Resonant ocean current responses driven by coastal winds near the critical latitude

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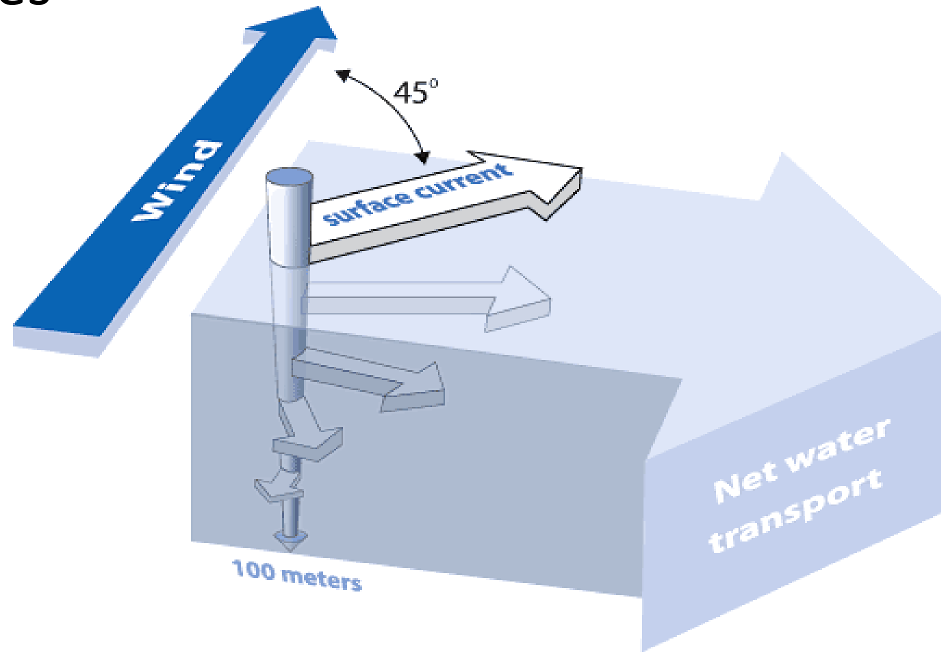
- Wind and current responses
  - Ekman theory..



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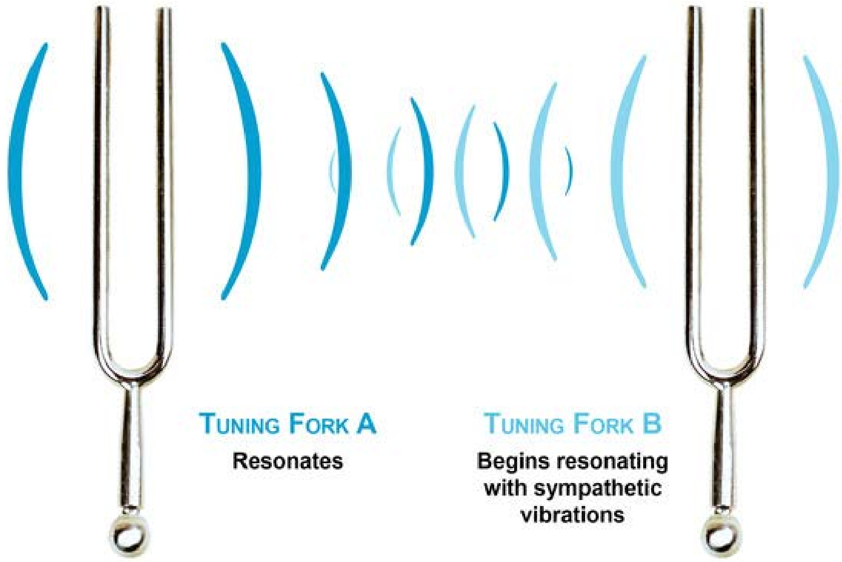
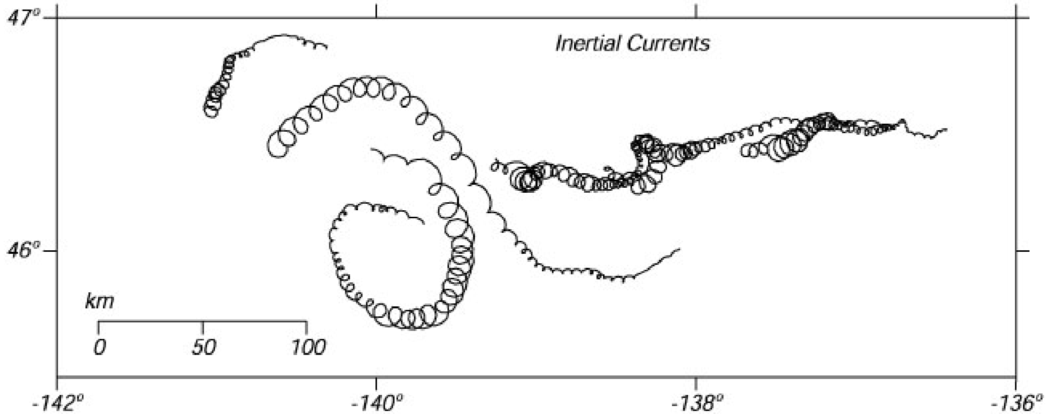
  - Ekman theory..
- Resonance
  - Forcing-response in the frequency domain
  - Natural frequency – Coriolis frequency

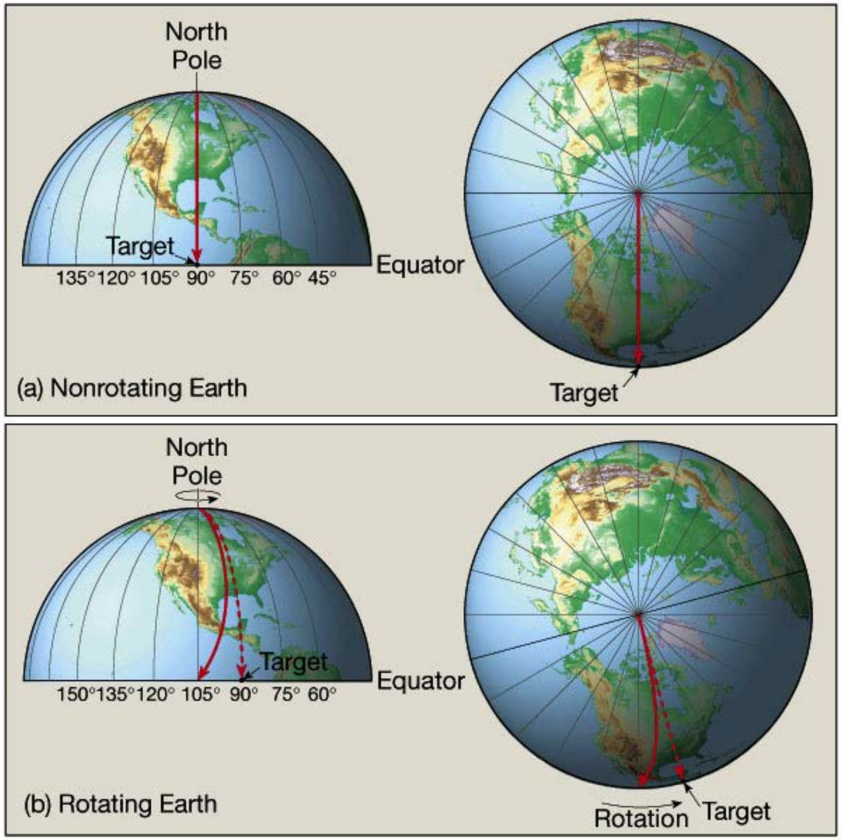


f_c =2 sin (latitude) [cycles per day]

A moving object in a rotating frame

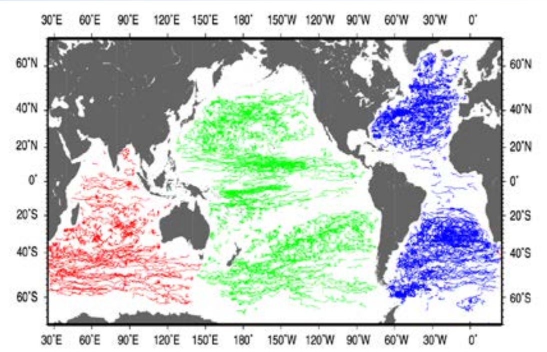
- Coriolis force deflects the path of a moving object in a rotating frame.
- f_c = 2 sin (latitude); [cycles per day]
  - A unique period as a function of latitude
  - A natural resonant frequency in the dynamic system

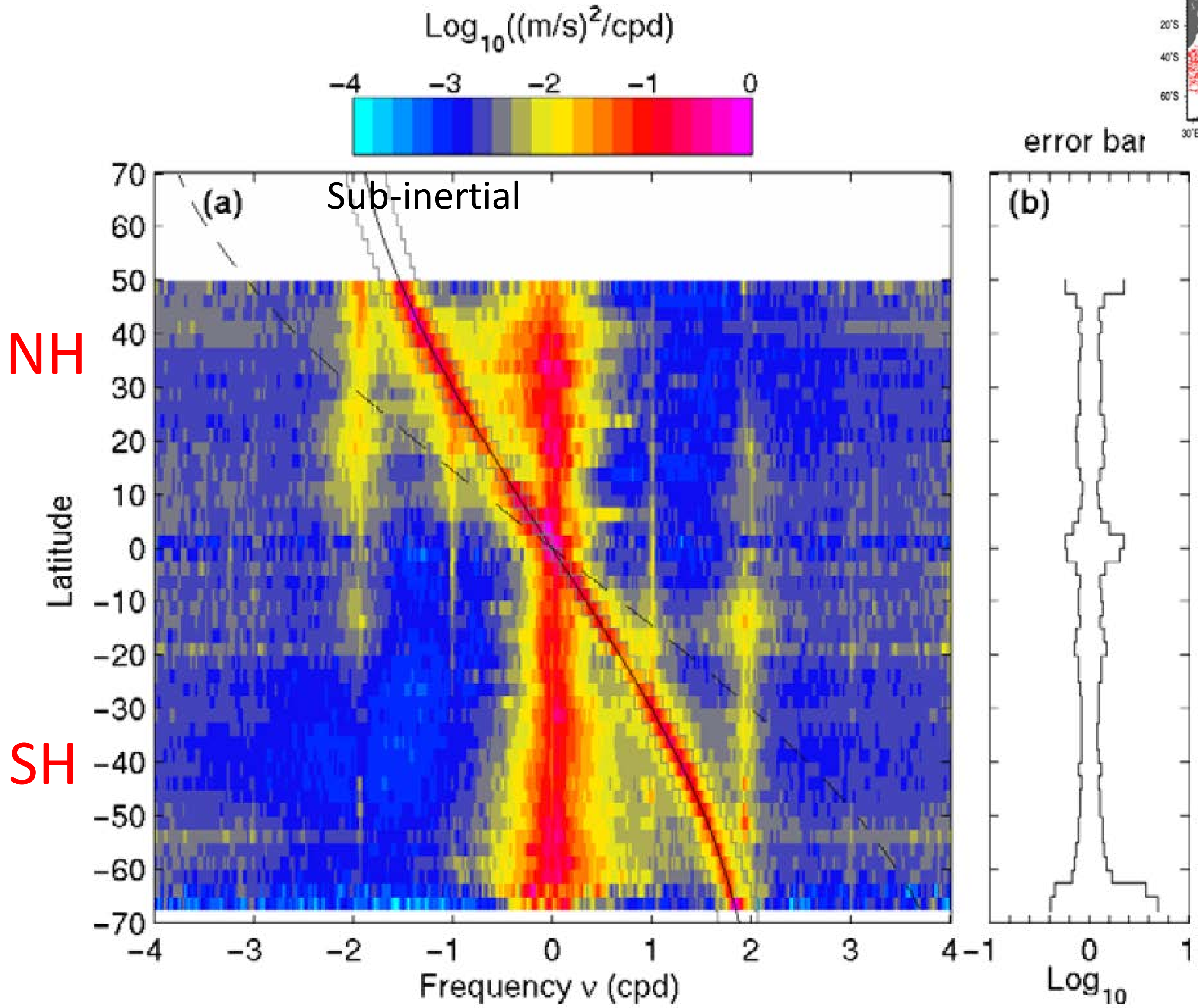




- At 30°N, f_c = 1 cpd is equal to the diurnal frequency.

NI variance distribution

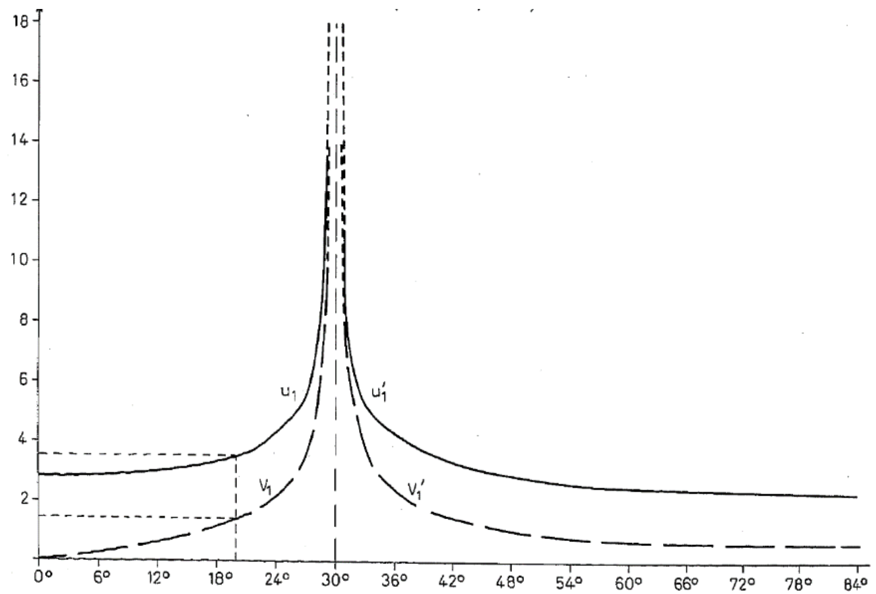




Elipot and Lumpkin (GRL 2008)

Resonant ocean current responses driven by coastal winds near the critical latitude

- Wind and current responses
  - Ekman theory..
- Resonance
  - Forcing-response in the frequency domain
  - Natural frequency – Coriolis frequency
- Critical latitude
  - Observations at different latitudes – wind and surface currents off the USWC



Shaffer, 1972; Ekman model

Resonant ocean current responses driven by coastal winds near the critical latitude

- At a given latitude, what would be the wind-current response in the frequency domain?
- At a given frequency, what would be the wind-current response as a function of latitude?

How can we identify a system?

How can we identify a system?

sys·tem
/ˈsistəm/

noun

1. a set of connected things or parts forming a complex whole, in particular.
2. a set of principles or procedures according to which something is done; an organized scheme or method.
   "a multiparty system of government"
   synonyms: method, methodology, technique, process, procedure, approach, practice; More

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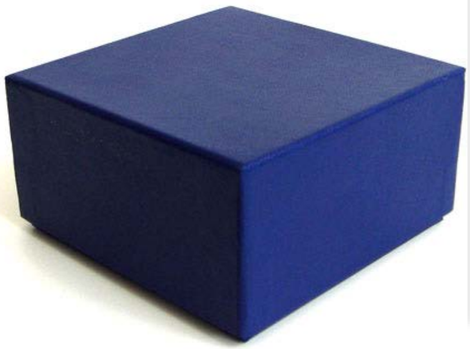

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- Governing equations

$$F = ma = m\left(\frac{\Delta v}{\Delta t}\right)$$

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$$F = ma = m\left(\frac{\Delta v}{\Delta t}\right)$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2\vec{v} + \frac{\rho}{\rho_0}\vec{g} - 2(\vec{\Omega}\times\vec{v})$$

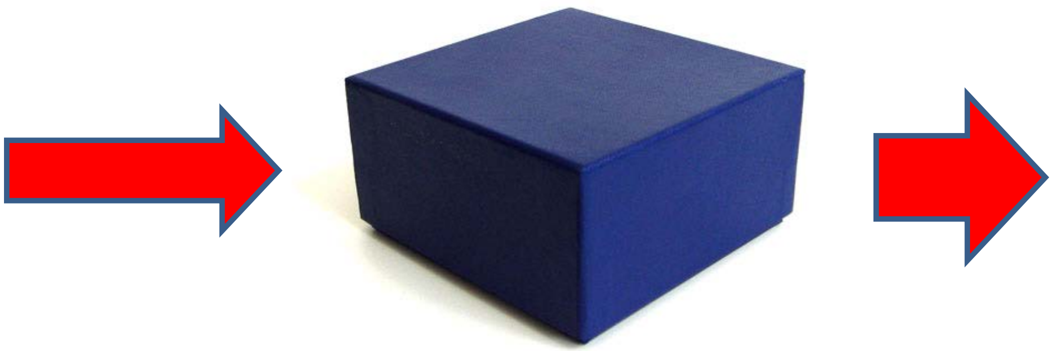
The movement of fluid depends upon: pressure, viscosity, gravity, rotation

How can we identify a system?

- Governing equations

$$F = ma = m\left(\frac{\Delta v}{\Delta t}\right)$$

Input(s)



Output(s)

- A statistical relationship between inputs and outputs
  - Transfer function or response function

$$\hat{\mathbf{u}}(z,\omega) = \mathbf{H}(z,\omega)\hat{\boldsymbol{\tau}}(\omega) \qquad \mathbf{u}(z,t) = \int_{t'} \mathbf{G}(z,t-t')\boldsymbol{\tau}(t')\,\mathrm{d}t',$$

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- Examples of a linear ‘ocean’ system using two approaches

Wind-current responses in the freq. domain

$$\frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z}$$

Ekman theory

Wind-current responses in the freq. domain

$$\frac{\partial \mathbf{u}}{\partial t} + if_c\mathbf{u} + r\mathbf{u} = \frac{1}{\rho}\frac{\partial \boldsymbol{\tau}}{\partial z}$$

Ekman theory

$$\mathbf{H_E}(z,\sigma) = \frac{\hat{\mathbf{u}}(z,\sigma)}{\hat{\boldsymbol{\tau}}(\sigma)} = \frac{e^{\lambda z}}{\lambda \rho \nu}$$

as a function of frequency

$$\lambda = \sqrt{[i(\sigma + f_c) + r]/\nu}$$

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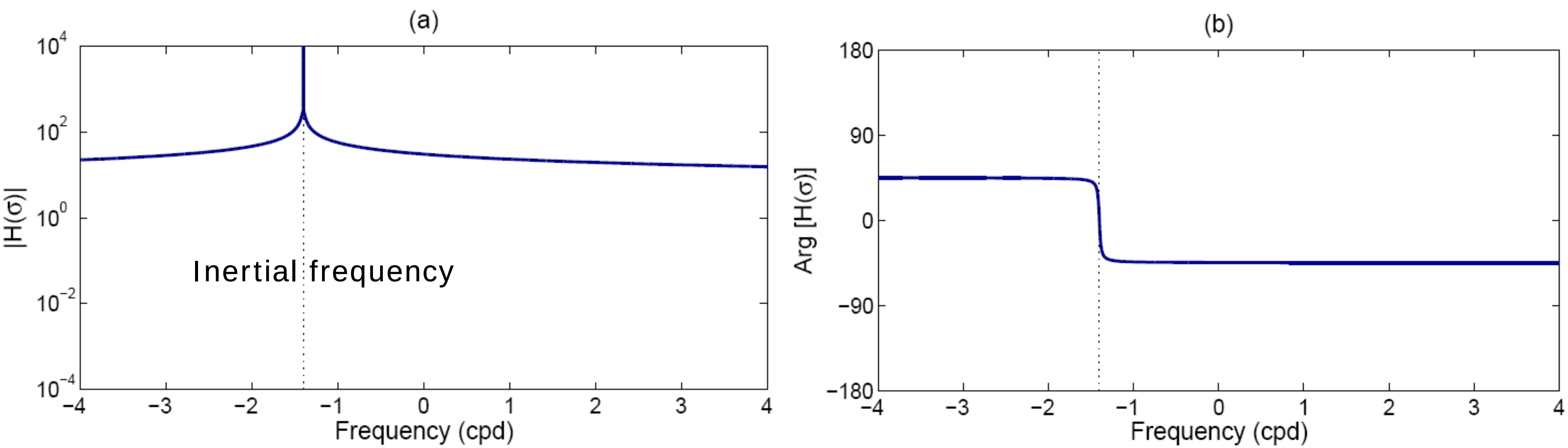

At a given latitude, the relationship between wind stress and surface currents is given as a transfer function in the frequency domain.

Wind-current responses in latitude

$$\frac{\partial \mathbf{u}}{\partial t} + i f_c \mathbf{u} + r\mathbf{u} = \frac{1}{\rho}\frac{\partial \boldsymbol{\tau}}{\partial z}$$

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$$\mathbf{H_E}(z, f_c) = \frac{\hat{\mathbf{u}}(z, f_c)}{\hat{\boldsymbol{\tau}}(f_c)} = \frac{e^{\lambda z}}{\lambda \rho \nu}$$

as a function of Coriolis freq. (latitude)

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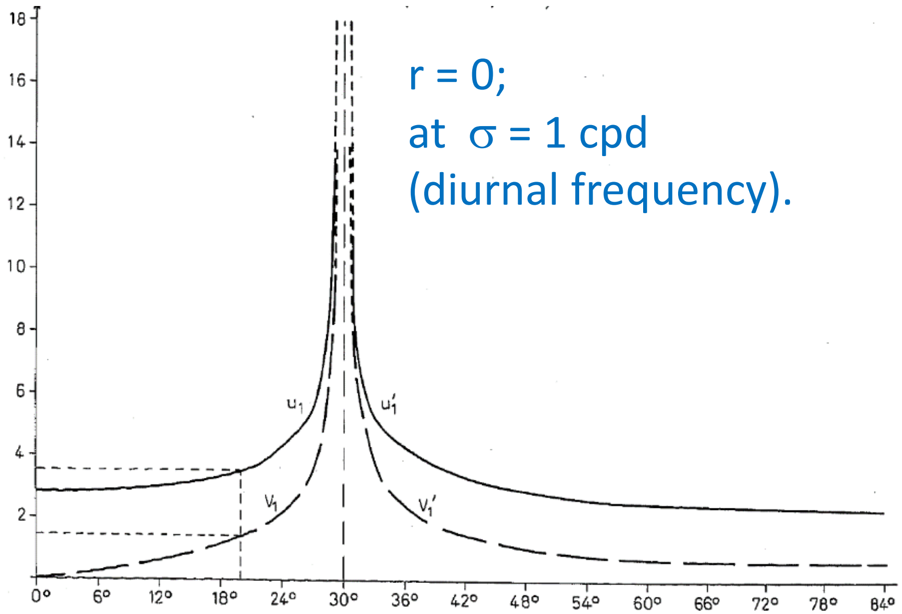

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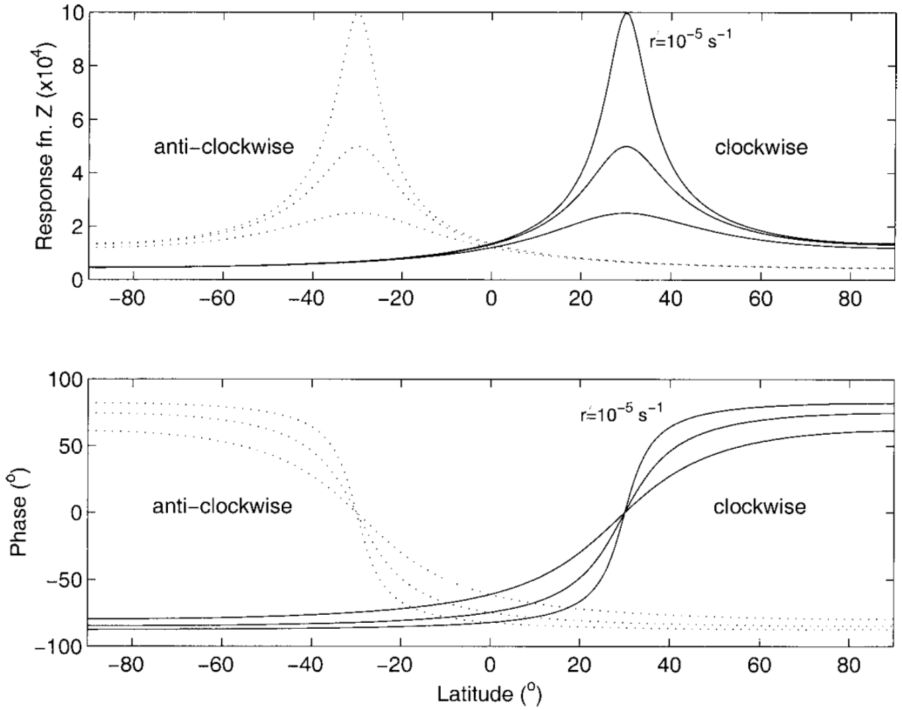
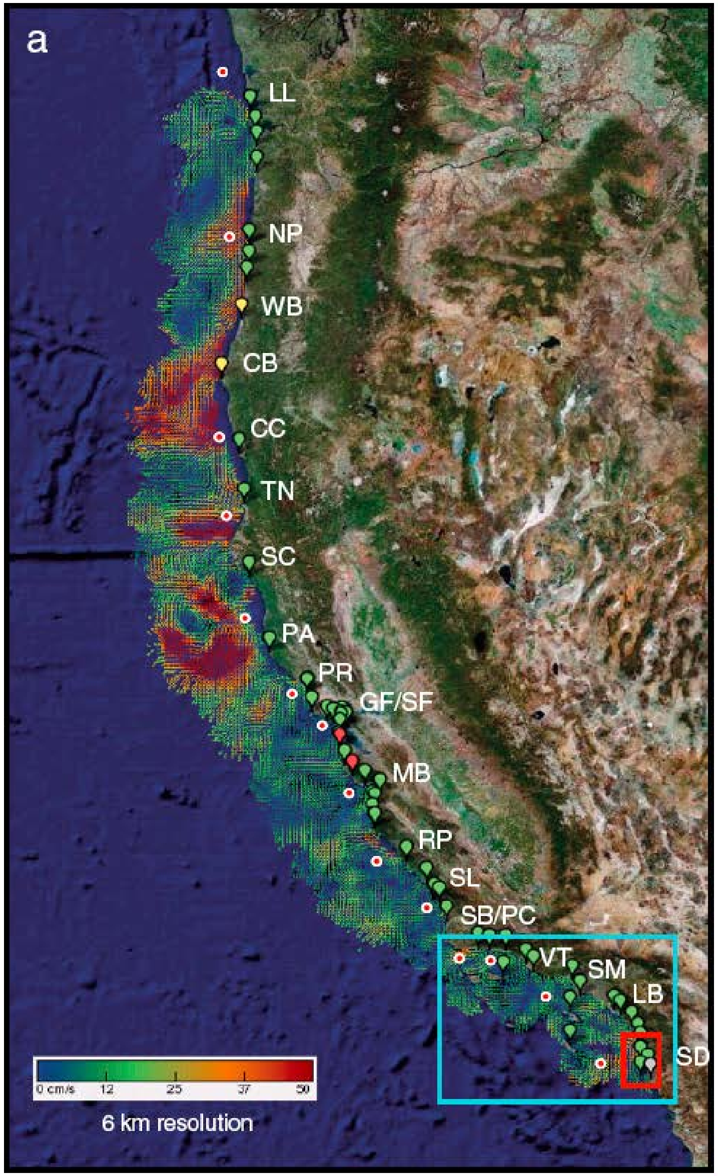


Simpson et al, JPO 2002 (Slab layer model)

Resonant latitude due to land/sea breeze: ±30°N

Latitudinal coastal observations



- US West Coast high-frequency radar network-derived surface currents and wind stress (red dots) at NDBC buoys.
- Latitudinal variation of 32°N to 47°N

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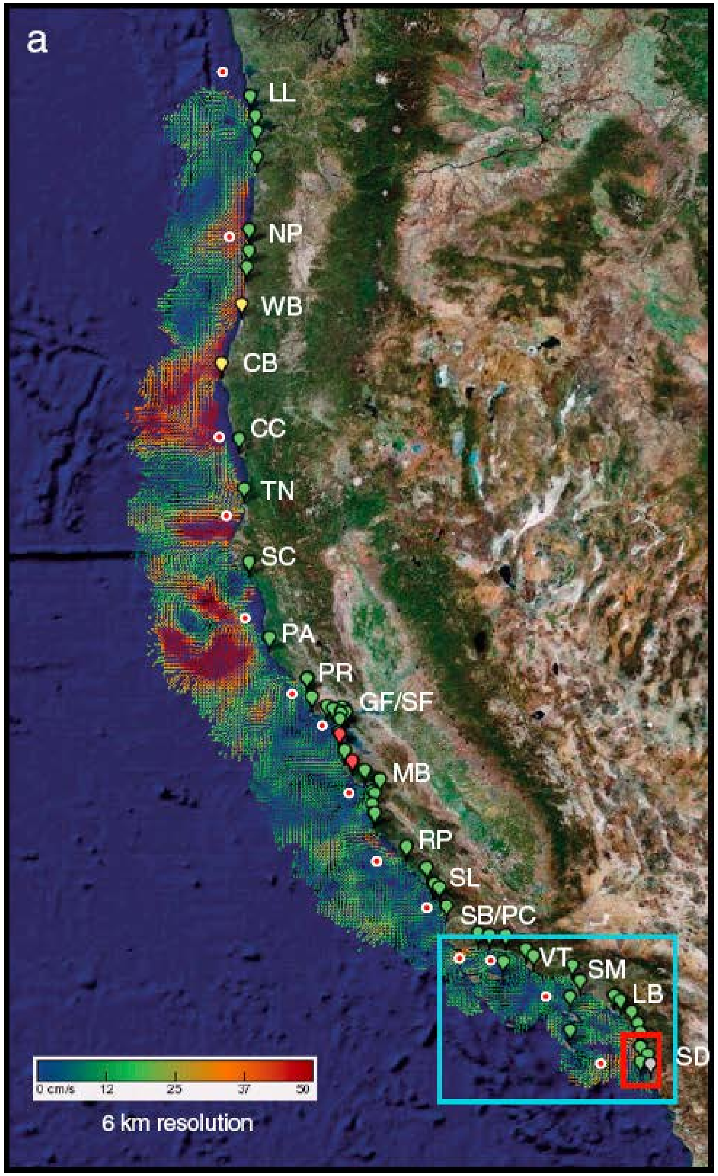

- US West Coast high-frequency radar network-derived surface currents and wind stress (red dots) at NDBC buoys.
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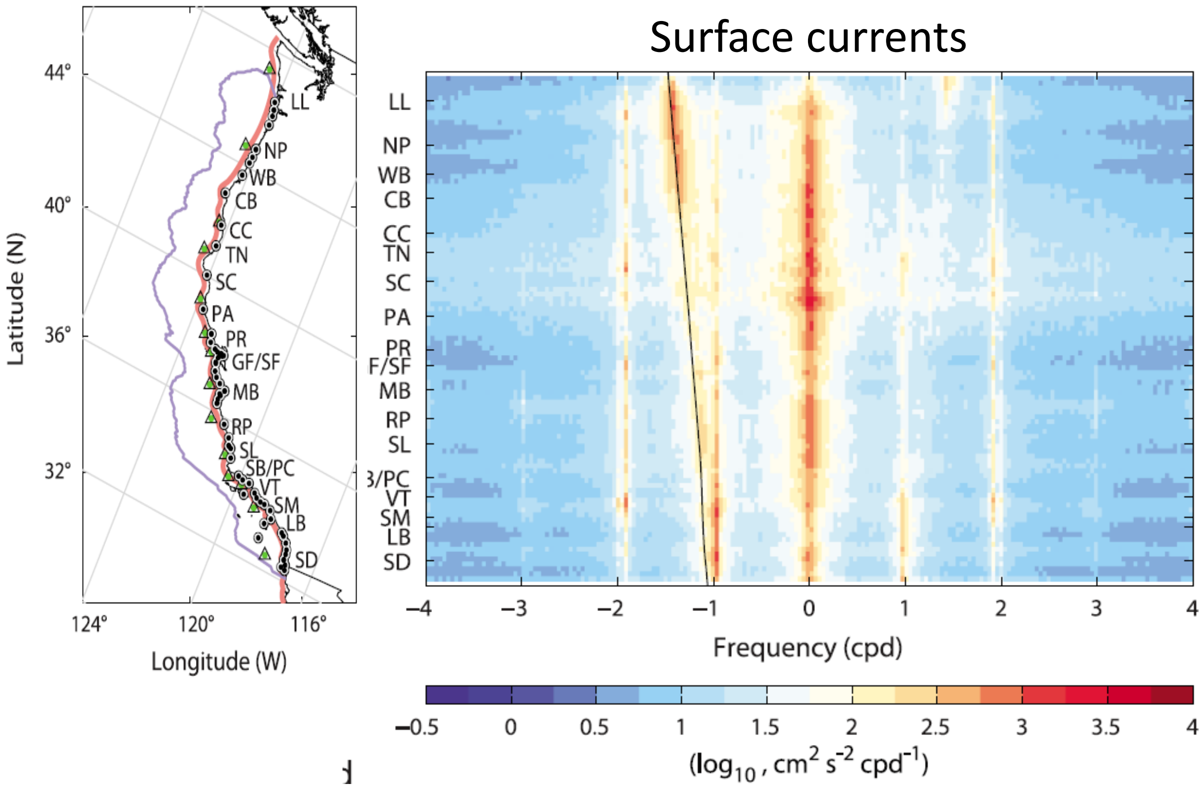
$$\hat{\mathbf{u}}(z,\omega) = \mathbf{H}(z,\omega)\hat{\boldsymbol{\tau}}(\omega)$$

$$\mathbf{H}(z,\omega) = \left(\langle \hat{\mathbf{u}}(z,\omega)\,\hat{\boldsymbol{\tau}}^{\dagger}(\omega)\rangle\right)\left(\langle \hat{\boldsymbol{\tau}}(\omega)\,\hat{\boldsymbol{\tau}}^{\dagger}(\omega)\rangle + \mathbf{R_a}\right)^{-1}$$

$\mathbf{R_a}$: Regularization matrix

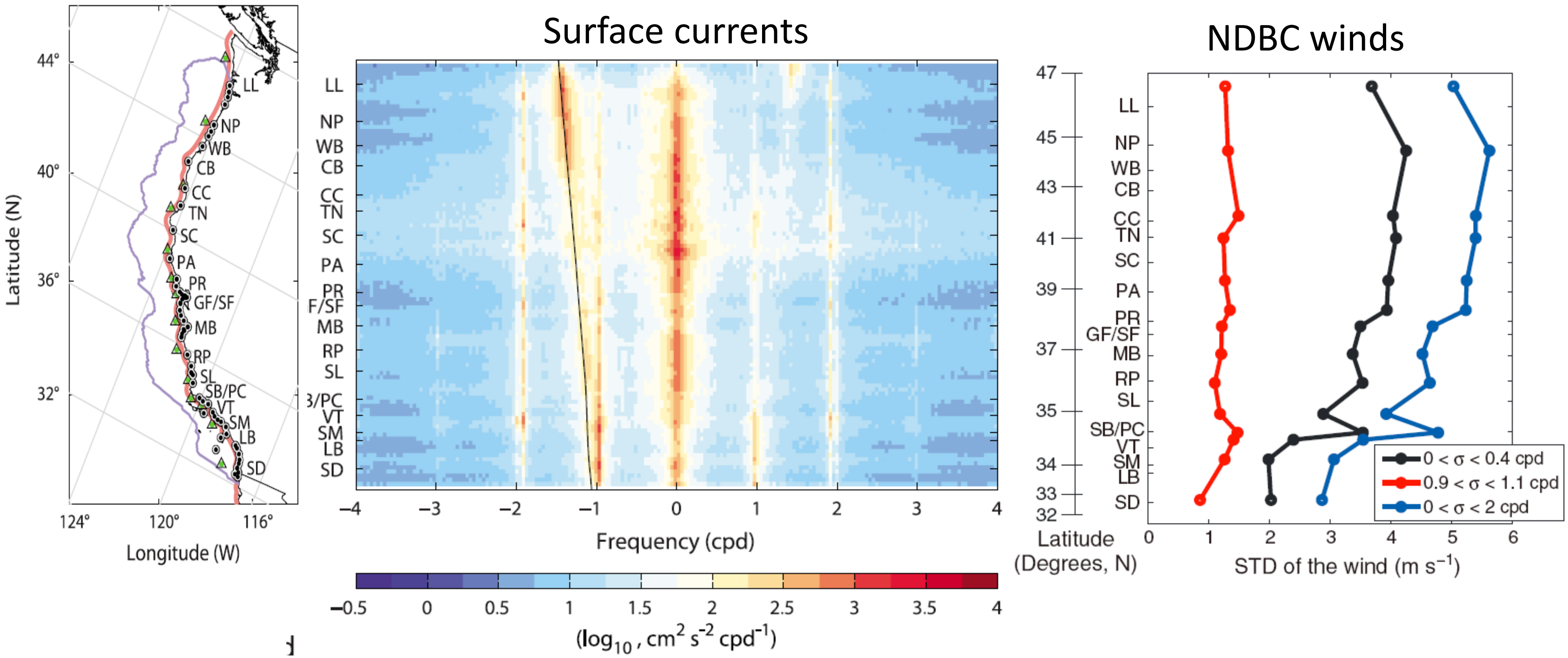
Variability of surface currents and wind



- Wind- and tide-coherent, low-frequency variance, and inertial variance

Kim *et al* (JGR, 2011)

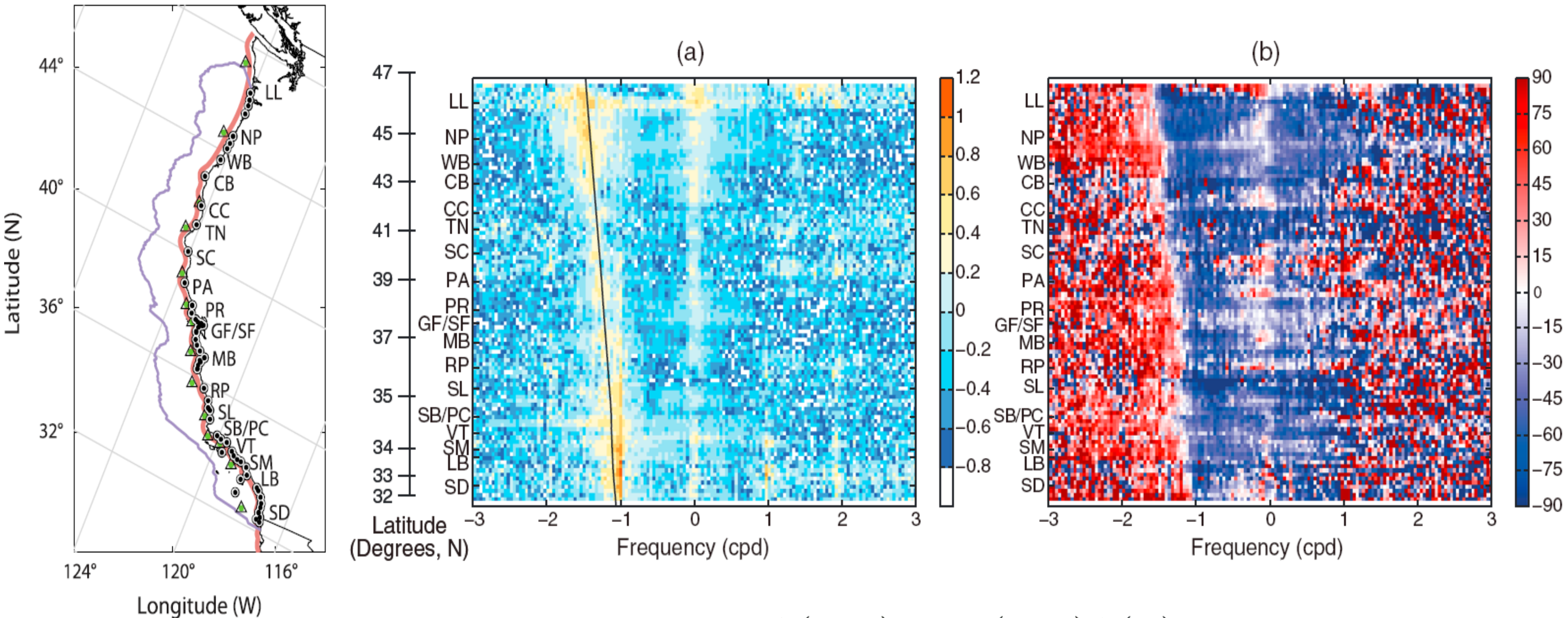
Variability of surface currents and wind



- Wind- and tide-coherent, low-frequency variance, and inertial variance
- Variance of the diurnal wind does not vary that much in the along-shore direction, but it is given as a function of distance from the shoreline (cross-shore direction).

Kim *et al* (JGR, 2011)

Coast-wide wind transfer functions



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(Kim and Crawford, *GRL 2014*)

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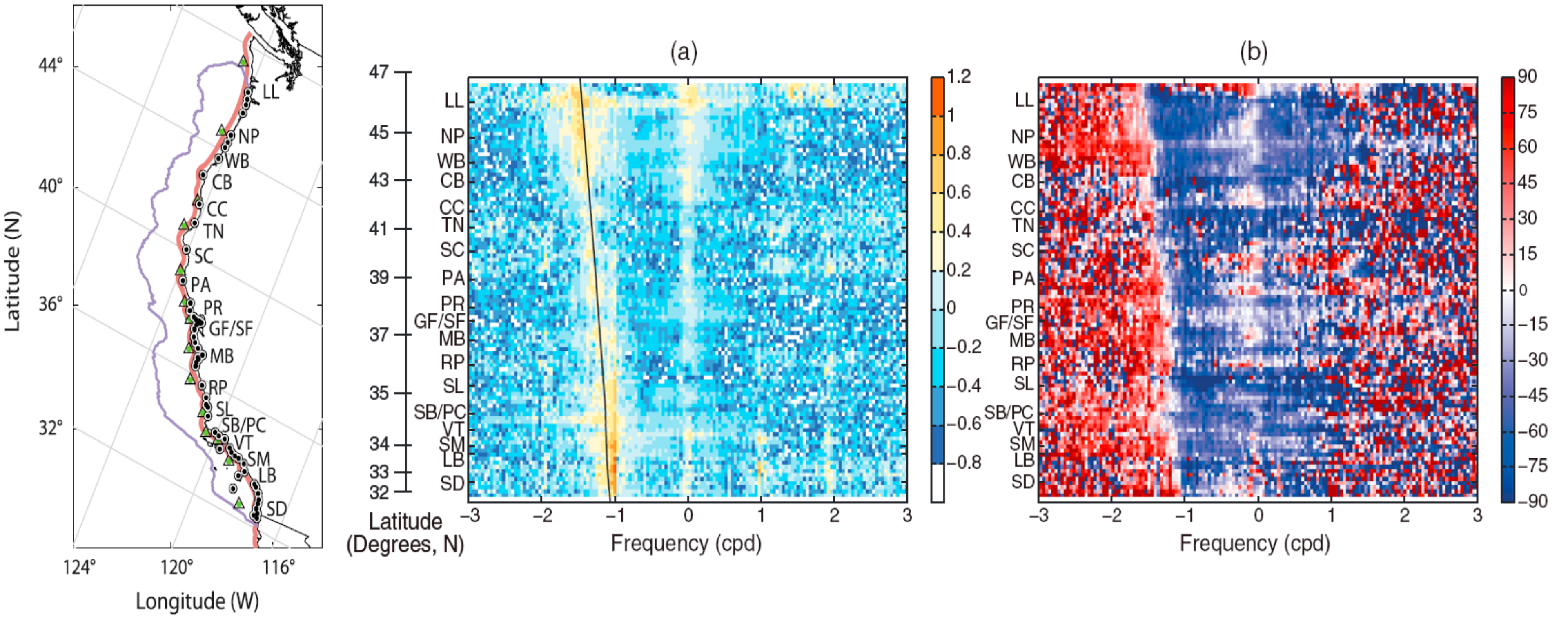

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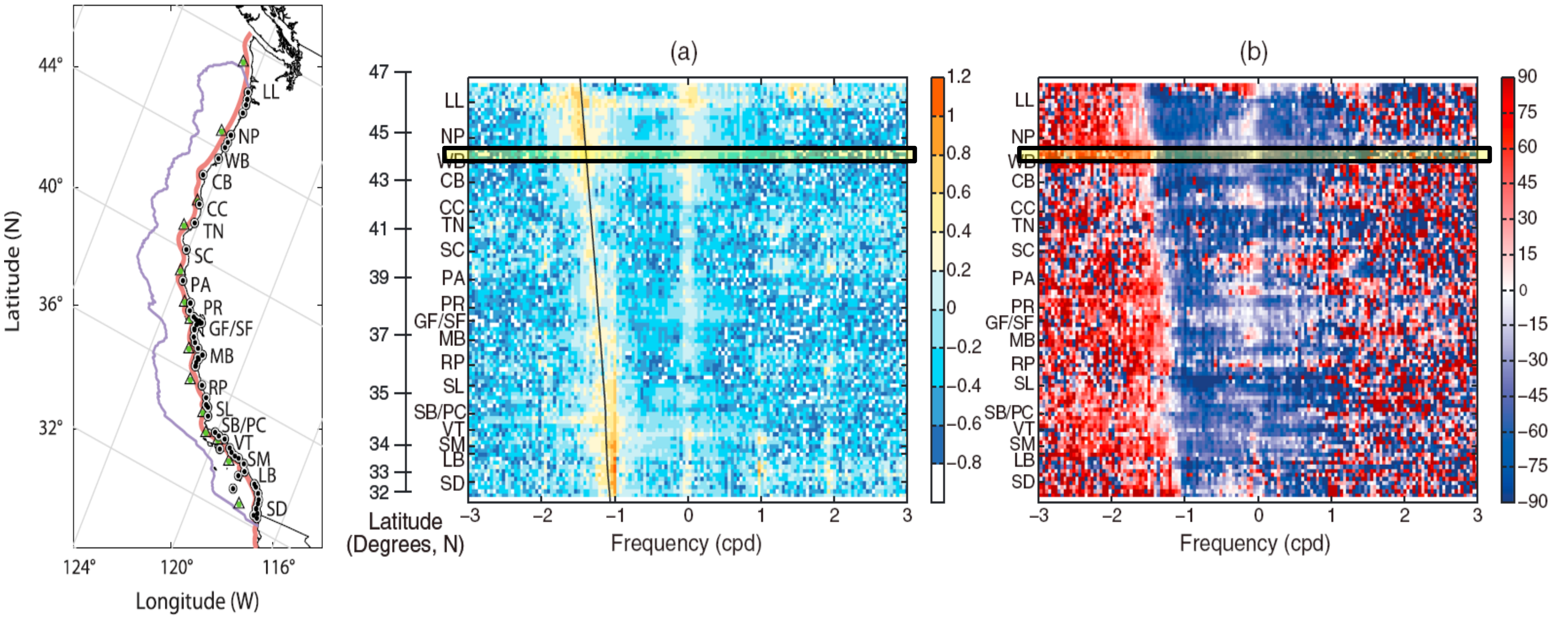

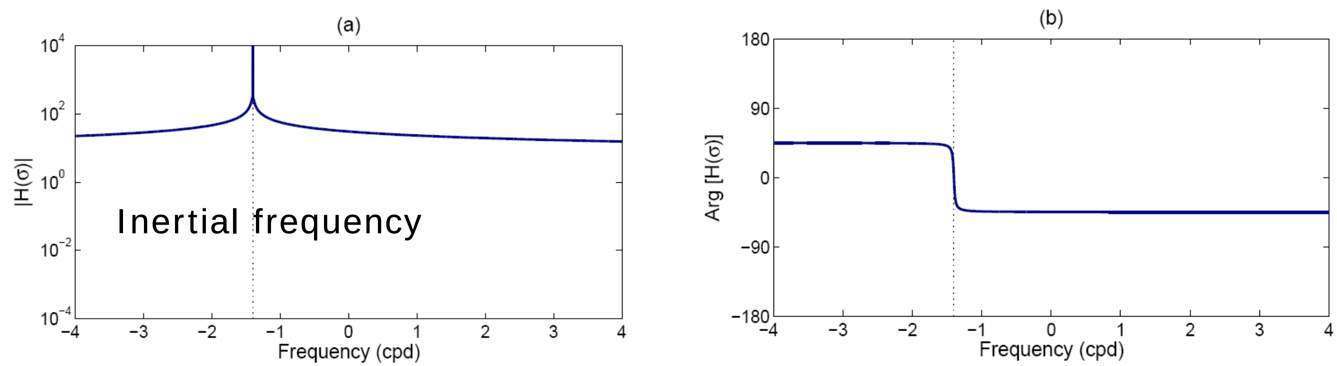

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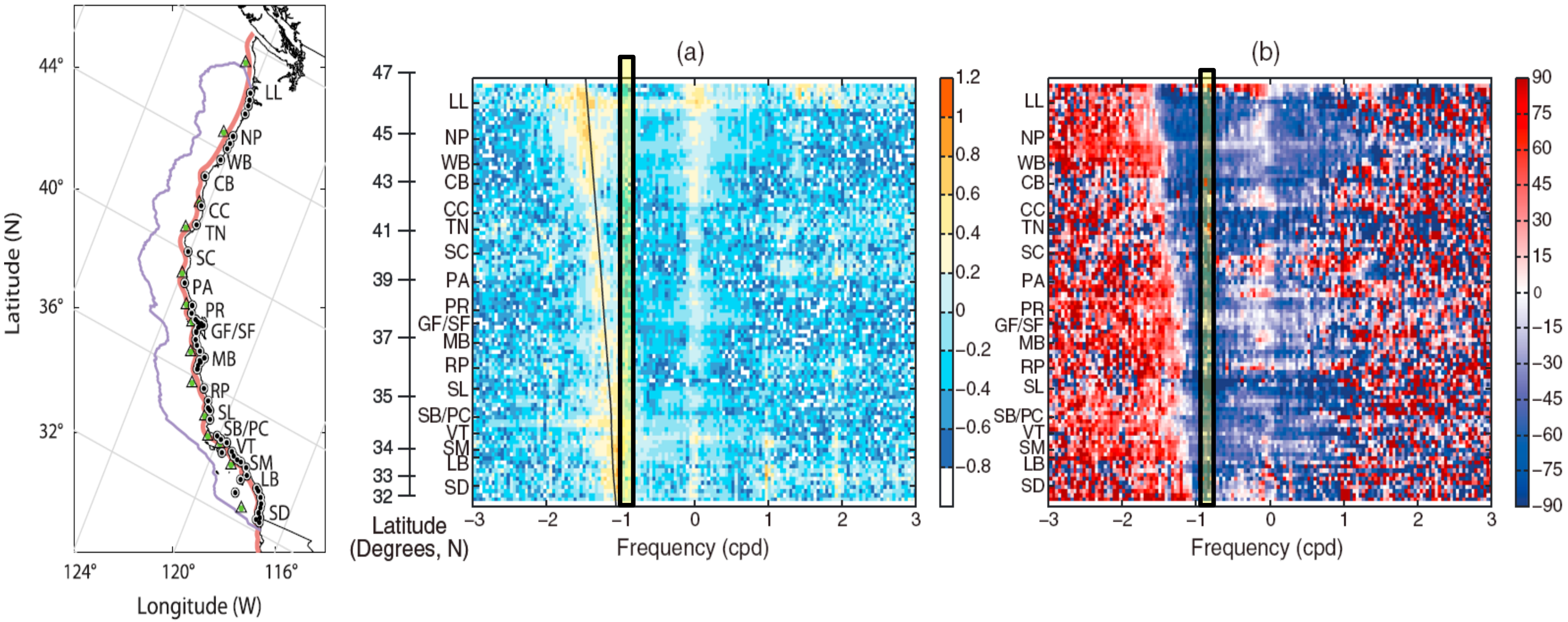
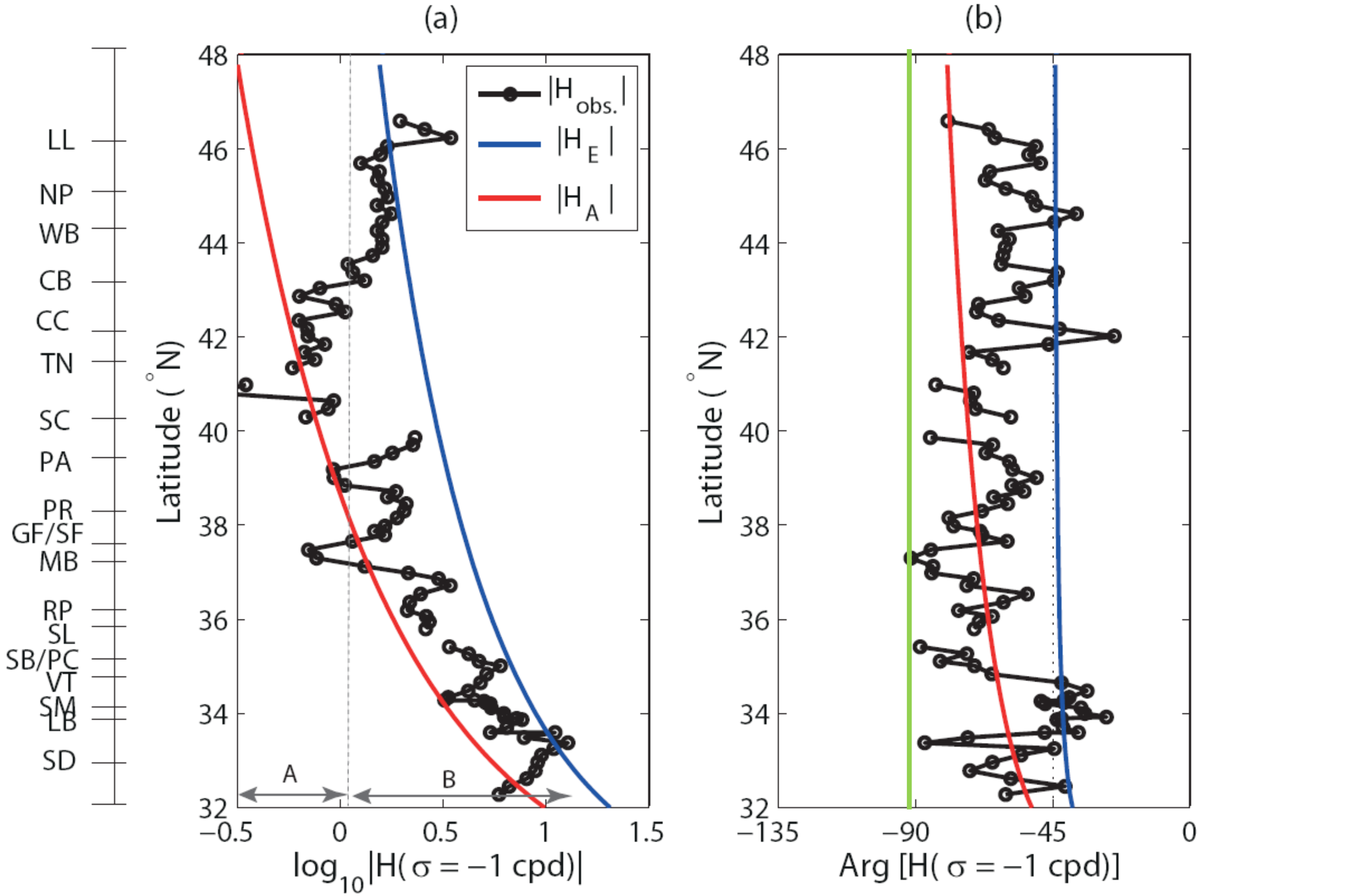


(Kim and Crawford, *GRL 2014*)

Resonant responses near the critical latitude



Slab layer model

Z = 0 (Ekman)

Z = 0.35δ_E(Near-surface avg. Ekman)

Resonant latitude due to land/sea breeze: ±30°N

Summary

- Wind-current responses are examined in the frequency domain and latitude using analytic solutions of Ekman model (and slab layer and surface-averaged Ekman models) and observations off the US West Coast.
- The current responses are enhanced at the local inertial frequency.
- Resonant responses can be expected at the +/-30^{o} latitude in the diurnal land-sea breeze environment.
- Energetic mixing and potential internal motions near the critical latitude are expected.
- Ocean responses to the diurnal wind and relevant bio-physical interactions can be a potential topic to pursue.
- ?