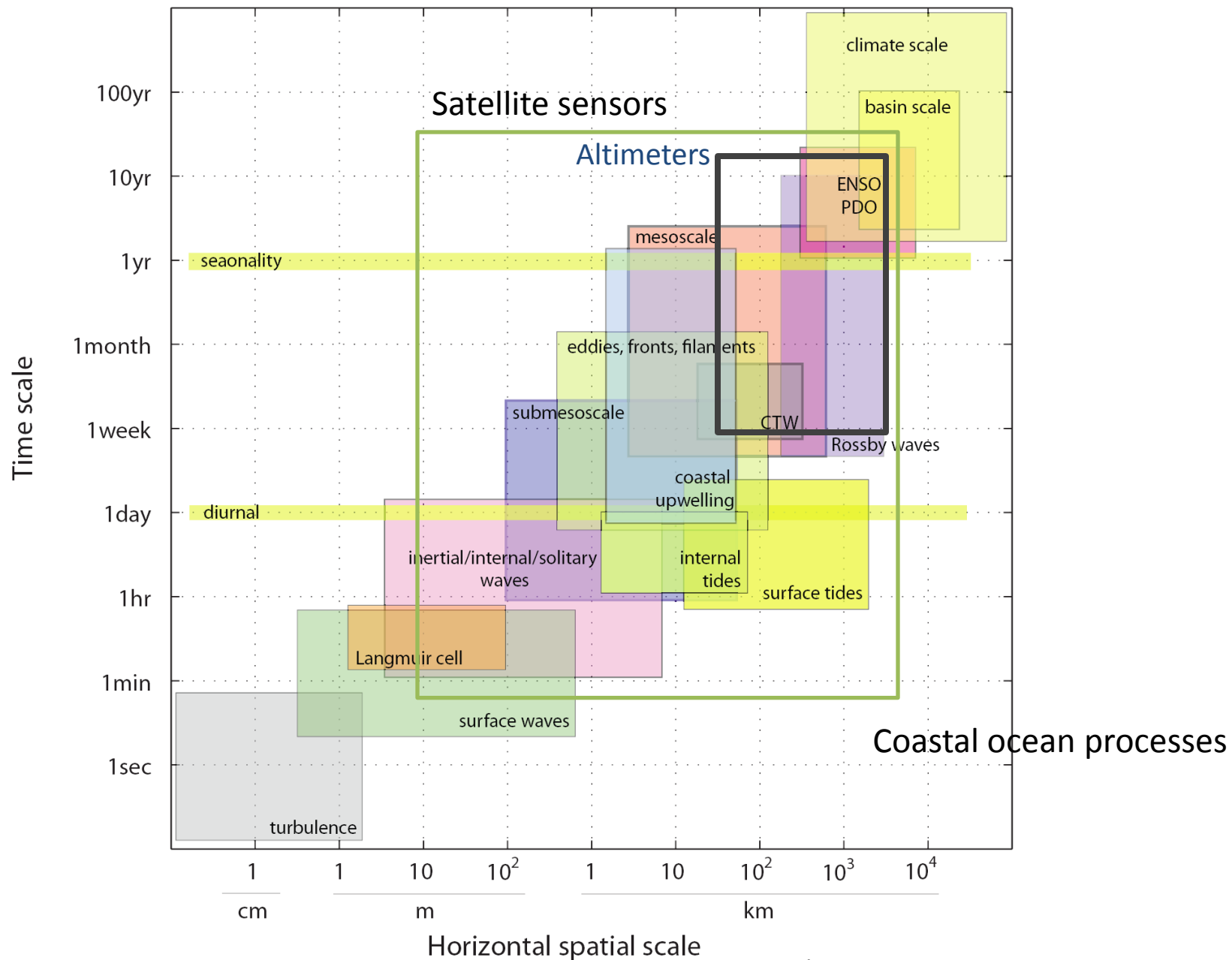


# Oceanic processes in time and spatial scales



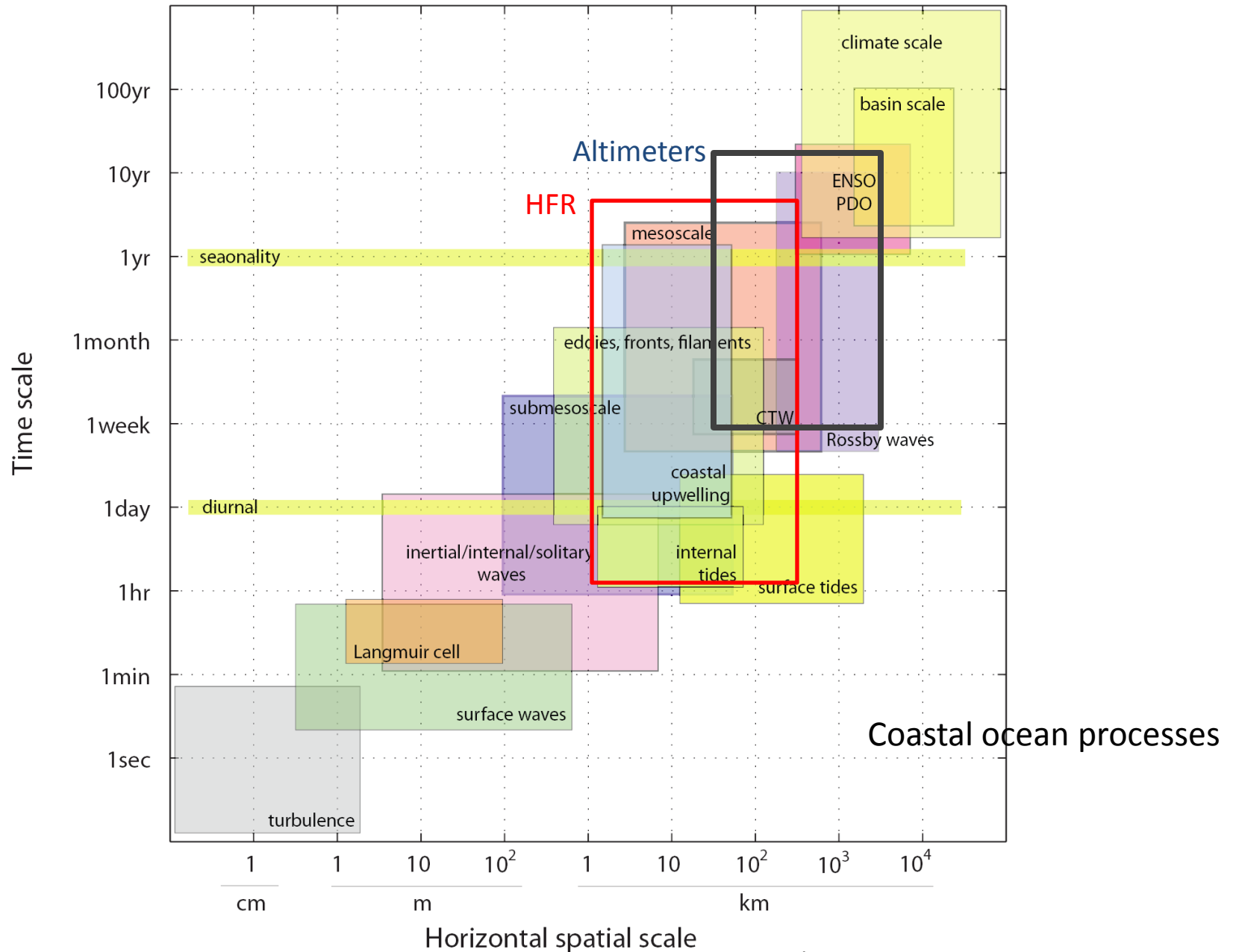
(Chelton 2001, Dickey *et al*, RG 2006)

# Oceanic processes in time and spatial scales



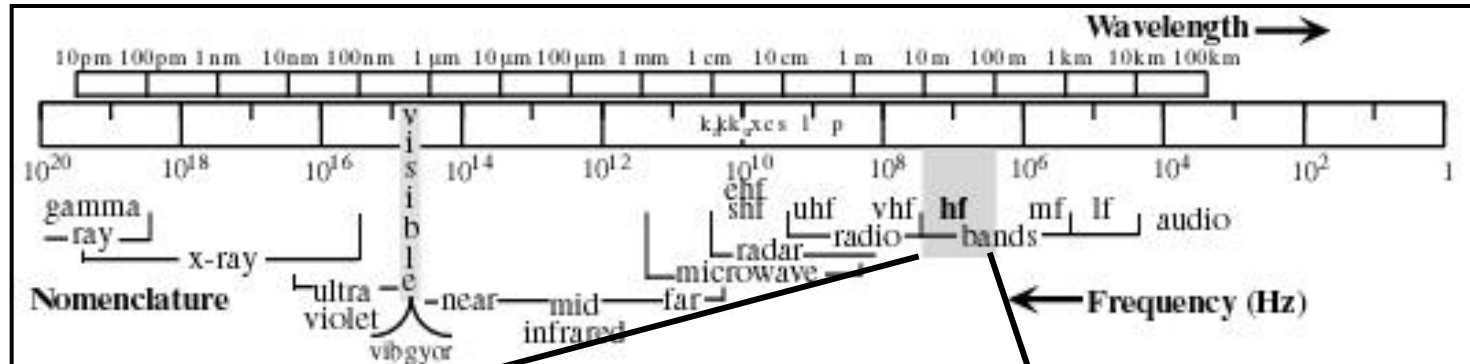
(Chelton 2001, Dickey *et al*, RG 2006)

# Oceanic processes in time and spatial scales



(Chelton 2001, Dickey *et al*, RG 2006)

# Radio signals used in high-frequency radar



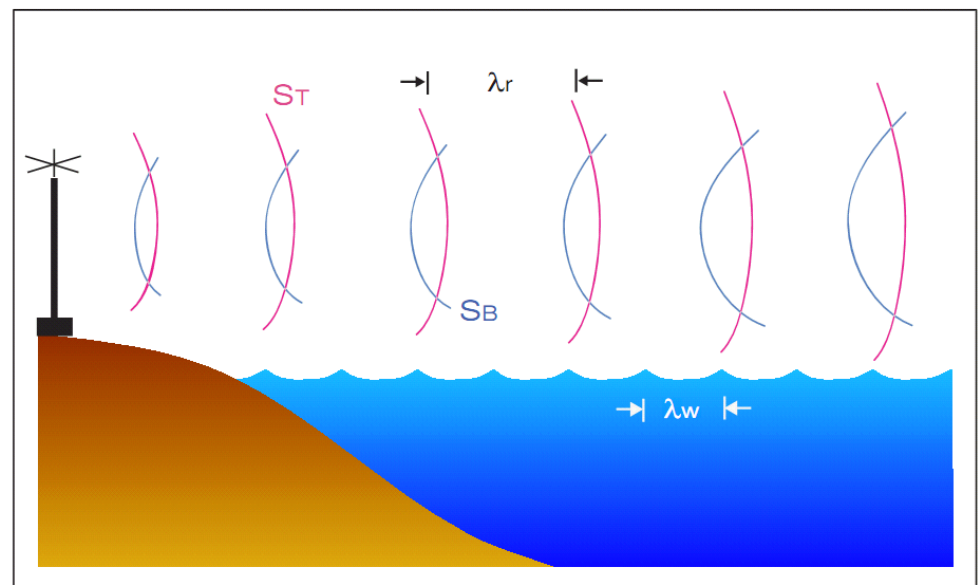
(Paduan and Graber, Oceanography 1997)

3-30 MHz (between AM radio and TV)  
 Wavelength ( $\lambda_r$ ) : 10 ~ 100 (m)

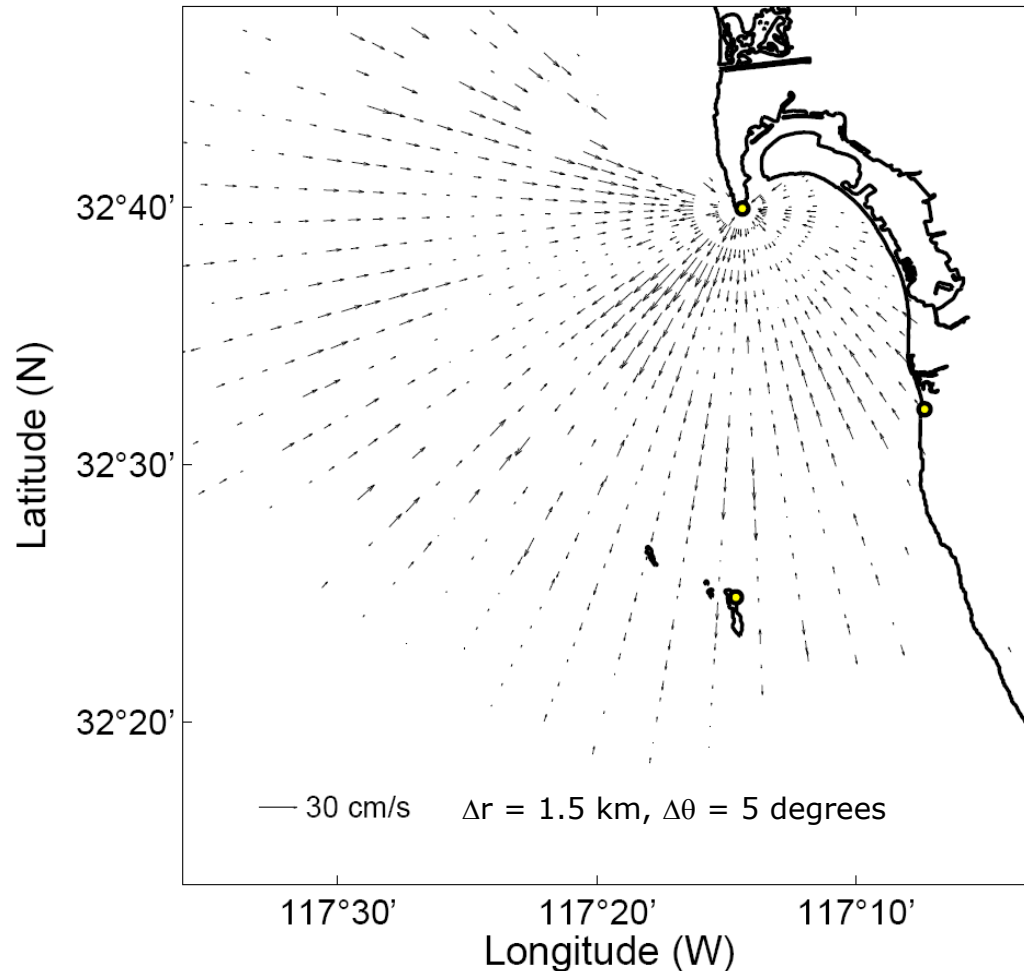
## Bragg backscattering

When the radar signals are backscattered in phase,

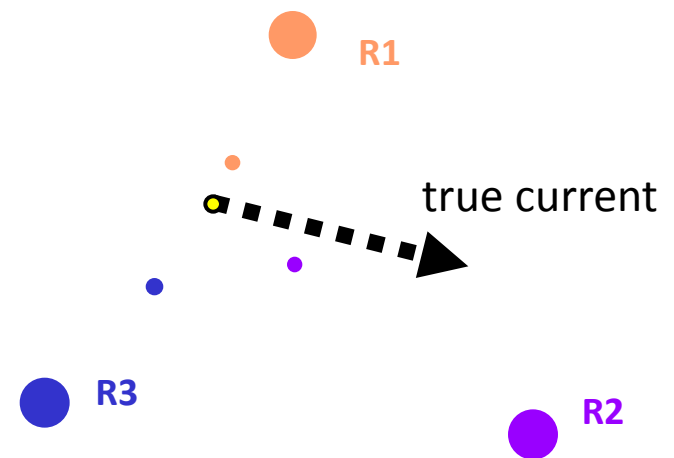
$$\lambda_w = \lambda_r / 2$$



# Surface radial current map

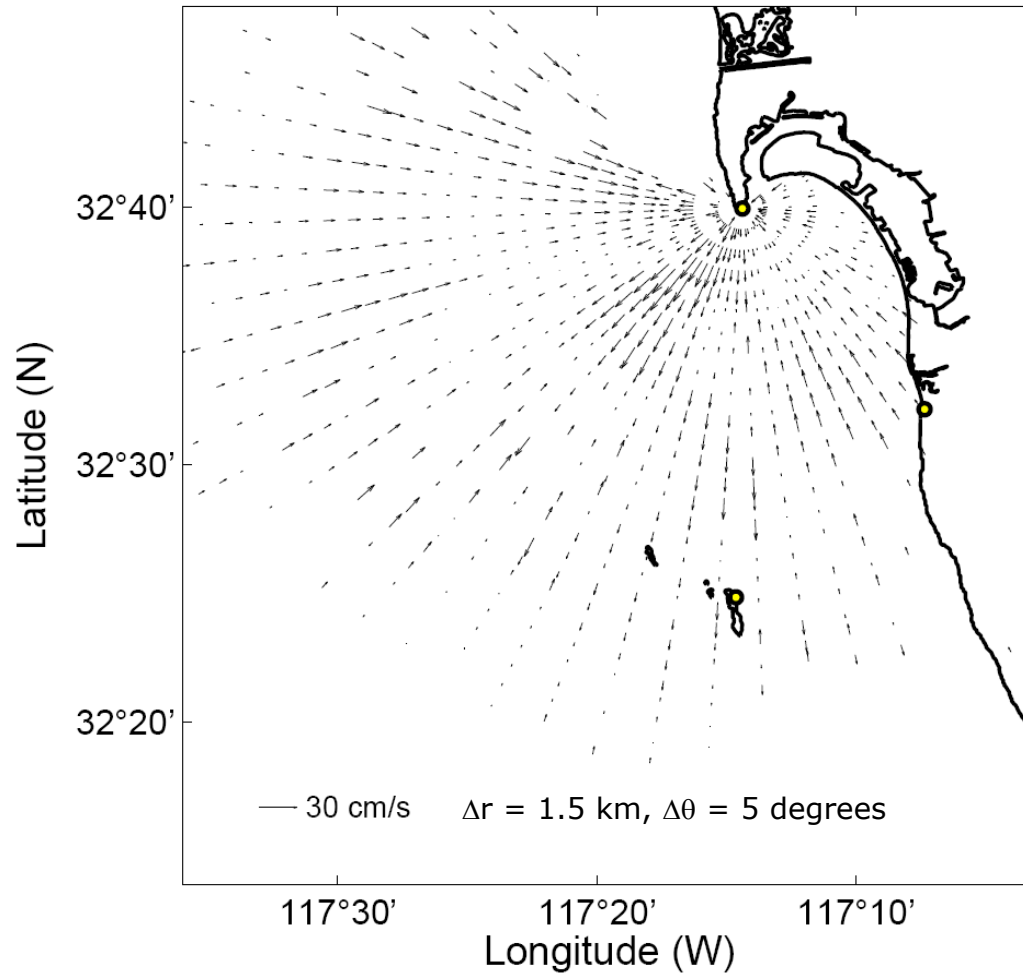


- Range
- Operating and sweeping frequency
- Angle
- Direction finding v.s. MUSIC
- Radial velocity
- Doppler shift
- Projected current component

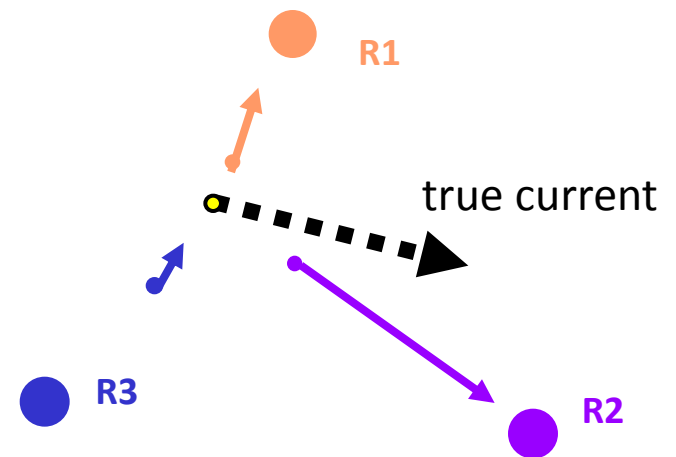




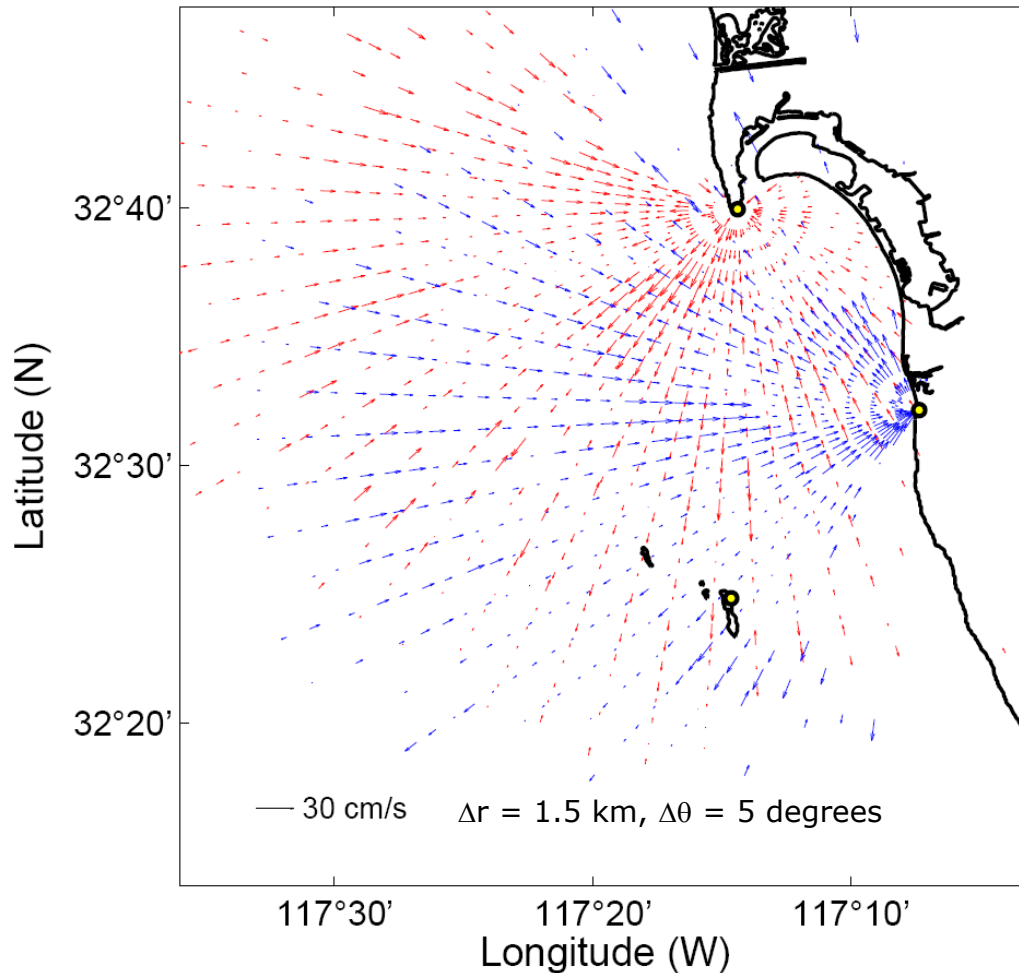
# Surface radial current map



- Range
  - Operating and sweeping frequency
- Angle
  - Direction finding v.s. MUSIC
- Radial velocity
  - Doppler shift
  - Projected current component

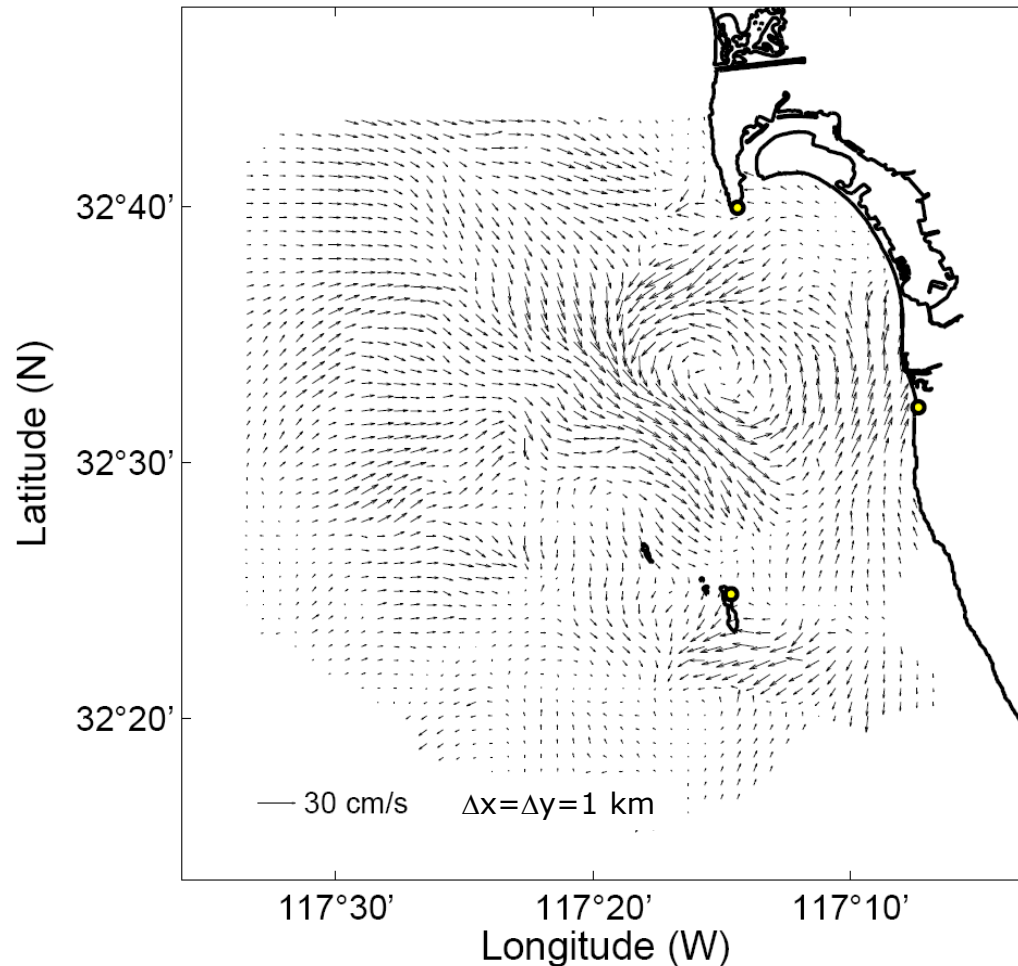


# Multiple surface radial current maps



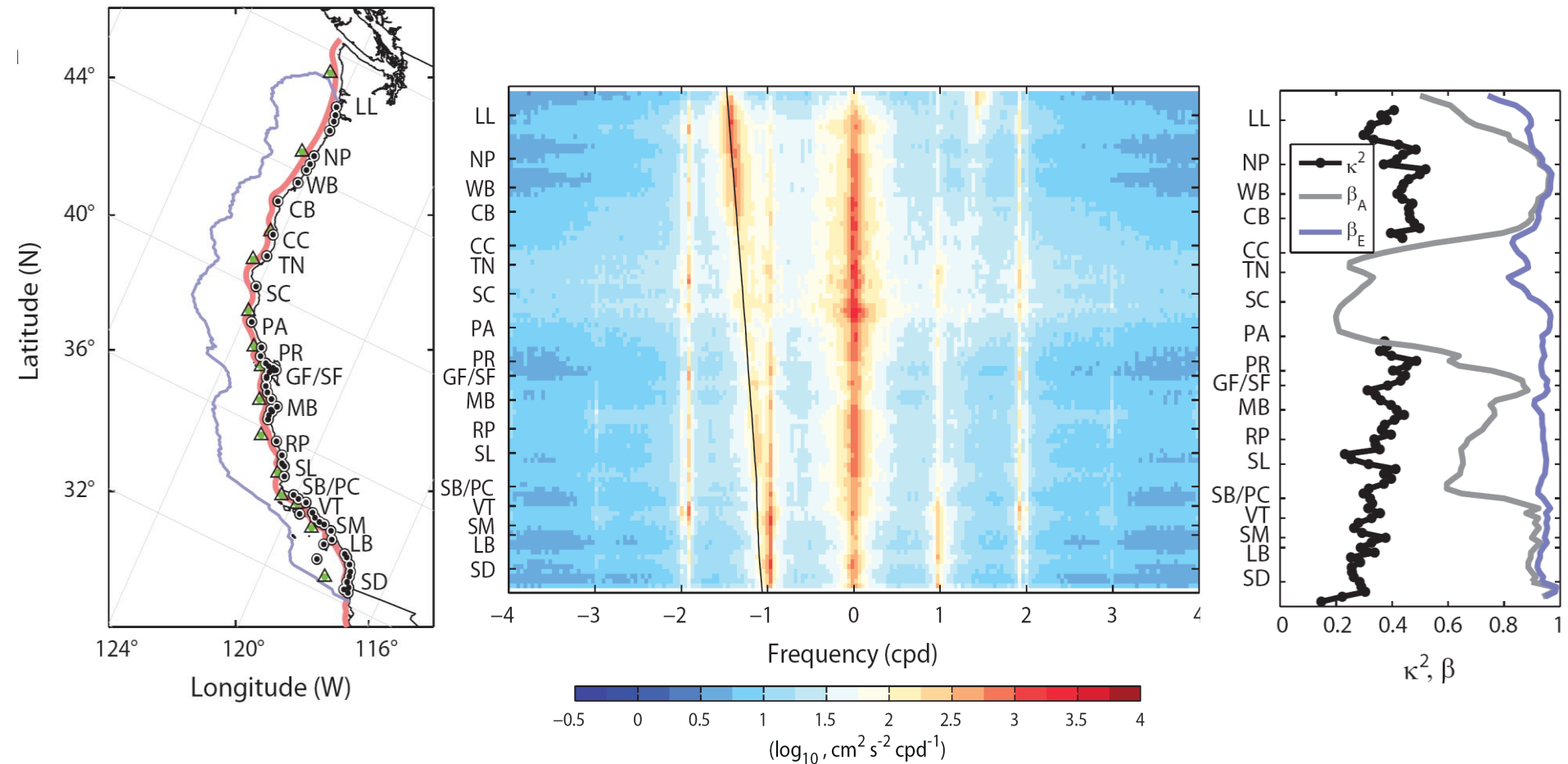
- Vector current map estimates
  - Un-weighted least squares fit (UWLS)
  - Optimal interpolation (OI)

# Improved vector current map



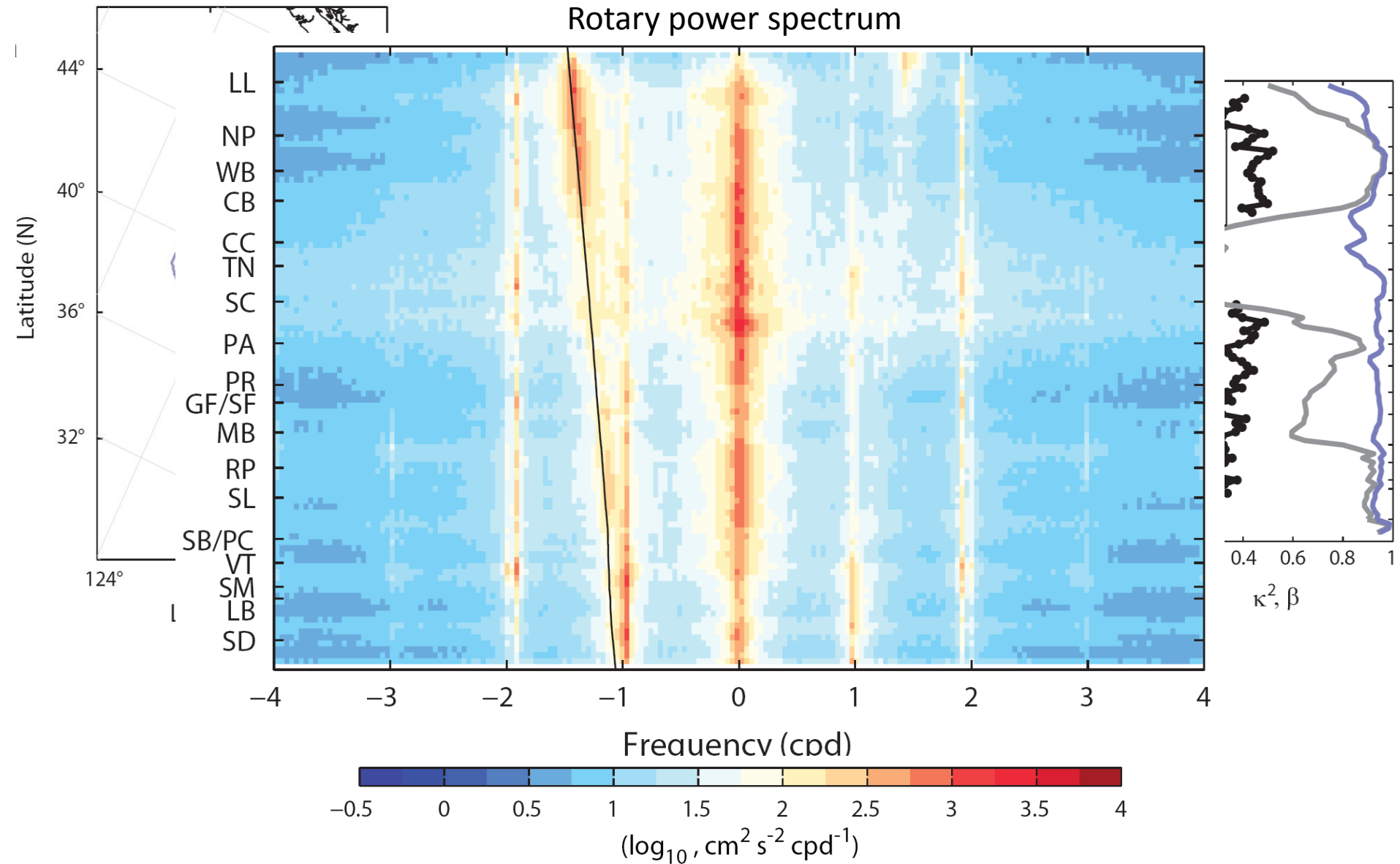
- Optimal interpolation
  - Minimize baseline inconsistency
  - A unified uncertainty definition
  - Divergence and vorticity
  - Velocity potential and stream function
- Exponential correlation function (based on data covariance matrix) leads to minimum level of spatial smoothing.

# Variance of surface currents (alongshore view)



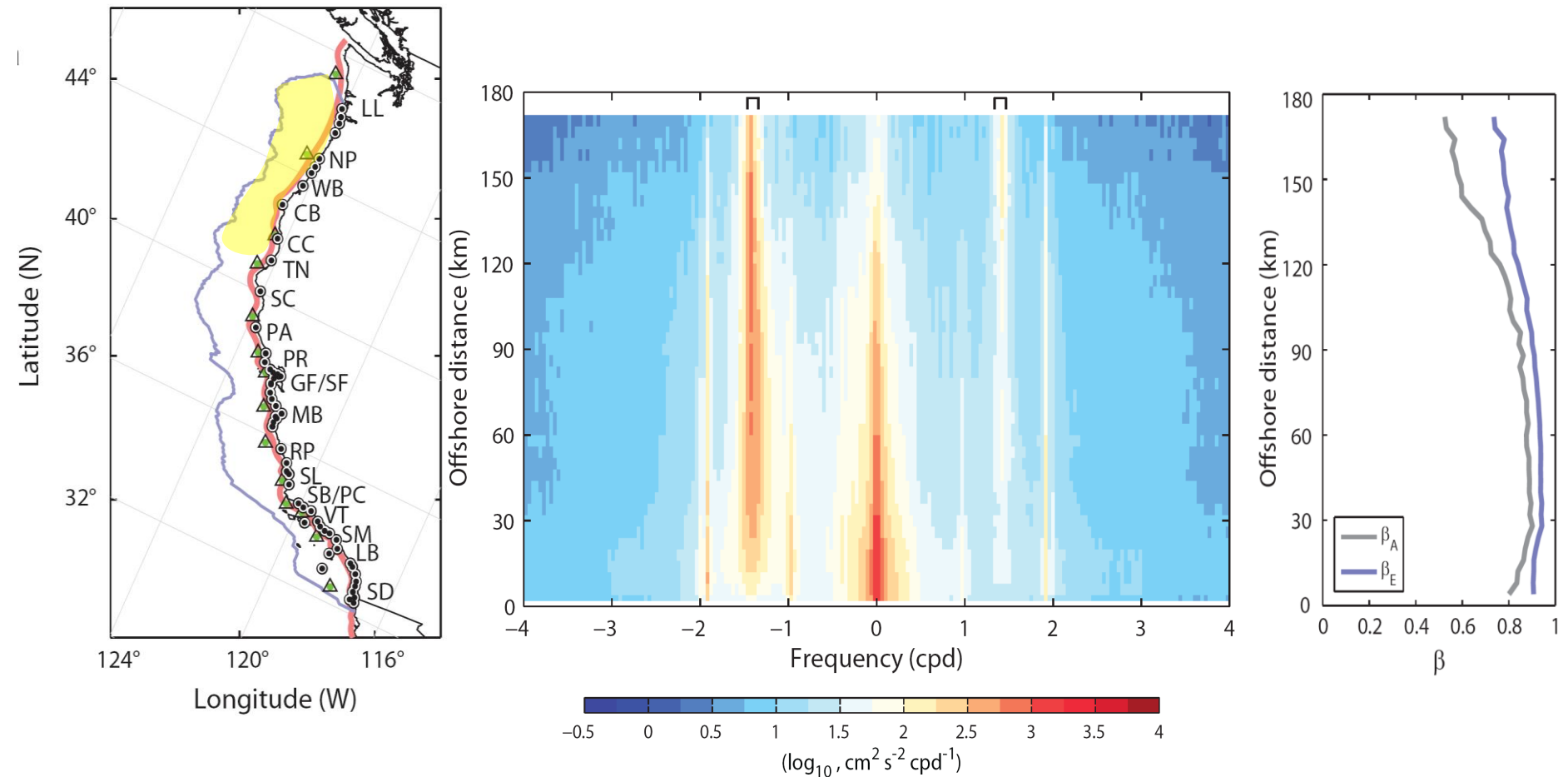
- 60+ compact array HFR (CODAR) system
- Hourly surface current maps (0.5, 1, 2, and 6 km resolution)
- Upper 1 m depth averaged currents
- From nearshore to 50 - 150 km offshore

# Variance of surface currents (alongshore view)



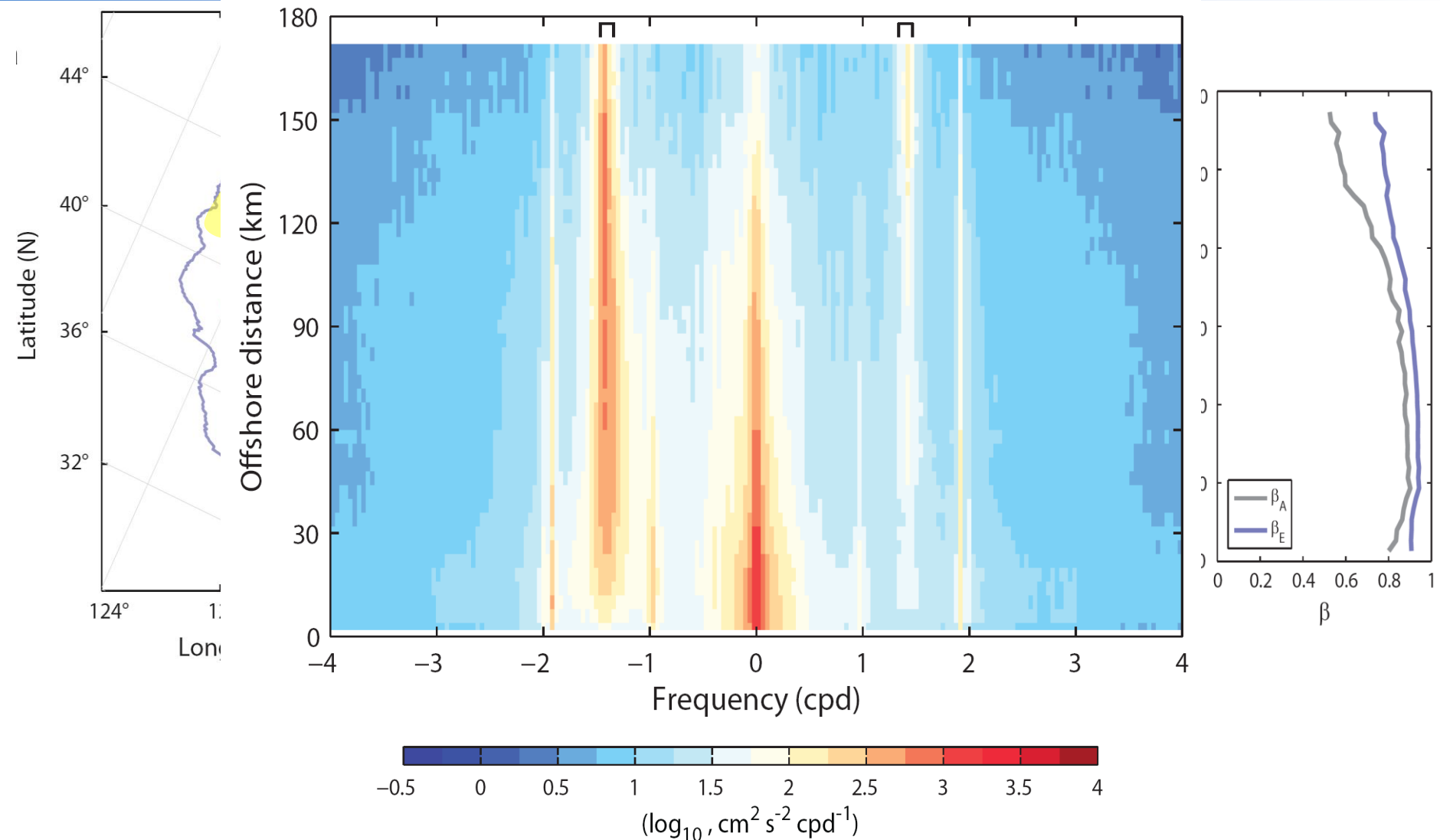
- Variance coherent with tides, wind, low frequency signals, and Coriolis force.
- Regional noise levels

# Variance of surface currents (cross-shore view)



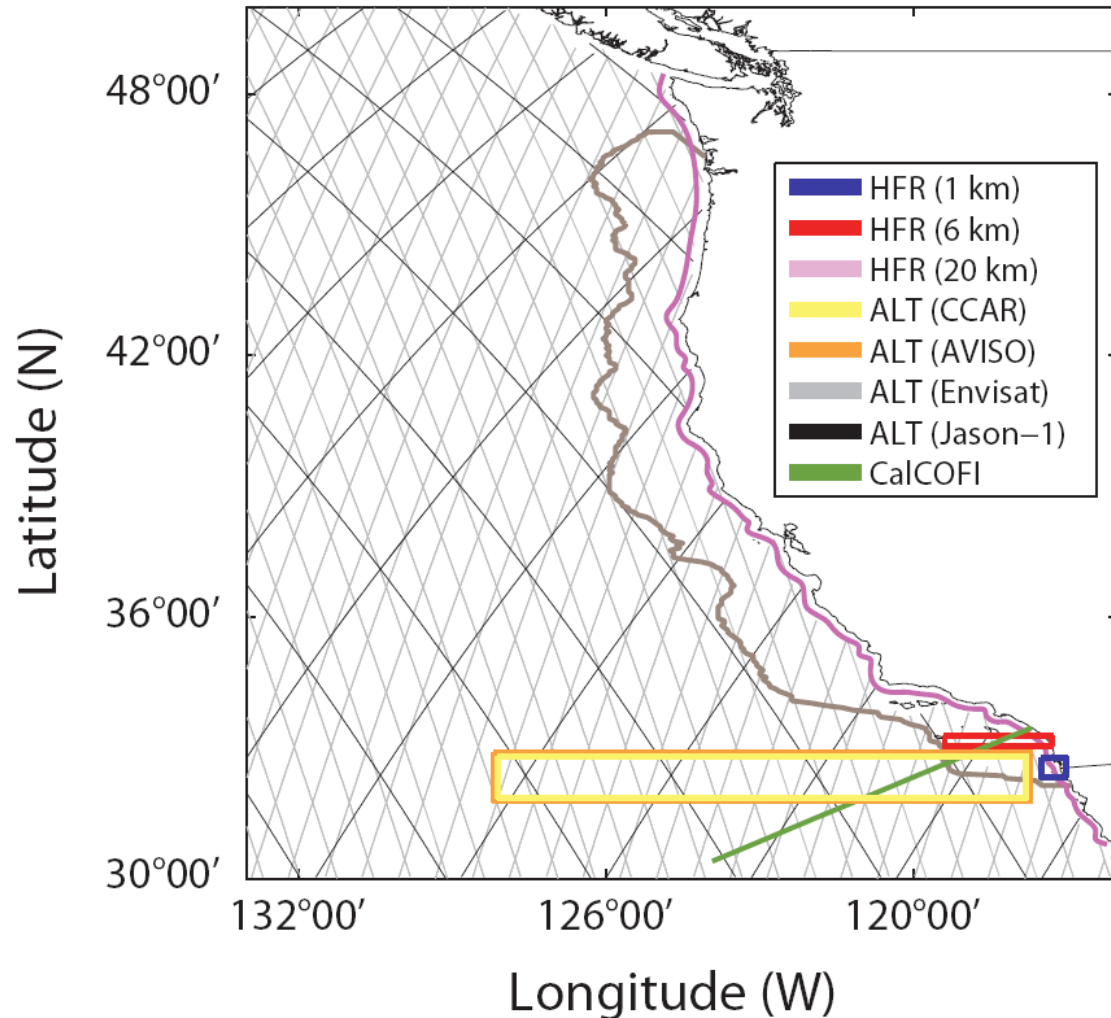
- Cross-shore variation of tide-, wind-, low frequency-forced energy
  - Low frequency pressure setup against the coast
  - Inertial variance gets narrow offshore
  - Variance of tide-coherent currents decrease with offshore distance
- (Kim *et al*, JGR 2011)

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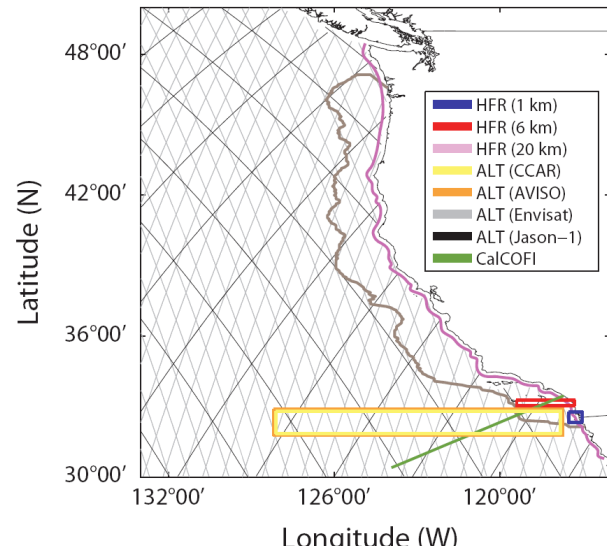
# Sampling domain in computation of energy spectra



- HFR surface currents (1, 6, and 20 km resolution; hourly) off southern California and on coastline axis (USWC)
- Gridded ALT products [CCAR (daily) and AVISO (weekly)] and along-track altimeter (ALT; Envisat/Jason-1; weekly) on NE Pacific
- CalCOFI shipboard ADCP (Line 90; quarterly)
- SoCAL was chosen because it contains relatively minimum ageostrophic components.



# Energy spectra in the wavenumber domain (1D)

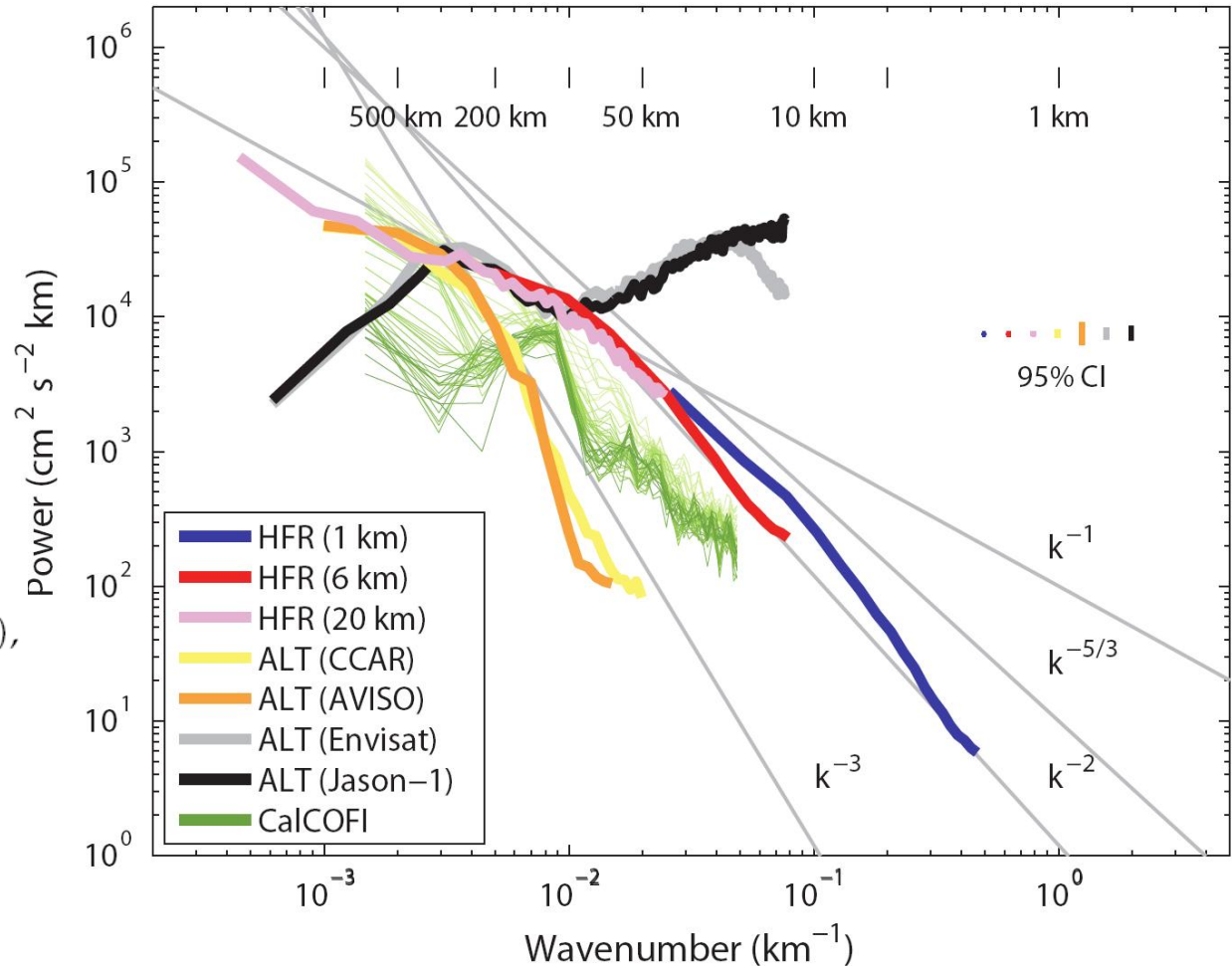


$$S_{u_{\perp}}(k_{\parallel}) = \left(\frac{g}{f_c}\right)^2 (2\pi k_{\parallel})^2 S_{\eta_{\parallel}}(k_{\parallel}),$$

Power spectrum of cross-track geostrophic currents from along-track SSHAs

$k^{-2}$  power law related to sub-mesoscale.

Robust estimate on  $k^{-2}$  spectra with data in other regions.



Two kinds of ALT data: Envisat and Jason-1

HFR data with three resolutions:

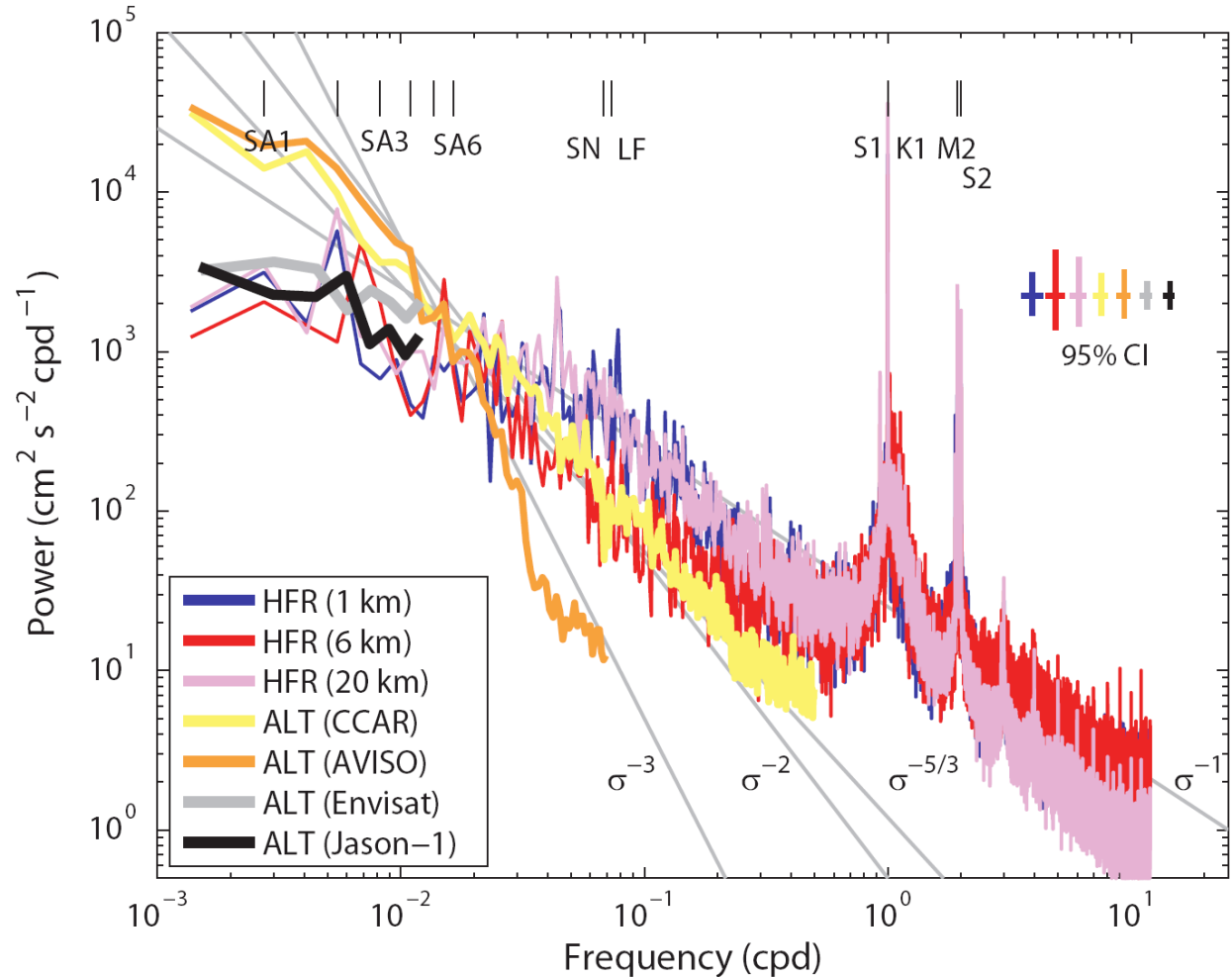
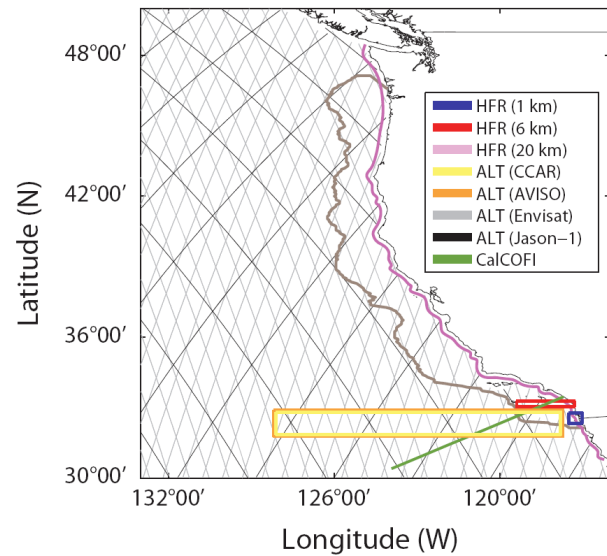
1 km and 6 km data are sampled from SoCAL,

because minimum ageostrophic components are expected.

20 km data are from the coastline axis.

(Kim *et al*, JGR 2011)

# Energy spectra in the frequency domain



Along-track altimeter data are binned in  $2^\circ \times 2^\circ$  grid boxes and averaged in time (7-daily  $\rightarrow$  30 daily time series) to increase signal to noise ratio.

# Scale-by-scale energy budget equation

$$\frac{\partial}{\partial t} E(k^*) + \Pi(k^*) = -2\nu\Omega(k^*) + F(k^*), \quad (\text{Frisch 1995})$$

where

$$E(k^*) = \frac{1}{2} \sum_{|\mathbf{k}| < k^*} |\hat{\mathbf{u}}(\mathbf{k})|^2, \quad \text{Cumulative kinetic energy}$$

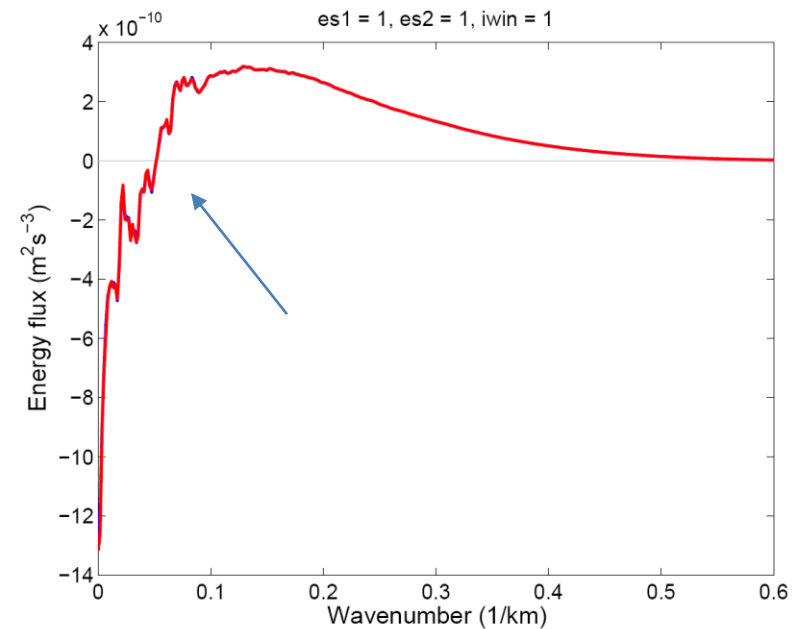
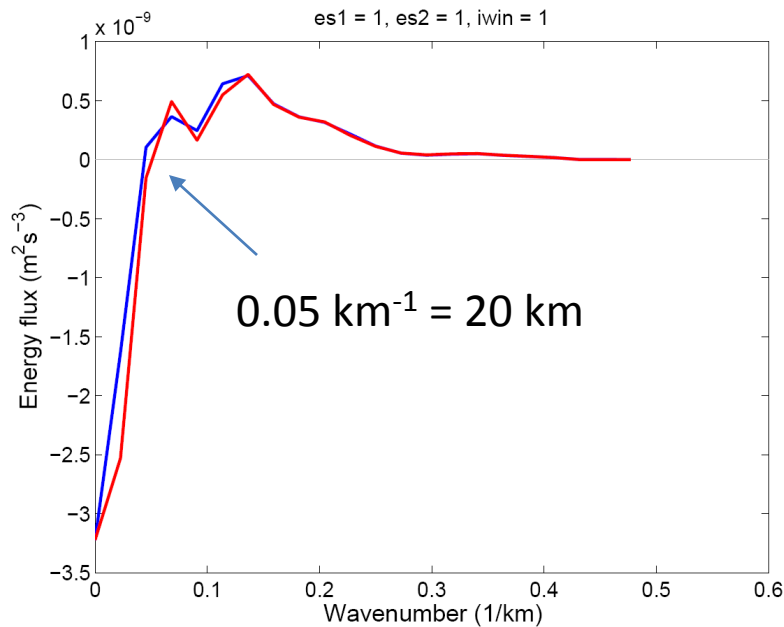
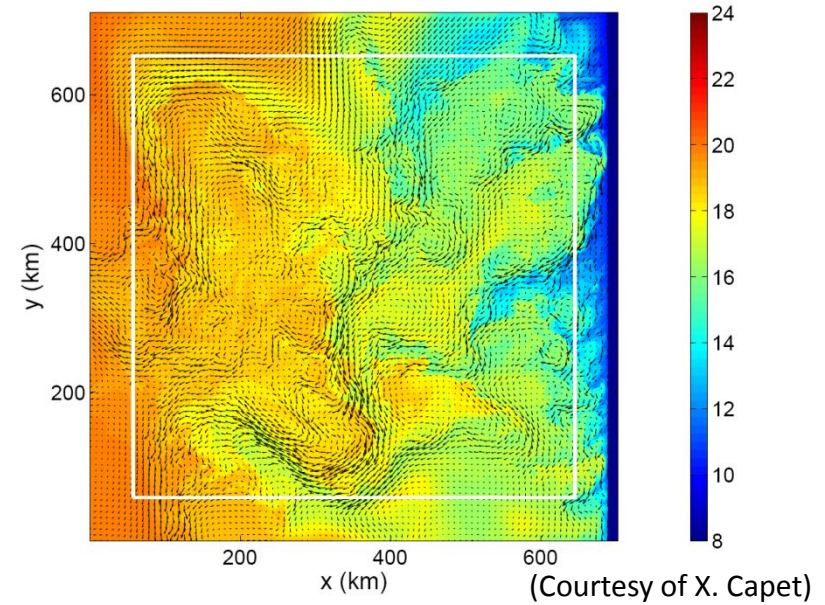
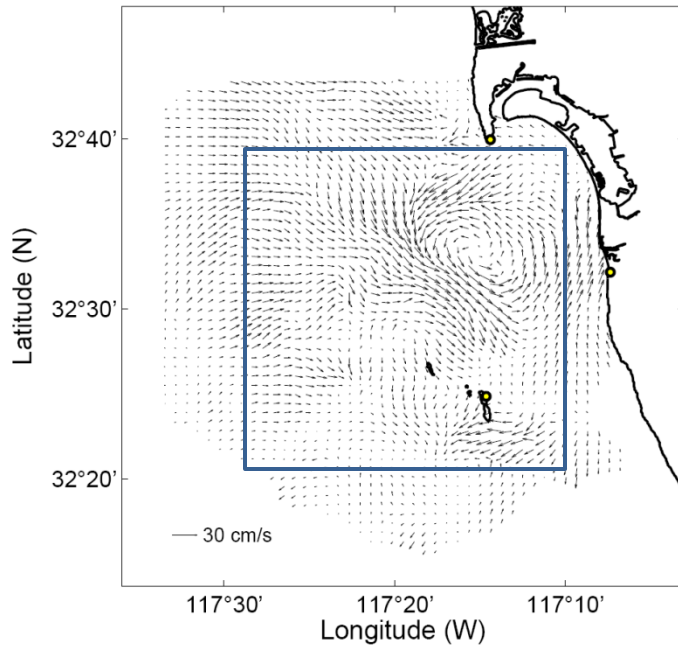
$$\begin{aligned} \Pi(k^*) &= \langle \mathbf{u}_{<} \cdot (\mathbf{u} \cdot \nabla \mathbf{u}) \rangle, \quad \text{Cumulative advective kinetic energy flux} \\ &= \langle \mathbf{u}_{<} \cdot (\mathbf{u}_{<} \cdot \nabla \mathbf{u}_{>}) \rangle + \langle \mathbf{u}_{<} \cdot (\mathbf{u}_{>} \cdot \nabla \mathbf{u}_{>}) \rangle, \end{aligned}$$

$$\Omega(k^*) = \frac{1}{2} \sum_{|\mathbf{k}| < k^*} \mathbf{k}^2 |\hat{\mathbf{u}}(\mathbf{k})|^2, \quad \text{Cumulative enstrophy}$$

$$\begin{aligned} \mathbf{u}(\mathbf{x}) &= \mathbf{u}_{<}(\mathbf{x}) + \mathbf{u}_{>}(\mathbf{x}), \\ &= \sum_{|\mathbf{k}| < k^*} \hat{\mathbf{u}}(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}} + \sum_{|\mathbf{k}| > k^*} \hat{\mathbf{u}}(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}}, \end{aligned}$$

- Surface currents from HFR observations (1 km) and sub-mesoscale model (0.75 km; X. Capet *et al*, 2009) off southern California

# Comparison of advective kinetic energy flux [ $\Pi(k^*)$ ]



# Summary

- Energy spectra at mesoscale and sub-mesoscale are examined with altimeter-, high-frequency radar-, shipboard ADCP-derived (coastal) currents.
- The operational HFR network provides the detailed aspects of coastal surface circulation and ocean dynamics at a resolution (km in space and hourly in time) containing responses to the low frequency, tides, wind forcing, and Earth rotation.
- Due to the noise at 100 km scale in altimeter observations, studies on energy spectra and flux below that scale can be explored with sub-mesoscale observations.
- The spatial covariance appears as an anisotropic exponential shape with decorrelation length scales of 20 km nearshore and 100 km offshore parallel to the shoreline, consistent with approximate  $k^{-2}$  decay behavior.