ID: P188

# Estimates of diffusion coefficients derived from satellite imagery


Sung Yong Kim[1,†]

[1]Division of Ocean Systems Engineering Korea Advanced Institute of Science and Technology
syongkim@kaist.ac.kr[†]



## Abstract

The high-resolution satellite-derived images of 500 m spatial and hourly temporal resolutions are examined to quantify the diffusion coefficients of the sea surface chlorophyll concentration. Based on advection-diffusion equations, the optimal solution of horizontal diffusivity is estimated using advection by the observed currents (e.g., tidal and geostrophic currents) and the derivatives of concentrations in time and space. We implement the estimated diffusion coefficients in the random walk and flight models, which have been used for tracking of the concentration of phytoplankton, zooplankton, and individuals of fishes as a statistical approach, through comparison of the data and model outputs. This work will be applied to the ecosystem process studies at submesoscale [*O*(1) km spatial scale and *O*(1) hour time scale] and improve the limitation of the present-day statistical modeling.


## Introduction

As an important topic in the physical-biological interactions in ocean science and engineering, the tracking of harmful algae booms such as green and red tides, the pollutants and oceanic trash including micro-plastics, the larvae and fish has been addressed with advection and diffusion problems in the environmental fluid [e.g., [1]]. Besides their horizontal movement, the vertical migration transport associated with mixing and transport has been addressed with vertical diffusivity and turbulence studies [e.g.,[2]]. Particulary, the estimates of horizontal and vertical diffusivity and their parameterizations have been investigated with observations [e.g., [3]] and numerical simulations [e.g., [2]].

The transport of waterborne materials should be taken into account in the spatial and temporal scales of background flows and diffusion. The periodic currents at tidal scales may generate the closed path of particles. However, the low frequency or mean circulation can govern the footprint of spatial and temporal visitation of targeting waterborne materials as tracking the particles is associated with the long-term time integration [e.g., [4]].

## An idealized simulation

### Surface advection and diffusion

Using the decorrelation length scales and spectral contents in the frequency domain of the surface current fields, estimated from their long-term observations, the surface current maps $[\mathbf{u}(\mathbf{x},t)]$ can be generated. The surface current field in the one- or two-dimensional domain can be used to yield the corresponding dimensional concentration maps $[\mathrm{C}(\mathbf{x},t)]$ when the particles having a constant number are released and tracked in space and time continuously with a random walk model, which contains a random parameter ($\boldsymbol{\epsilon}$). Two primary approaches in Lagrangian stochastic modeling such as random walk and random flight models have been used to simulate the random processes in nature.

The model surface currents ($u^*$ and $v^*$) based on a spectral model are defined as

$$u^*(x,y,t) = \sum_{m=-M^*}^{M^*}\sum_{n=-N^*}^{N^*}\sum_{s=-S^*}^{S^*} \hat{A}_{mns}\cos\vartheta_{mns} + \hat{B}_{mns}\sin\vartheta_{mns}, \quad (1)$$

$$v^*(x,y,t) = \sum_{m=-M^*}^{M^*}\sum_{n=-N^*}^{N^*}\sum_{s=-S^*}^{S^*} \hat{C}_{mns}\cos\vartheta_{mns} + \hat{D}_{mns}\sin\vartheta_{mns}, \quad (2)$$

where

$$\vartheta_{mns} = 2\pi\left(k_m x + l_n y - \sigma_s t\right) = 2\pi\left(\frac{m}{L_x^*}x + \frac{n}{L_y^*}y - \sigma_s t\right), \quad (3)$$

and $k_m$ and $l_n$ denote the wavenumber, and $L_x^*$ and $L_y^*$ are the length of the domain in the $x$ and $y$ directions, respectively. As the spatial covariance in the physical space is equivalent to the power spectrum in the wavenumber domain [e.g., [5]], the coefficients ($\hat{A}_{mn}$, $\hat{B}_{mn}$, $\hat{C}_{mn}$, and $\hat{D}_{mn}$) are assumed as random variables of normal distribution having zero mean and unit standard deviation, [$\mathsf{N}(0,1)$], for instance,

$$\hat{A}_{mns} = (\xi_{mn})^{\frac{1}{2}}\,\xi_s \mathsf{N}(0,1), \quad (4)$$

with the power spectrum of the current field in the wavenumber [$\xi_{mn} = \xi(k_m,l_n)$] and frequency [$\xi_s = \xi(\sigma_s)$] domains. The wavenumber spectrum can be approximated with two-dimensional Gaussian or exponential function:

$$\xi(k_m,l_n) = \pi\lambda_x\lambda_y \exp\left(-\pi^2 k_m^2\lambda_x^2 - \pi^2 l_n^2\lambda_y^2\right), \quad (5)$$

$$\xi(k_m,l_n) = \frac{4\lambda_x\lambda_y}{\left(1+4\pi^2k_m^2\lambda_x^2+4\pi^2l_n^2\lambda_y^2\right)^{3/2}}, \quad (6)$$

and $\lambda_x$ and $\lambda_y$ denote the de-correlation length scales in the $x$ and $y$ directions, respectively.

Conversely, the spectrum in the frequency domain is approximated with background energy and variance at peak frequencies,

$$\xi_s(\sigma_s) = A\sigma_s^{-\alpha} + \sum_{n=1}^{N} B_n \exp\left(-\frac{|\sigma_s - \nu_n|}{(\lambda_t)_n}\right), \quad (7)$$

where $A$ and $B_n$ are amplitudes of the spectrum in the frequency domain, and $\alpha$ is the slope of background energy. $\nu_n$ and $\lambda_t$ denote the frequencies having peaks and their bandwidths, respectively ($n = 1, 2, \cdots, N$). The bandwidth can be determined with the width to have $a$ dB reduced from the peak, where $a$ can be 1, 5, or 10 depending on the decay pattern of the local peak.

The location of $m$-th particle ($\mathbf{x}_m$) is estimated cumulative time integration of the currents at the particle's location:

$$\mathbf{x}_m(t_N) = \mathbf{x}_m(t_0) + \int_0^{t_N}\left[\mathbf{u}(\mathbf{x}_m,t') + \boldsymbol{\epsilon}_m(t')\right]\mathrm{d}t', \quad (8)$$

$$= \mathbf{x}_m(t_0) + \sum_{n=1}^{N}\left[\mathbf{u}(\mathbf{x}_m,t_n) + \boldsymbol{\epsilon}_m(t_n)\right]\Delta t, \quad (9)$$

where the random variable ($\boldsymbol{\epsilon}_m$) is generated as many as the number of particles at each time step ($m = 1, \cdots, M$).

$$C(\mathbf{x},t;\boldsymbol{\epsilon}) = \frac{H(\mathbf{x},t;\boldsymbol{\epsilon})}{\sum_{\mathbf{x}} C(\mathbf{x},t;\boldsymbol{\epsilon})}, \quad (10)$$

where $H(\mathbf{x},t)$ is the histogram of the number of particles within a given bin ($\delta x$ and $\delta y$) and is a function of $\boldsymbol{\epsilon}$ in the particle trajectory (Equations 8 and 9).

### Advection-diffusion equations

Given the surface current field $[\mathbf{u}(\mathbf{x},t)]$ and the derived concentration $[\mathrm{C}(\mathbf{x},t)]$ from the particle trajectory using a random walk mode, which is implemented with a random variable ($\boldsymbol{\epsilon}$), the diffusion coefficients ($\nu$) can be estimated in the advection-diffusion equations in the one-dimensional domain:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \nu_x\frac{\partial^2 C}{\partial x^2}. \quad (11)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \kappa_x\frac{\partial^2 C}{\partial x^2} + \kappa_y\frac{\partial^2 C}{\partial^2 y^2} \quad (12)$$


Acknowledgement Sung Yong Kim is supported by the Basic Science Research Program through the National Research Foundation (NRF), Ministry of Education (NRF-2013R1A1A2057849), the Human Resources Development of the Korea Institute of Energy Technology Evaluation and Planning (KETEP), Ministry of Trade, Industry and Energy (no. 20114030200040), and the program on Management of Marine Organisms Causing Ecological Disturbance and Harmful Effects through Korea Integrated Marine Science Technology (KIMST) and Ministry of Fishery (MOF), Republic of Korea.


## References


[1] T Dickey, S Zedler, X Yu, SC Doney, D Frye, H Jannasch, D Manov, D Sigurdson, JD McNeil, L Dobeck, et al. Physical and biogeochemical variability from hours to years at the Bermuda Testbed Mooring site: June 1994–March 1998. *Deep Sea Research Part II: Topical Studies in Oceanography*, 48(8):2105–2140, 2001.

[2] Z. Schlag, E. W. North, and K. Smith. *Larval TRANSport Lagrangian model (LTRANS) User¡¯s Guide.* University of Maryland Center for Environmental Science, Horn Point Laboratory,Cambridge, MD, 2008.

[3] Kurt Polzin and Raffaele Ferrari. Isopycnal dispersion in NATRE. *J. Phys. Oceanogr.*, 34(1):247–257, 2004.

[4] P. J. W. Roberts. Modeling Mamala Bay outfall plumes. I: Near field. *J. Hydr. Engr. ASCE*, 126(6):564–573, 1999.

[5] L. Cohen. Convolution, filtering, linear systems, the Wiener-Khinchin theorem: Generalizations. *Int'l. Soc. Opt. Eng.*, 1770:378–393, 1992.

[6] E. O. Brigham. *The Fast Fourier Transform and its Applications.* Prentice Hall Signal Processing Series. Prentice-Hall, Inc., Englewood Cliffs, NJ 07632, 1988.