A statistical description on the wind-coherent responses of sea surface heights off the U.S. West Coast

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Questions

- Could we explain and isolate the wind-coherent sea surface heights (SSHs) using a statistical model and discern local and remote wind-forced SSHs?

- How are the statistical model and analytic model consistent?

- What will be the implications of the wind transfer function analysis (or multivariate wind regression)?

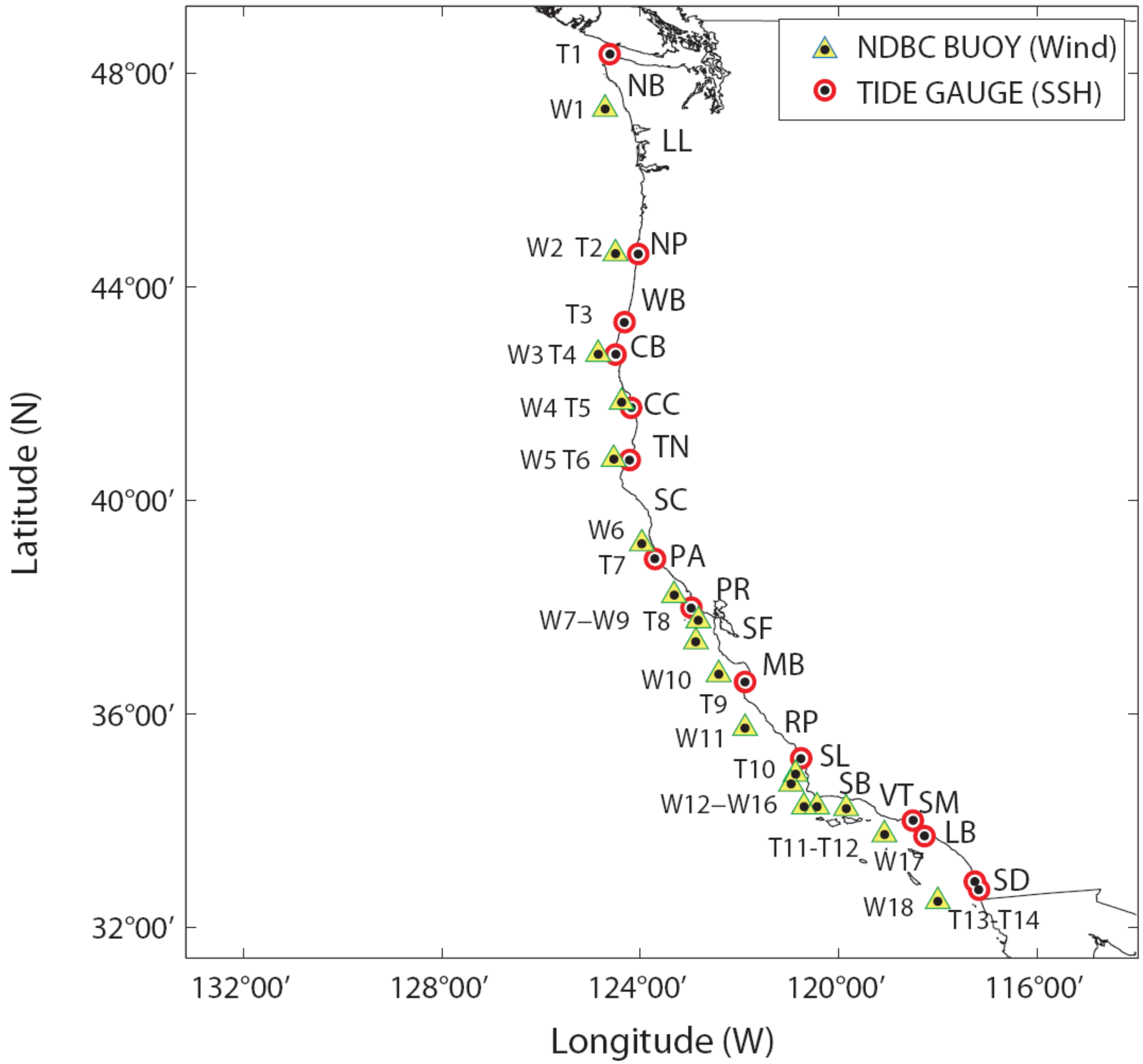
Outline

- An overview of study domain (U.S. West Coast)
  - Mean, standard deviations, steadiness of wind and SSHs
  - Energy spectra of wind and SSHs

- Statistical and analytic models
  - Comparison of two models: Transfer functions
  - 2D model's diagnostics - Distribution of terms in momentum equations (analytic model only)
  - Local vs. remote wind-forced SSHs (statistically model only)

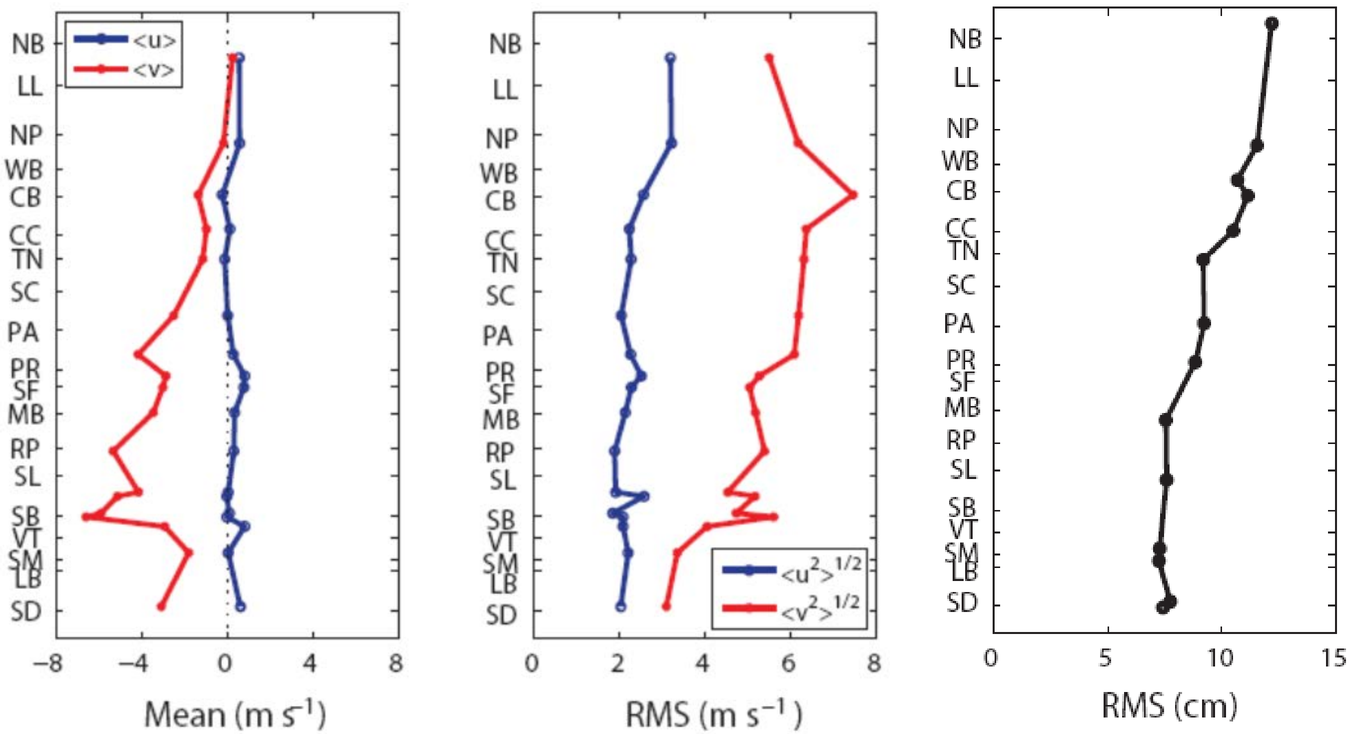
- Summary

Study domain



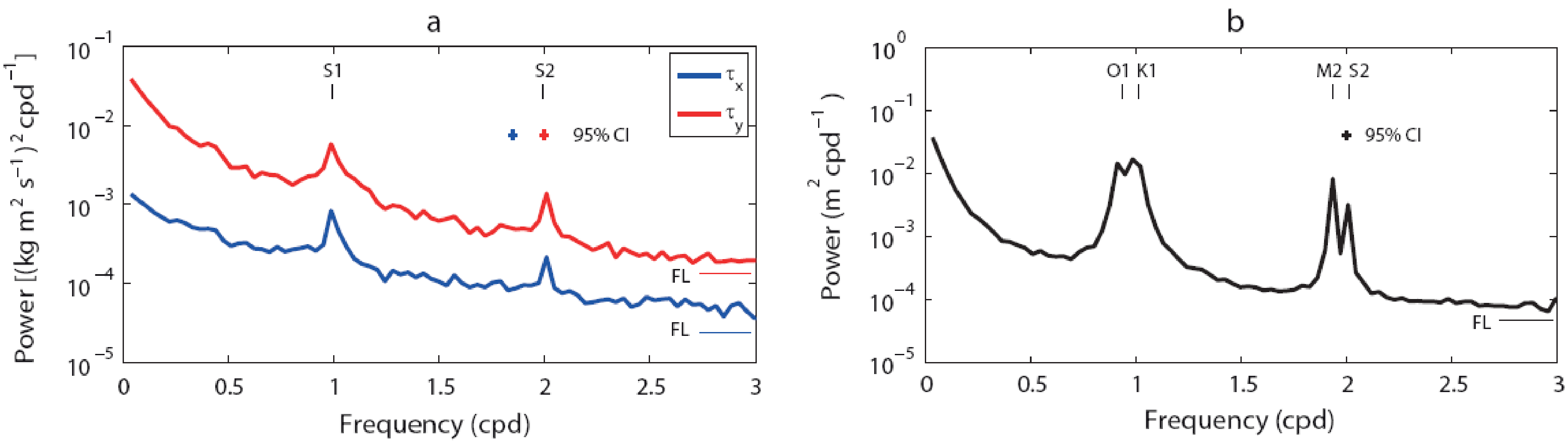
- Winds at NDBC (National Date Buoy Center) buoys
- Sea surface heights (SSHs) at NOAA tide gauges
- Hourly records for 16 years (1995-2010)
- Rotation of the wind vector along the principal axis (nearly parallel to the shoreline)

Mean and STD of wind & STD of SSHAs

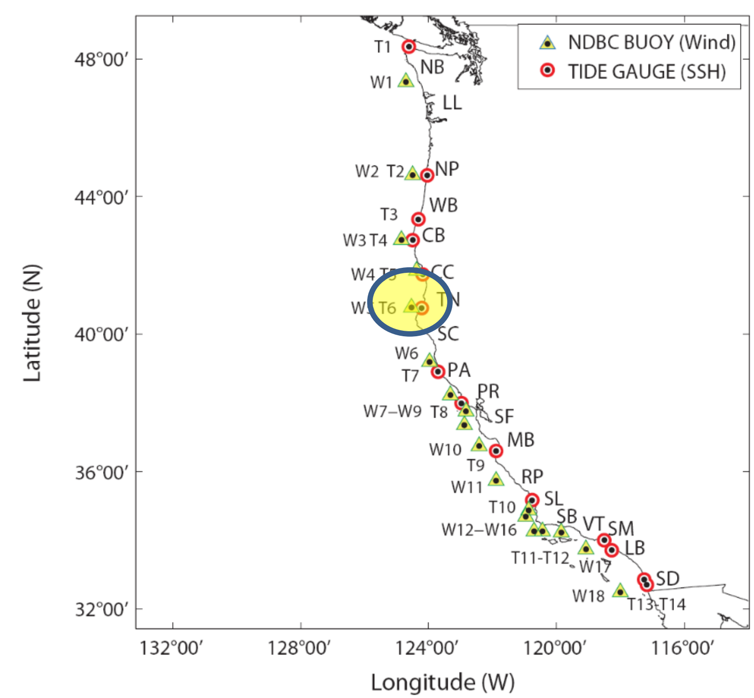


- Zero wind in the cross-shore direction and equatorward wind.
- 2-3 times higher variance of along-shore wind than cross-shore wind;
- Standard deviation (STD) of SSHAs (Sea surface height anomalies) – detided SSH [possibly barotropic tide removed]

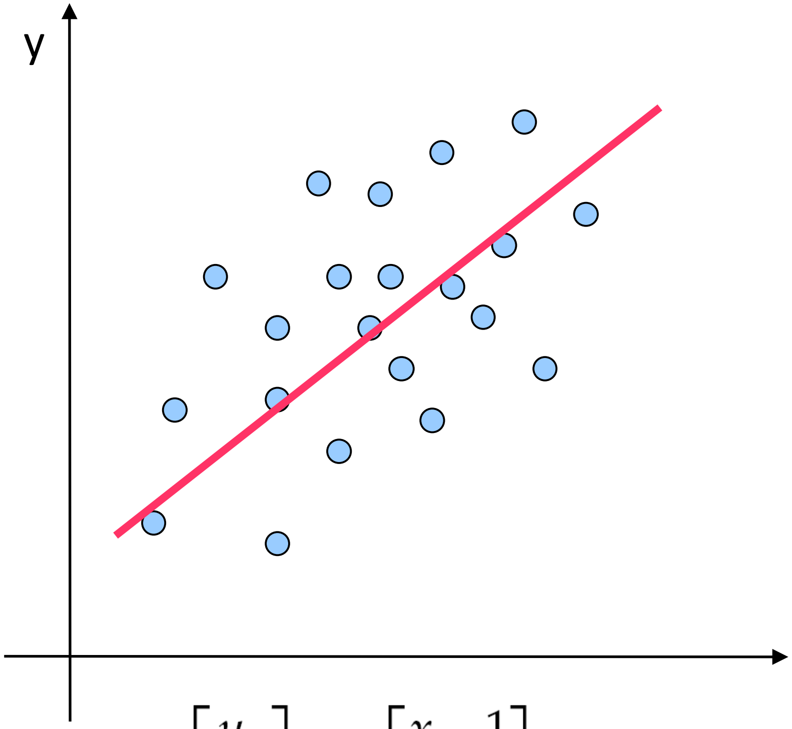
Spectral contents – an example



- An example of wind and SSHs nearby locations.
- Red spectra
- Alongshore wind has higher energy than cross-shore wind.
- Tidal cups in the barotropic tide-removed SSHs
- Drag coefficients (Yelland+Taylor 1996)



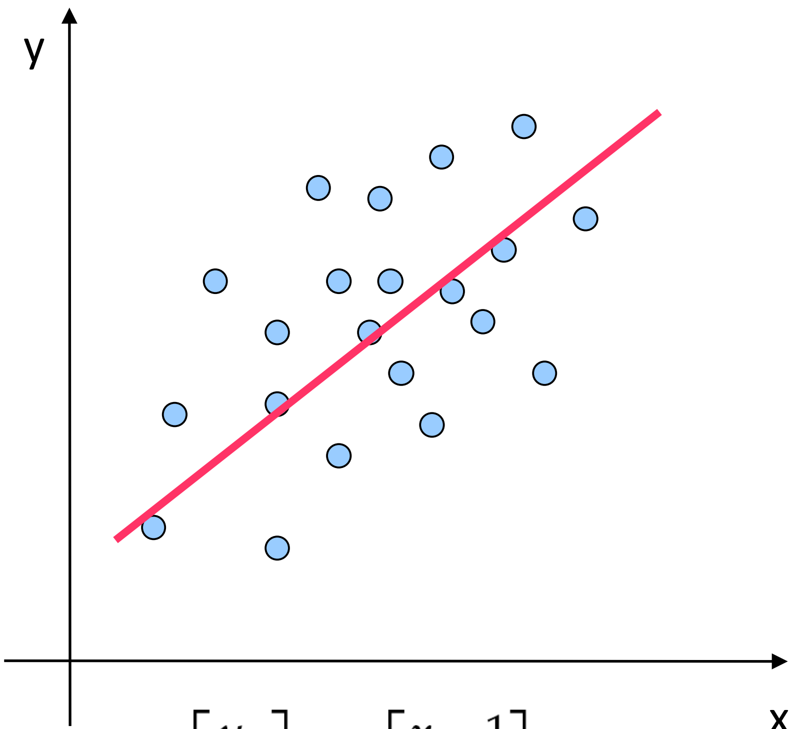
A statistical model – transfer function



$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\mathbf{d} = \mathbf{Zm}$$

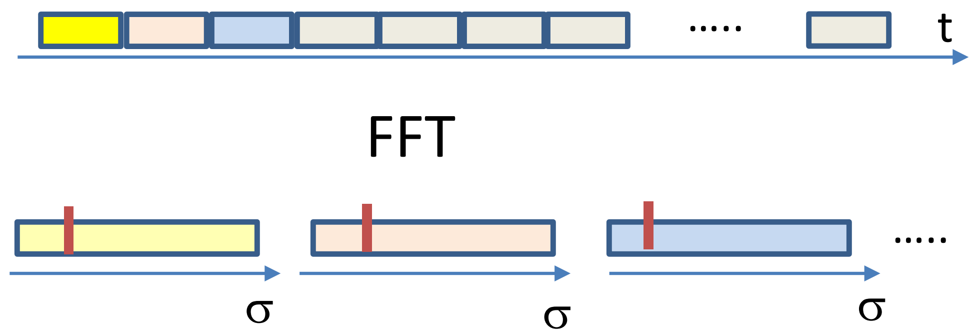
A statistical model – transfer function



$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\mathbf{d} = \mathbf{Zm}$$

- Ensemble time series of wind stress and SSHAs records
- Their Fourier coefficients in the individual frequency bins are regressed.



d: Data (SSHAs)
Z: Basis functions (wind stress)
m: Regression coefficients (Transfer function)

A statistical model – transfer function

- Decomposition of sea surface heights (SSHs)

$$\eta(\sigma) = \eta_{\mathrm{B}}(\sigma) + \eta_{\mathrm{T}}(\sigma) + \eta_{\mathrm{F}}(\sigma).$$

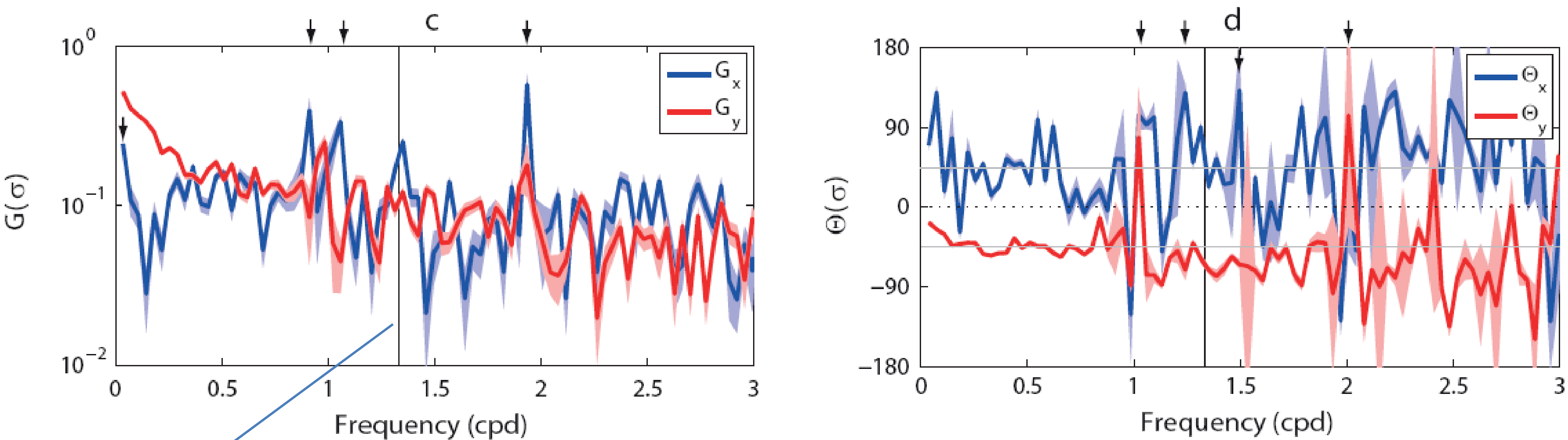
  - Inverted barometer effects (B); Tide-coherent (T); De-tided SSHs (F)

- Transfer function (G) in the frequency domain

$$\mathbf{G}(\sigma) = \left(\langle \eta_{\mathrm{F}}(\sigma)\, \boldsymbol{\tau}^{\dagger}(\sigma)\rangle\right)\left(\langle \boldsymbol{\tau}(\sigma)\, \boldsymbol{\tau}^{\dagger}(\sigma)\rangle + \mathbf{R_a}\right)^{-1}$$

  - Wind stress (τ), regularization matrix (Ra)
  - Estimates of wind-coherent SSHs based on a linear relationship between two variables in frequency-by-frequency
  - Regularization matrix adjusts misfitting and overfitting of the regression.

Data-derived transfer functions – magnitude and argument

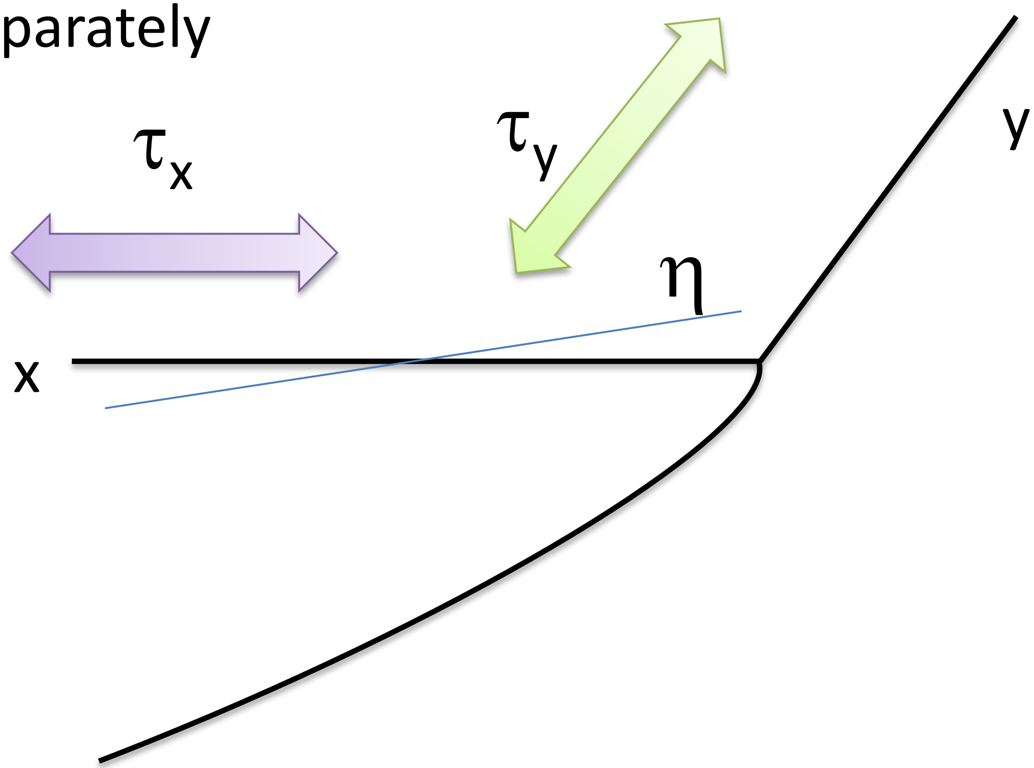


Local inertial frequency

- Low signal-to-noise ratio of the cross-shore wind.
- Red spectrum of G_y
- Slow transition (0 to 90), then nearly constant argument (Θ_y)

An analytical model

- 2D idealized model
- Straight coast (y direction);
- Small sea level slope in the alongshore direction compared to the slope in the cross-shore direction
- Depth-integrated cross-shore flux is closed.
- Periodic wind stress at each frequency and direction is applied separately
- .



$$\frac{\partial \eta}{\partial y} \ll \frac{\partial \eta}{\partial x},$$

$$\int u \, \mathrm{d}z = 0,$$

C. Winant, JPO(2007)
A. Ponte, JPO(2012)

Notations of individual terms (in the frequency domain)

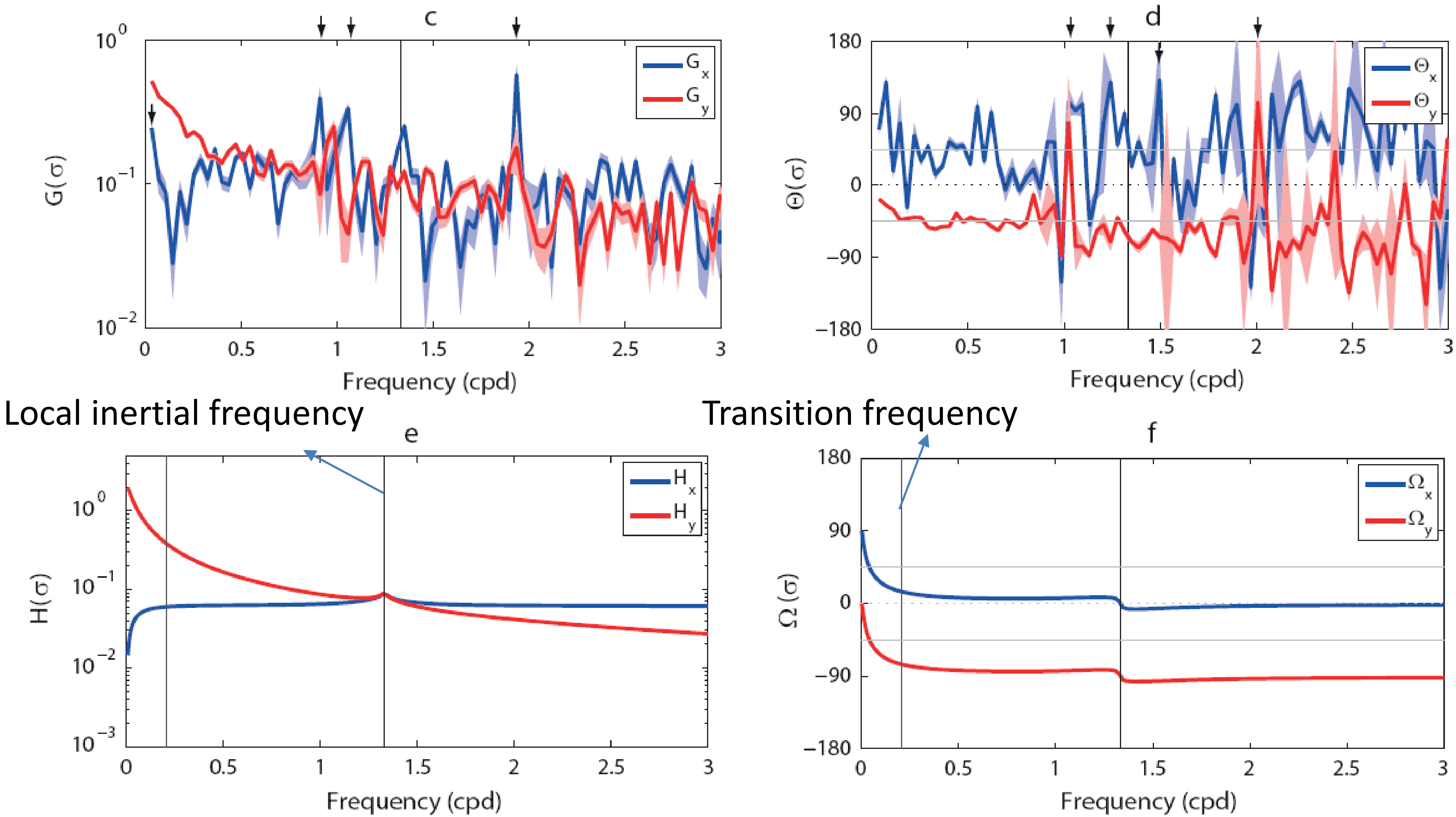
The wind-driven frictional currents ($W_{\{\cdot\}\{\cdot\}}$) can be described with the currents which are parallel and normal to the wind direction. These two primary wind-driven currents may generate the geostrophic currents ($P_{\{\cdot\}\{\cdot\}}$) due to local pressure gradients ($\partial\eta/\partial x$ and $\partial\eta/\partial y$) near the coastal boundaries [e.g., [59]]. As we compute the current response to τ_x and τ_y separately, their decomposed terms at the surface ($z = 0$) are expressed as [e.g., [85]]

$$\hat{u}(\sigma) = P_{xx}\hat{\tau}_x + P_{xy}\hat{\tau}_y + W_{xx}\hat{\tau}_x + W_{xy}\hat{\tau}_y, \tag{14}$$

$$\hat{v}(\sigma) = P_{yx}\hat{\tau}_x + P_{yy}\hat{\tau}_y + W_{yx}\hat{\tau}_x + W_{yy}\hat{\tau}_y, \tag{15}$$

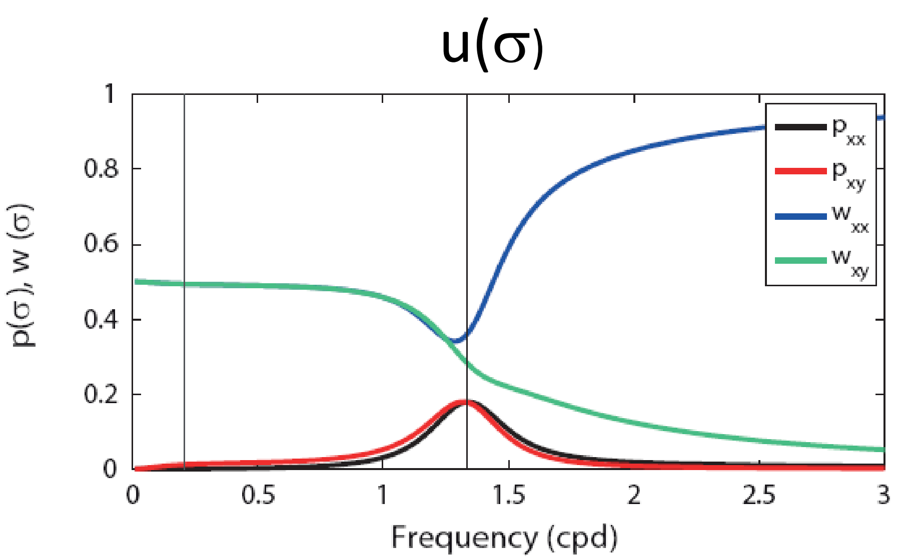
where the first and second subscripts of currents ($P_{\{\cdot\}\{\cdot\}}$, $W_{\{\cdot\}\{\cdot\}}$) denote the direction of the current and forcing (wind stress), respectively. For example, P_{yx} denotes Fourier coefficients of the v-component driven by $\partial\eta/\partial x$ that is related to frictional currents (W_{xx}), generated by τ_x.

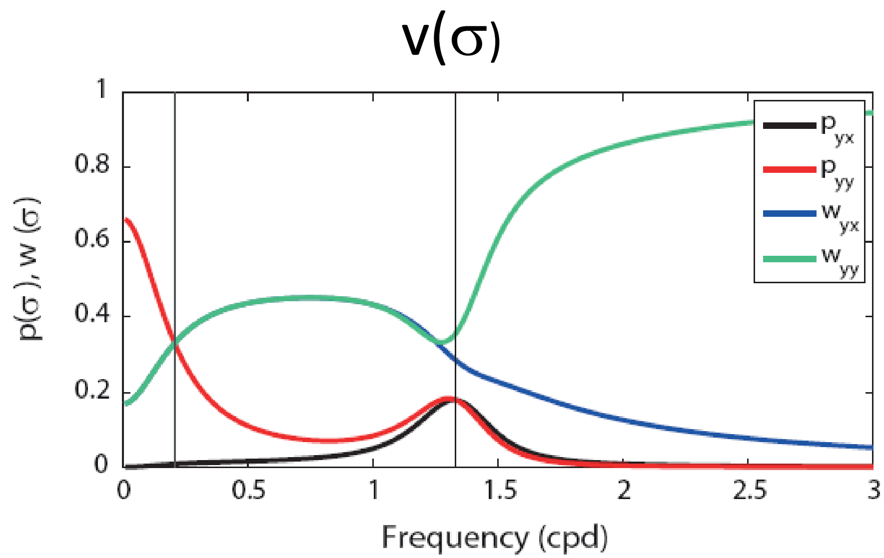
Model-derived transfer functions – magnitude and argument



- Consistent results in the alongshore direction
- The alignment of the wind and shoreline, bottom bathymetry, stratification may change the argument

2D model's diagnostic

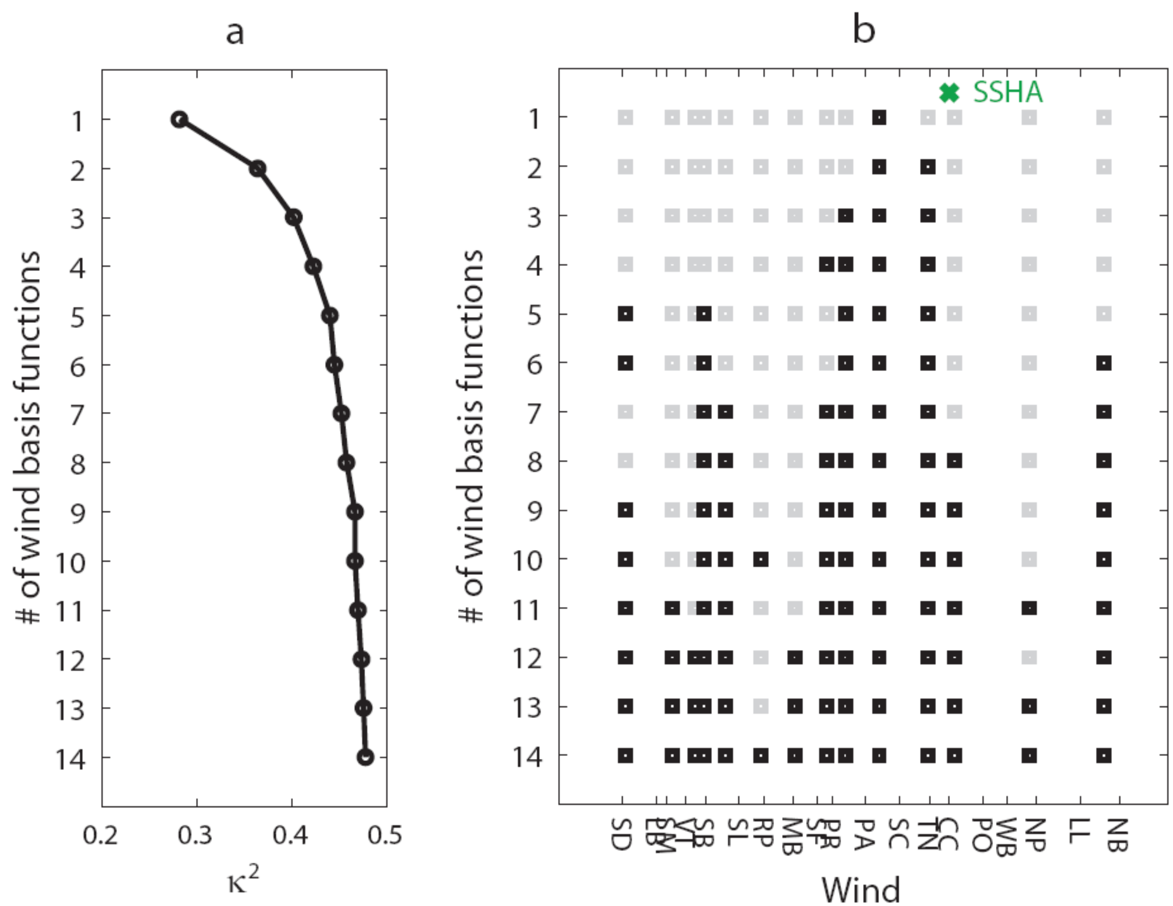




- Distribution of variance (squared quantity) of momentum terms in the frequency domain.
- Geostrophic currents due to pressure gradients in the cross-shore direction
- Frictional currents become dominant at higher than transition freq.

$$\hat{u}(\sigma) = P_{xx}\hat{\tau}_x + P_{xy}\hat{\tau}_y + W_{xx}\hat{\tau}_x + W_{xy}\hat{\tau}_y,$$
$$\hat{v}(\sigma) = P_{yx}\hat{\tau}_x + P_{yy}\hat{\tau}_y + W_{yx}\hat{\tau}_x + W_{yy}\hat{\tau}_y,$$

Multivariate wind regression (local vs. remote winds?)



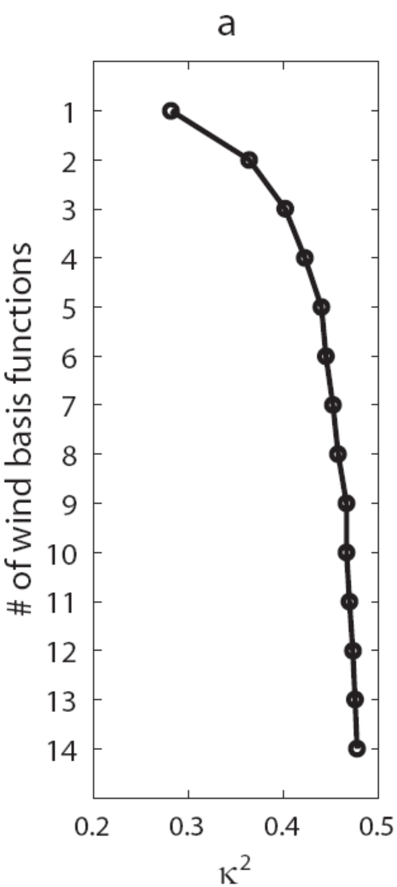
$$\eta_{\mathrm{W}}(\sigma) = \sum_{l=1}^{L} \mathbf{G}_l(\sigma)\boldsymbol{\tau}_l(\sigma),$$

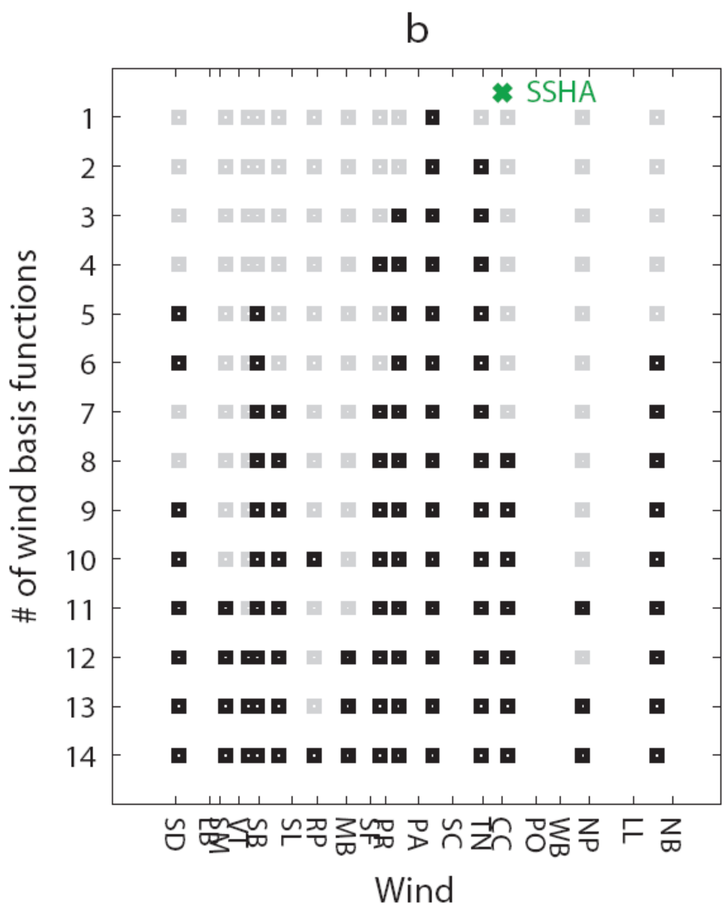
$$\kappa^2(\mathbf{x}) = 1 - \frac{\sum_n |\eta_{\mathrm{R}}(\mathbf{x}, \sigma_n)|^2}{\sum_n |\eta_{\mathrm{F}}(\mathbf{x}, \sigma_n)|^2},$$

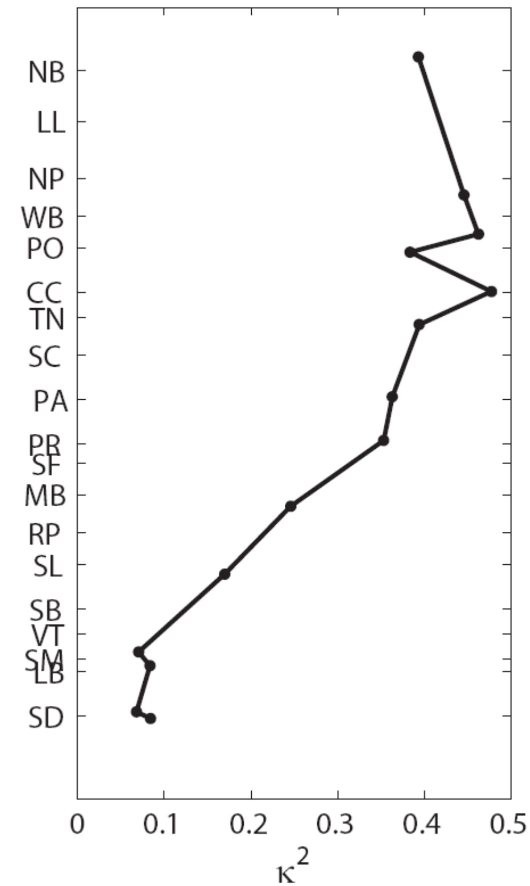
Cross-validated wind skill using incremental wind basis functions

Which basis provides a better skill?

Multivariate wind regression (local vs. remote winds?)







Cross-validated wind skill using incremental wind basis functions

Which basis provides a better skill?

Cross-validated wind skill using all available winds

Summary

- Wind-driven sea surface heights are examined with the statistical and analytic models by comparing transfer functions derived from individual approaches.

- The model diagnostics show how the terms in the momentum equations are balanced and how the transfer function anslysis are relevant to the analytic model.

- The multivariate wind regression can provide a tool to discern the local and remote wind-forced SSHs.