Revisit: Optimal interpolation in mapping of high-frequency radar-derived surface radial velocity maps

Sung Yong Kim
Department of Mechanical Engineering
Korea Advanced Institute of Science and Technology (KAIST)
Daejeon, 34141 Republic of Korea
syongkim@kaist.ac.kr

***Abstract*—This paper revisits optimal interpolation in mapping of high-frequency radar-derived surface radial velocity maps. OI has been used in the estimate of the current vector as an alternative to un-weighted least-squares fit [see [1] for more details]. OI is a biased estimator and assumes a (continuous) spatial covariance function, derived from the observed spatial scale and structure. It improves both baseline consistency and the uncertainty definition in the estimates.**

I. Introduction

High-frequency (HF) radar using Bragg-backscattered radar signals ([2], [3], [4]) has matured as an oceanographic observation tool. It can provide hourly high resolution surface current fields covering 50 to 150 km from the coastline with 0.5–6 km spatial resolution. It can serve as part of the infrastructure of the coastal ocean observing system to integrate other in-situ observations and numerical modeling products. Research on surface current measurements using HF radars can be classified by technical issues, scientific interpretations, and environmental applications: the processing of the backscattered radar signal and the generation of the vector current map from multiple radial current measurements; understanding of the ocean surface circulation through analysis in time and space; water quality monitoring, larvae spreading, search and rescue, and oil spill tracking [[5]].

An un-weighted least-squares fitting (UWLS) method has been used by many authors to extract the vector currents from the radial velocities ([6], [7], [8]). Implicit in this approach is an assumption of a uniform vector velocity producing the radial velocities within the search radius for a given vector grid point. In other words, the correlation of the vector current is assumed to be one everywhere within the search radius and zero outside. The method also assumes an unlimited signal variance, which may create spurious estimates when combining nearly aligned noisy radial velocities due to the singularity of the geometric covariance matrix. The terms 'signal variance' and 'error variance' are defined here as the expected variance of the surface currents and the expected observational error variance of the surface currents, respectively. In operation, spurious vector solutions most often occur near the baseline between two radars or near the maximum range. The segmented correlation function in the UWLS method can also produce a discontinuous current field. The proposed optimal interpolation (OI) method uses a correlation for the surface currents which more accurately describes the spatial relationship between radial velocity measurements [[1]].

The technical and mathematical descriptions of un-weighted least-squares fit and optimal interpolation are presented in the following section.

II. Methods

A. Radial velocity maps

The radial velocity (r) at a radial grid point with a bearing angle (θ) is presented as a sum of the projection of two orthogonal components (u and v) onto that angle and the observational error (ϵ) [e.g., [9]]:

$$r = \mathbf{u}^{\dagger}\mathbf{g} + \epsilon = u\cos\theta + v\sin\theta + \epsilon, \qquad (1)$$

where $\mathbf{g} = [\cos\theta \;\; \sin\theta]^{\dagger}$ is the directional unit vector, θ is the bearing angle, and $\mathbf{u} = [u \;\; v]^{\dagger}$ is the vector current at the sampling location († is the vector or matrix transpose).

B. Un-weighted least squares fit

A vector current ($\hat{\mathbf{u}}$; 2×1 vector) estimated from radials ($\mathbf{r}$; $L_a \times 1$ vector) using unweighted LSF is

$$\hat{\mathbf{u}} = \left(\mathbf{G}^{\dagger}\mathbf{G}\right)^{-1}\mathbf{G}^{\dagger}\mathbf{r} = \mathbf{H_a}\mathbf{r}, \qquad (2)$$

where $\mathbf{G} = [\mathbf{g}_1\,\mathbf{g}_2\cdots\mathbf{g}_{L_a}]^{\dagger}$ ($\mathbf{G}$; $L_a \times 2$ matrix; equation 1).

In surface current measurements using HFRs, GDOP depends on the number (L_a) of available radial velocities within a search radius [e.g., [1]]. The GDOP indicates the diagonal components of the inverse of a geometric covariance matrix ($\mathbf{G}^{\dagger}\mathbf{G}$; 2×2 matrix), which corresponds to equation 12 in OI:

$$\boldsymbol{\nu} = \left(\mathbf{G}^{\dagger}\mathbf{G}\right)^{-1} = \begin{bmatrix} \nu_{uu} & \nu_{uv} \\ \nu_{vu} & \nu_{vv} \end{bmatrix}, \qquad (3)$$

where ν_{uu} and ν_{vv} are the GDOP in the x and y directions, respectively,

$$\nu_{uu} = \frac{1}{\det(\mathbf{G}^\dagger\mathbf{G})} \sum_{l=1}^{L_a} \sin^2\theta_l \geq \frac{1}{L_a}, \tag{4}$$

$$\nu_{vv} = \frac{1}{\det(\mathbf{G}^\dagger\mathbf{G})} \sum_{l=1}^{L_a} \cos^2\theta_l \geq \frac{1}{L_a}, \tag{5}$$

and $\det$ denotes the determinant of a matrix. Thus, the GDOP associated with L_a radials is given by

$$\nu = \nu_{uu} + \nu_{vv} \geq \frac{4}{L_a}. \tag{6}$$

The GDOP has been used as a cutoff value for spurious and inconsistent vector estimates [e.g., [8], [10]]. However, since the GDOP only has a lower bound without an upper limit (equations 4 to 6), it may not be appropriate to be chosen as a criterion. In particular, GDOP can vary in time and space as the available radials vary in the same way. However, GDOP has been used as a fixed value with the assumption that there are no missing radials [e.g., [11], [12], [13]]. For instance, it has been mispresented as a unit quantity in the operational quality assurance and quality control (QAQC) [e.g., http://cordc.ucsd.edu/projects/mapping/maps/].

C. *Optimal interpolation*

A vector current ($\hat{\mathbf{u}}$) is OI-mapped from radials ($\mathbf{r}$; $L_b \times 1$ vector) using a data-model covariance ($\mathrm{cov}_{\mathrm{dm}}$; $L_b \times 2$ matrix) and data-data covariance ($\mathrm{cov}_{\mathrm{dd}}$; $L_b \times L_b$ matrix):

$$\hat{\mathbf{u}} = \mathrm{cov}_{\mathrm{dm}}^\dagger \mathrm{cov}_{\mathrm{dd}}^{-1} \mathbf{r} = \mathbf{H_b}\mathbf{r}, \tag{7}$$

where the data-model covariance is the covariance between a vector current at the grid point of interest and radials, and the data-data covariance is the covariance between radials themselves:

$$\hat{\mathbf{u}} = \left(\langle \mathbf{r}\mathbf{u}^\dagger \rangle\right)^\dagger \left(\langle \mathbf{r}_s \mathbf{r}_s^\dagger \rangle + \langle \boldsymbol{\epsilon}\boldsymbol{\epsilon}^\dagger \rangle\right)^{-1} \mathbf{r}, \tag{8}$$

$$= \left(\mathbf{g}_j^\dagger \langle \mathbf{u}_j \mathbf{u}_i^\dagger \rangle\right)^\dagger \left(\mathbf{g}_j^\dagger \langle \mathbf{u}_j \mathbf{u}_k^\dagger \rangle \mathbf{g}_k + \langle \boldsymbol{\epsilon}\boldsymbol{\epsilon}^\dagger \rangle\right)^{-1} \mathbf{r}, \tag{9}$$

where $\mathbf{u} = \mathbf{u}_i = [u_i \ v_i]^\dagger$ is a vector current at the i-th grid point of interest and $\mathbf{r} = [r_1 \ r_2 \ \cdots \ r_{L_b}]^\dagger$ is the radial velocities, participating in the estimate of the vector current ($j, k = 1, 2, \cdots, L_b$).

The current covariance ($\langle \mathbf{u}\mathbf{u}^\dagger \rangle$) can be simplified with the signal variance (σ^2) and spatial correlation (ρ) as a function of spatial lags, and the error covariance ($\langle \boldsymbol{\epsilon}\boldsymbol{\epsilon}^\dagger \rangle$) can be simplified as a diagonal matrix scaled by a scalar of γ^2, which can retain dependence on locations of the vector current grid or be a constant regardless of their locations:

$$\hat{\mathbf{u}} = \left[\mathbf{g}_j^\dagger \sigma_{ij}^2 \rho\left(\Delta x_{ij}, \Delta y_{ij}\right)\right]^\dagger \left[\mathbf{g}_j^\dagger \sigma_{jk}^2 \rho(\Delta x_{jk}, \Delta y_{jk}) \mathbf{g}_k + \delta_{jk}\gamma_k^2\right]^{-1} \tag{10}$$

where δ_{jk} denotes the Kronecker delta and, for instance, an exponential correlation function, frequently used for mapping submesoscale surface current fields [e.g., [1], [14]], is given by

$$\rho(\Delta x, \Delta y) = \exp\left(-\sqrt{\frac{\Delta x^2}{\lambda_x^2} + \frac{\Delta y^2}{\lambda_y^2}}\right), \tag{11}$$

and λ_x and λ_y denote the decorrelation length scales in the x and y directions, respectively.

The root-mean-square estimated from the covariance of nearby radial pairs obtained from multiple radars [see [1], [9] for more details], i.e., the uncertainty of the radial observations in the area of interest, can be used as γ^2 in equation 10. However, the error covariance of the radials ($\langle \boldsymbol{\epsilon}\boldsymbol{\epsilon}^\dagger \rangle$; $L_b \times L_b$ matrix) may not be a diagonal matrix because the noise of the radials may not be independent.

The uncertainty ($\boldsymbol{\kappa}$) of the OI-mapped vector current ($\hat{\mathbf{u}}$), which corresponds to equation 3 in LSF, is defined as

$$\boldsymbol{\kappa} = \frac{\gamma^2}{\sigma^2}\left(\mathrm{cov}_{\mathrm{mm}} - \mathrm{cov}_{\mathrm{dm}}^\dagger \mathrm{cov}_{\mathrm{dd}}^{-1} \mathrm{cov}_{\mathrm{dm}}\right) = \begin{bmatrix} \kappa_{uu} & \kappa_{uv} \\ \kappa_{vu} & \kappa_{vv} \end{bmatrix}, \tag{12}$$

where

$$0 \leq \kappa_{uu} \leq \gamma^2, \tag{13}$$

$$0 \leq \kappa_{vv} \leq \gamma^2. \tag{14}$$

Note that $\boldsymbol{\kappa}$ has a unit of the square of the radial velocities. Additionally, a normalized uncertainty index ($\hat{\boldsymbol{\kappa}}$; $\hat{\boldsymbol{\kappa}} = \boldsymbol{\kappa}/\gamma^2$; $0 \leq \hat{\boldsymbol{\kappa}} \leq 1$) can be used as a consistent criterion for quality assurance and quality control (QAQC) of estimated vector currents.

III. Conclusion

In mapping of high-frequency radar-derived surface radial velocity maps, OI has been used in the estimate of the current vector as an alternative to un-weighted least-squares fit. As a biased estimator, OI assumes a (continuous) spatial covariance function, derived from the observed spatial scale and structure in contrast of the segmented correlation function within a search range. OI improves both baseline consistency and the uncertainty definition in the estimates.

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