Resonant ocean current responses driven by coastal winds near the critical latitude

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- Wind and current responses
  - Ekman theory.
Resonant ocean current responses driven by coastal winds near the critical latitude

- Wind and current responses
  - Ekman theory.
- Resonance
  - Forcing-response in the frequency domain
  - Natural frequency – Coriolis frequency

\[ f_c = 2 \sin (\text{latitude}) \text{ [cycles per day]} \]
A moving object in a rotating frame

- Coriolis force deflects the path of a moving object in a rotating frame.
  - $f_c = 2 \sin (\text{latitude}); \text{ [cycles per day]}$
    - A unique period as a function of latitude
    - A natural resonant frequency in the dynamic system

- At $30^\circ N$, $f_c = 1 \text{ cpd}$ is equal to the diurnal frequency.
NI variance distribution

Elipot and Lumpkin (GRL 2008)
Resonant ocean current responses driven by coastal winds near the critical latitude

- Wind and current responses
  - Ekman theory
- Resonance
  - Forcing-response in the frequency domain
  - Natural frequency – Coriolis frequency
- Critical latitude
  - Observations at different latitudes – wind and surface currents off the USWC

Shaffer, 1972; Ekman model
Resonant ocean current responses driven by coastal winds near the critical latitude

• At a given latitude, what would be the wind-current response in the frequency domain?
• At a given frequency, what would be the wind-current response as a function of latitude?
How can we identify a system?
How can we identify a system?

**system**

/ˈsɪstəm/  

noun

1. a set of connected things or parts forming a complex whole, in particular.

2. a set of principles or procedures according to which something is done; an organized scheme or method.
   "a multiparty system of government"

**synonyms:** method, methodology, technique, process, procedure, approach, practice; More
How can we identify a system?

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How can we identify a system?

- Governing equations

\[ F = ma = m \left( \frac{\Delta v}{\Delta t} \right) \]
How can we identify a system?

- Governing equations

\[ F = ma = m\left(\frac{\Delta v}{\Delta t}\right) \]

\[ \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \vec{v} + \frac{\rho}{\rho_0} \vec{g} - 2(\vec{\Omega} \times \vec{v}) \]

The movement of fluid depends upon: pressure, viscosity, gravity, rotation.
How can we identify a system?

• Governing equations

\[ F = ma = m\left(\frac{\Delta v}{\Delta t}\right) \]

• A statistical relationship between inputs and outputs
  • Transfer function or response function

\[ \hat{u}(z, \omega) = H(z, \omega) \hat{\tau}(\omega) \quad u(z, t) = \int_{t'} G(z, t - t') \tau(t') \, dt', \]
How can we identify a system?

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- Examples of a linear ‘ocean’ system using two approaches
Wind-current responses in the freq. domain

\[
\frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad \text{Ekman theory}
\]
Wind-current responses in the freq. domain

\[
\frac{\partial u}{\partial t} + if_c u + ru = \frac{1}{\rho} \frac{\partial \tau}{\partial z}
\]

Ekman theory

\[
H_E(z, \sigma) = \frac{\hat{u}(z, \sigma)}{\hat{\tau}(\sigma)} = \frac{e^{i\lambda}}{\lambda \rho \nu}
\]

as a function of frequency

\[
\lambda = \sqrt{\left[i(\sigma + f_c) + r\right]/\nu}
\]
Wind-current responses in the freq. domain

\[ \frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]

\[ H_E(z, \sigma) = \frac{\hat{\mathbf{u}}(z, \sigma)}{\hat{\tau}(\sigma)} = \frac{e^{i z}}{\lambda \rho v} \]

\[ \lambda = \sqrt{[i(\sigma + f_c) + r]/v} \]

At a given latitude, the relationship between wind stress and surface currents is given as a transfer function in the frequency domain.
Wind-current responses in latitude

\[ \frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]

Ekman theory

\[ H_E(z, f_c) = \frac{\hat{u}(z, f_c)}{\hat{\tau}(f_c)} = \frac{e^{i z}}{\lambda \rho v} \]

as a function of Coriolis freq. (latitude)

\[ \lambda = \sqrt{[i(\sigma + f_c) + r]/v} \]
Wind-current responses in latitude

\[ \frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]

\[ \mathbf{H}_E(z, f_c) = \frac{\hat{u}(z, f_c)}{\hat{\tau}(f_c)} = \frac{e^{i\lambda z}}{\lambda \rho v} \]

\[ \lambda = \sqrt{[i(\sigma + f_c) + r]/v} \]

\[ \begin{align*}
r &= 0; \\
\text{at } \sigma &= 1 \text{ cpd} \\
(\text{diurnal frequency}).
\end{align*} \]

Shaffer, 1972; Ekman model

Simpson et al, JPO 2002 (Slab layer model)

Resonant latitude due to land/sea breeze: ±30°N
Latitudinal coastal observations

- US West Coast high-frequency radar network-derived surface currents and wind stress (red dots) at NDBC buoys.
- Latitudinal variation of 32°N to 47°N
Latitudinal coastal observations

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\[
\hat{u}(z, \omega) = H(z, \omega) \hat{\tau}(\omega)
\]

\[
H(z, \omega) = \left( \langle \hat{u}(z, \omega) \hat{\tau}^\dagger(\omega) \rangle \right) \left( \langle \hat{\tau}(\omega) \hat{\tau}^\dagger(\omega) \rangle + R_a \right)^{-1}
\]

\( R_a \): Regularization matrix
Variability of surface currents and wind

- Wind- and tide-coherent, low-frequency variance, and inertial variance

Kim et al. (JGR, 2011)
• Wind- and tide-coherent, low-frequency variance, and inertial variance

• Variance of the diurnal wind does not vary that much in the along-shore direction, but it is given as a function of distance from the shoreline (cross-shore direction).

Kim et al (JGR, 2011)
Coast-wide wind transfer functions

\[ \hat{u}(z, \omega) = H(z, \omega) \hat{\tau}(\omega) \]

(Kim and Crawford, GRL 2014)
Coast-wide wind transfer functions

- At a given latitude, what would be the wind-current response in the frequency domain?
- At a given frequency, what would be the wind-current response as a function of latitude?

(Kim and Crawford, GRL 2014)
Coast-wide wind transfer functions

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Resonant responses near the critical latitude

Slab layer model

\[ Z = 0 \text{ (Ekman)} \]
\[ Z = 0.35 \delta_E \text{ (Near-surface avg. Ekman)} \]

Resonant latitude due to land/sea breeze: \( \pm 30^\circ \text{N} \)
Summary

• Wind-current responses are examined in the frequency domain and latitude using analytic solutions of Ekman model (and slab layer and surface-averaged Ekman models) and observations off the US West Coast.
• The current responses are enhanced at the local inertial frequency.
• Resonant responses can be expected at the +/-30° latitude in the diurnal land-sea breeze environment.
• Energetic mixing and potential internal motions near the critical latitude are expected.
• Ocean responses to the diurnal wind and relevant bio-physical interactions can be a potential topic to pursue.