FLUID MECHANICS

FIFTH EDITION

Pijush K. Kundu
Ira M. Cohen
David R. Dowling

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Dedication

This revision to this textbook is dedicated to my wife and family who have patiently helped chip many sharp corners off my personality, and to the many fine instructors and students with whom I have interacted who have all in some way highlighted the allure of this subject for me.

D.R.D.

In Memory of Pijush Kundu

Pijush Kanti Kundu was born in Calcutta, India, on October 31, 1941. He received a BS degree in Mechanical Engineering in 1963 from Shibpur Engineering College of Calcutta University, earned an MS degree in Engineering from Roorkee University in 1965, and was a lecturer in Mechanical Engineering at the Indian Institute of Technology in Delhi from 1965 to 1968. Pijush came to the United States in 1968, as a doctoral student at Penn State University. With Dr. John L. Lumley as his advisor, he studied instabilities of viscoelastic fluids, receiving his doctorate in 1972. He began his lifelong interest in oceanography soon after his graduation, working as Research Associate in Oceanography at Oregon State University from 1968 until 1972. After spending a year at the University de Oriente in Venezuela, he joined the faculty of the Oceanographic Center of Nova Southeastern University, where he remained until his death in 1994.

During his career, Pijush contributed to a number of sub-disciplines in physical oceanography, most notably in the fields of coastal dynamics, mixed-layer physics, internal waves, and Indian-Ocean dynamics. He was a skilled data analyst, and, in this regard, one of his accomplishments was to introduce the "empirical orthogonal eigenfunction" statistical technique to the oceanographic community.

I arrived at Nova Southeastern University shortly after Pijush, and he and I worked closely together thereafter. I was immediately impressed with the clarity of his scientific thinking and his thoroughness. His most impressive and obvious quality, though, was his love of science, which pervaded all his activities. Some time after we met, Pijush opened a drawer in a desk in his home office, showing me drafts of several chapters to a book he had always wanted to write. A decade later, this manuscript became the first edition of *Fluid Mechanics*, the culmination of his lifelong dream, which he dedicated to the memory of his mother, and to his wife Shikha, daughter Tonushree, and son Joydip.

Julian P. McCreary, Jr.,
University of Hawaii

In Memory of Ira Cohen

Ira M. Cohen earned his BS from Polytechnic University in 1958 and his PhD from Princeton University in 1963, both in aeronautical engineering. He taught at Brown University for three years prior to joining the University of Pennsylvania faculty as an assistant professor in 1966. He served as chair of the Department of Mechanical Engineering and Applied Mechanics from 1992 to 1997.

Professor Cohen was a world-renowned scholar in the areas of continuum plasmas, electrostatic probe theories and plasma diagnostics, dynamics and heat transfer of lightly ionized gases, low current arc plasmas, laminar shear layer theory, and matched asymptotics in fluid mechanics. Most of his contributions appear in the *Physics of Fluids* journal of the American Institute of Physics. His seminal paper, "Asymptotic theory of spherical electrostatic probes in a slightly ionized, collision dominated gas" (1963; *Physics of Fluids, 6,* 1492–1499), is to date the most highly cited paper in the theory of electrostatic probes and plasma diagnostics.

During his doctoral work and for a few years beyond that, Ira collaborated with a world-renowned mathematician/physicist, the late Dr. Martin Kruskal (recipient of National Medal of Science, 1993) on the development of a monograph called "Asymptotology." Professor Kruskal also collaborated with Professor Cohen on plasma physics. This was the basis for Ira's strong foundation in fluid dynamics that has been transmitted into the prior editions of this textbook.

In his forty-one years of service to the University of Pennsylvania before his death in December 2007, Professor Cohen distinguished himself with his integrity, his fierce defense of high scholarly standards, and his passionate commitment to teaching. He will always be remembered for his candor and his sense of humor.

Professor Cohen's dedication to academics was unrivalled. In addition, his passion for physical fitness was legendary. Neither rain nor sleet nor snow would deter him from his daily bicycle commute, which began at 5:00 AM, from his home in Narberth to the University of Pennsylvania. His colleagues grew accustomed to seeing him drag his forty-year-old bicycle, with its original

three-speed gearshift, up to his office. His other great passion was the game of squash, which he played with extraordinary skill five days a week at the Ringe Squash Courts at Penn, where he was a fierce but fair competitor. During the final year of his life, Professor Cohen remained true to his bicycling and squash-playing schedule, refusing to allow his illness get in the way of the things he loved.

Professor Cohen was a member of Beth Am Israel Synagogue, and would on occasion lead Friday night services there. He and his wife, Linda, were first married near Princeton, New Jersey, on February 13, 1960, when they eloped. They were married a second time four months later in a formal ceremony. He is survived by his wife, his two children, Susan Cohen Bolstad and Nancy Cohen Cavanaugh, and three grandchildren, Melissa, Daniel, and Andrew.

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About the Third Author

David R. Dowling was born in Mesa, Arizona, in 1960 but grew up in southern California where early practical exposure to fluid mechanics—swimming, surfing, sailing, flying model aircraft, and trying to throw a curve ball—dominated his free time. He attended the California Institute of Technology continuously for a decade starting in 1978, earning a BS degree in Applied Physics in 1982, and MS and PhD degrees in Aeronautics in 1983 and 1988, respectively. After graduate school, he worked at Boeing Aerospace and Electronics and then took a post-doctoral scientist position at the Applied Physics Laboratory of the University of Washington. In 1992, he started a faculty career in the Department of Mechanical Engineering at the University of Michigan where he has since taught and conducted research in fluid mechanics and acoustics. He has authored and co-authored more than 60 archival journal articles and more than 100 conference presentations. His published research in fluid mechanics includes papers on turbulent mixing, forced-convection heat transfer, cirrus clouds, molten plastic flow, interactions of surfactants with water waves, and hydrofoil performance and turbulent boundary layer characteristics at high Reynolds numbers. From January 2007 through June 2009, he served as an Associate Chair and as the Undergraduate Program Director for the Department of Mechanical Engineering at the University of Michigan. He is a fellow of the American Society of Mechanical Engineers and of the Acoustical Society of America. He received the Student Council Mentoring Award of the Acoustical Society of America in 2007, the University of Michigan College of Engineering John R. Ullrich Education Excellence Award in 2009, and the Outstanding Professor Award from the University of Michigan Chapter of the American Society for Engineering Education in 2009. Prof. Dowling is an avid swimmer, is married, and has seven children.

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